The 3rd International Conference on
″Computational Mechanics
and Virtual Engineering″
COMEC 2009
29 – 30 OCTOBER 2009, Brasov, Romania

# THE SYSTEMS BEHAVIOR AT VIBRATIONS THROUGH THE MODAL ANALYSIS

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***Abstract****: The behavior description of the systems subjected at vibrations with the modal analysis is an important experimental modality.*
*Modal analysis can be explained in terms of vibrations modes for a simple, freely hung flat plate; vibration characteristics of the system are measured with the input – output data.*
*Then the Fast Fourier Transform will be used to convert the time data in frequency domain data.*
***Keywords****: modal analysis. Vibration, mechanical system.*

## 1. INTRODUCTION

High performance criteria of the machines and the technological equipments impose the introduction of some special working conditions in dynamic regime; any of machine found into motion/work becomes a generating source of vibrations either as structural form, or as radiant form.
The working principle of machine and the working imperfections are operations are causes generated of vibrations about which is possible to directly intervene through the good design or working.
The technological process represents an another generating cause of vibrations about which isn't possible to have a direct intervention, fact for that must be identified technical resolutions for the anti-vibratory isolation; this situation is known specially at the machines and equipments what use shocks and/or vibrations into technological process, for example: crushers, vibrating screens, pile drivers for the cast iron smashing, forging hammers, mechanic presses.

## 2. ANALYSIS OF MECHANICAL SYSTEMS

For an mechanical system without damping described of equation

$$M\ddot{y} + ky = 0 \tag{1}$$

(in which M and K are the inertial, respectively stiffness matrix, square matrixes with "u" order; 0 is a null matrix, with (u,1) type and $y = [y_1\ y_2\ y_3\ \ldots\ldots y_n]^T$ is the transposed matrix of the displacements).
it finds a solutions of type

$$y = y_\ell = \begin{bmatrix} y_{\ell 1} \\ y_{\ell 2} \\ y_{\ell 3} \\ . \\ y_{\ell n} \end{bmatrix} \cdot \cos(\omega_n t - \varphi_n) = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ . \\ A_n \end{bmatrix} \cos(\omega_n t - \varphi_n) \tag{2}$$

Through the substitution of equation (I.2.) into (I.1) will obtain:

$$\left[ -\omega_n^2 \cdot M \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ . \\ A_n \end{bmatrix} + K \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ . \\ A_n \end{bmatrix} \cdot \cos(\omega_n t - \varphi_n) = 0 \right] \tag{3}$$

In the case in which, in necessary mode $\cos(\omega_n t - \varphi_n) \neq 0$, for that the system (I.3) to allow the solutions different of these ordinary must be fulfilled the condition:

$$\left| -\omega_n^2 M + K \right| = 0 \tag{4}$$

which represents the eigen pulsations equation of the u dimensional mechanic system which are considered.
In accordance with eigen pulsation values $\omega$ and amplitudes --- , the response of mechanical linear system, invariant in time, without damping it write:

$$y_\ell = \begin{bmatrix} y_{\ell 1} \\ y_{\ell 2} \\ y_{\ell 3} \\ . \\ y_{\ell n} \end{bmatrix} = [A_1, A_2, \ldots\ldots\ldots\ldots\ldots. A_r, A_n] \cdot \begin{bmatrix} \cos(\omega_{n1} t - \varphi_1) \\ \cos(\omega_{n2} t - \varphi_2) \\ \cos(\omega_{n3} t - \varphi_3) \\ . \\ \cos(\omega_{nn} t - \varphi_n) \end{bmatrix} \tag{5}$$

where

$$A_r = [A_{1r}, A_{2r} \ldots\ldots\ldots\ldots\ldots. A_{nr}]^T, \qquad r = \overline{1,n}$$

are named eigen or modal vectors.
Eigen pulsation $\omega_{nr}$ and modal vector $A_r$ define the eigen shape “r” of vibration.

## 3. THE STUDY OF VIBRATION THROUGH THE MODAL ANALYSIS

In the case of the elastically structures representing the machine tools and technological equipments, the eigen values are possible to show through the frequency characteristics.
The aim of an experimental modal analysis is to measure the modal parameters, the resonance frequencies, of the disturbing factors, of the eigen modes shape; in fac these modal parameters describe the vibrating behavior of the mechanical system.
For to determine the eigen vibration shape according to one of the eigen pulsation, for example $\omega_{ni}$, the structure is excited with a harmonic force (with angular frequency $\omega = \omega_{ni}$) and with some vibrations captors or an afferent apparatus of these it measure the forced vibrations in many points of the structure.
In this procedure it use at least two vibrations captors: one in the reference point of the structure, and with another makes measurements in the points, emphasizes the vibrations in phase/or in opposition of phase according to the reference point.
The graphical representation (graph) of these amplitudes (in comparison with the structure static equilibrium position) makes up the eigen vibration shape proper the $\omega_{ni}$ pulsation.
Thus is shown in Fig. 1, the application of a dynamic excitation on the structure is the essential part of checking through modal analysis an excitatory system or an impact hammer serves this aim; in fact the structure is excited at the one single point or in many points so that the structure vibrates into interesting domain of frequency at the same time to be possible measure the applied dynamic force.
The force transducers will measure the input, the excitation force and accelerometers (displacement transducers) will measure the response of system, output.
These signal are processed by an analysis system that will digitize them through Fast Fourier Transform (FFT), evaluating the system response with frequency response function (FRF).
This procedure is repeated for several excitations (at the modern installation the number of the points exceeds forty points) and for response combinations.

