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THE COMPUTATIONAL METHOD IN DYNAMIC RIGID ROTATION WITH BIG CLEARANCE IN BEARING

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Abstract : *One studies the rotational motion of the rigid if the pin of radius r remains propped inside a radial-axial bearing of radius R and height h.*
One considers a permanent contact, the pin being propped in point A located on the plan what limit lower the bearing, in point B located on lateral cylindrical surface of the bearing and in point D located on the circle what limit higher the bearing.
Keywords : *rigid ,clearance,radial-axial bearing, pin.*

1. INTRODUCTION

In this paper [2] has studied the rotational motion of a rigid body with big clearance in bearing on a simplified model in which the pin is reduced a line segment and conventional radius of the bearing was given by the difference between the real radius of the bearing and the real radius of the pin.
In this paper will develop the mathematical model for real motion case of the pin with big clearance in a radial-axial bearing.

2. GENERAL ASPECTS

One considers the radial-axial bearing, of radius R and height h (fig. 1) and the pin, of radius r, jointly with the body with mass m and center of gravity C.

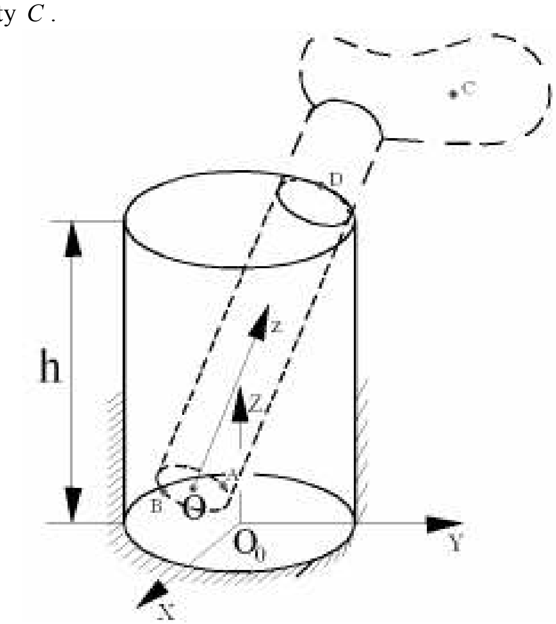

Figure 1

The pin is propped against bearing in contact point A, between the circle what limit lower the pin with the plane O_0XYZ what limit lower the bearing, in contact point B between the same circle what limit lower the pin with lateral cylindrical surface of the bearing and in contact point D between cylindrical surface what limit the pin with the circle what limit higher the bearing.

One chooses the fixed reference system O_0XYZ so that point O_0 be the center of the below circle of the bearing and the axis O_0Z being the axis of the bearing and one chooses the mobile reference system $Oxyz$ so that the point O be the center of the below circle of the pin and the axis Oz being the axis of the pin.

Considering the permanent contacts in points D, B, A and considering that the body jointly with the pin is acted by a given system of forces, it is required to develop a mathematical model of the motion.

3. THE EQUATIONS OF MOTION

In order to obtain the equation of motion is used the Lagrange's equations for the systems with constraints , the generalized co-ordinates defining the column matrix

$$\{q\} = (X_0 \quad Y_0 \quad Z_0 \quad \psi \quad \theta \quad \varphi)^T \tag{1}$$

where $(X_0 \quad Y_0 \quad Z_0)$ are the co-ordinates of point O relative to the fixed system O_0XYZ and $(\psi \quad \theta \quad \varphi)$ are the Euler's angles.

In this conditions the Lagrange's equations, using the scalar notations,

- E_c - kinetic energy;
- $F_k, k = 1,2.....6$ generalized forces ;
- $\lambda_j, j = 1,2....$ Lagrange's multipliers;
- $[B]$ the constraint matrix
- $\left\{\frac{\vartheta E_c}{\vartheta \dot{q}}\right\}$; $\{F\}$; $\{\lambda\}$ the column matrices defined by relations

$$\left\{\frac{\vartheta E_c}{\vartheta \dot{q}}\right\} = \left(\frac{\vartheta E_c}{\vartheta \dot{X}_0} \quad \frac{\vartheta E_c}{\vartheta \dot{Y}_0} \quad \frac{\vartheta E_c}{\vartheta \dot{Z}_0} \quad \frac{\vartheta E_c}{\vartheta \dot{\psi}} \quad \frac{\vartheta E_c}{\vartheta \dot{\theta}} \quad \frac{\vartheta E_c}{\vartheta \dot{\varphi}}\right)^T \tag{2}$$

$$\{F\} = (F_1 \quad F_2 \quad F_3 \quad F_4 \quad F_5 \quad F_6)^T \tag{3}$$

$$\{\lambda\} = (\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6)^T \tag{4}$$

is written as

$$\frac{d}{dt}\left\{\frac{