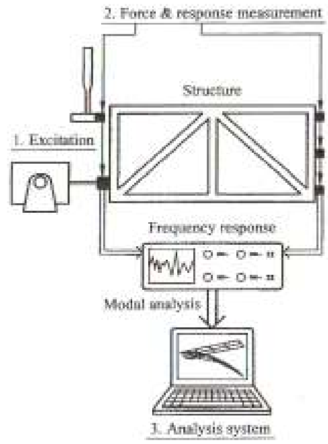


Figure 1: Modal analysis test setup

All frequencies response functions are stored on the disk memory of the analysis system.
In time of next phase, the analysis system will determine the modal characteristics ( the nodes of the system, modal vectors, participations factors) of the structure under investigation, based upon measurement frequency response functions.

## 3. EXPERIMENTAL MODAL ANALYSIS , PRACTICAL TESTINGS

An experimental modal analysis consists many phases:

- the first phase is the building of the test set – up, respectively the hanging of the test object;
- the second phase is the attaching of transducers (accelerometers) the connecting at the data acquisition and the calibration of the measured system;
- the third phase is the acquisition of data and most often, the estimation of the frequency response functions (FRF);

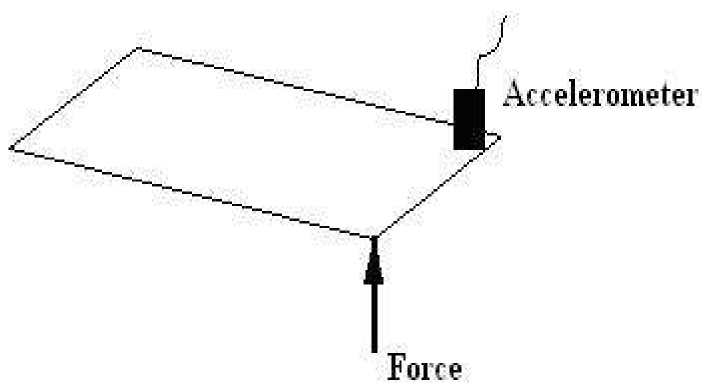


Figure 2: Simple flat plate subjected on vibrations

- the fourth phase is the system identification : the determination of the vibratory characteristic of the system through the measurement of the input – output data;
- the validation of the obtained results is the fifth phase.

All these phases are necessary in order to realize the last phase, respectively the using of the obtained results for the system improving as one whole (complete).
For the practical analysis was made the analysis of the eigen vibration modes for a simple freely hung flat plane.
The excitation (knocking with the impact hammer) is applied on the corner of the plate and the response of the plate is measured with an accelerometer or another point, see the Fig. 2.
The cinematic parameters analysis of the plate vibration is made through the response (displacement) registration in time, as a time evolution of the accelerometer received signal; the signal time analysis consists a principal study modality for the non – steady transitive signals, as well as random stead – fast signals for to evaluate some dominant frequencies.

This response in time has an informative role or if the same results are seen in frequency domain (the modern method of vibration control) may be noted another very interesting results.
With the aim visibility the results in frequency domain will use the Fast Fourier Transform (FFT) which that will obtain the frequency response function (FRF) proper the followed plate.

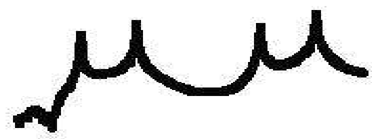


Figure 3: Frequency response function

The frequency response function (the transfer function) is defined as the simple ratio between direct Fourier transforms of the exit and respectively of the mechanic system excitation (entrance).
In fact we measure both the applied force values and the response of the structure due to the applied force (displacement, velocity, acceleration).
The frequency response function contains real and imaginary parts of the magnitude and phase components; the peaks in this plot correspond to the frequency of oscillation at which the magnitude of response is greatest; this fact may be easily seen if we overlay the response in time domain with that in frequency, as shown below, Fig. 4.

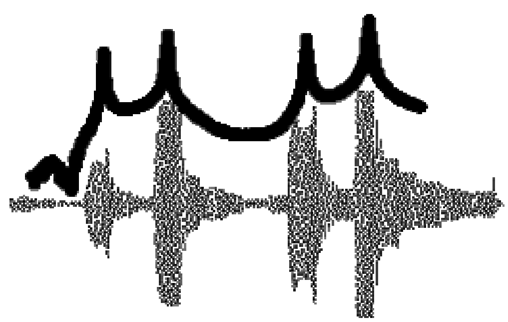

Figure 4: Overlay of the time/frequency response function

If the amplitude (magnitude) of response in time trace increases, the amplitude of the FRF also increases; these points of increased amplitude occur at the natural (eigen) frequencies of the system.
For a certain structure in the experimental techniques of the modal analysis,using the frequency response function will be estimated dynamic characteristic of the structure, respectively the inertial, damping and stiffness matrix or distribution in this structure.

## 4. CONCLUSIONS

The response of mechanical structure, for various excitation values with graph register make the possibility the amplitude – frequency diagram, from that results:

- the eigen frequency values, which coincide approximately with the frequency values for which amplitude – frequency diagram has peaked;
- the damping value (proper) according for every proper vibration mode.

The establishment of one optimal mathematic model of the mechanic structures and evaluation of dynamic parameters, allows to estimate the dynamic behavior of these and the establishment of structural changes for the decreasing of vibrations still in designing phase.

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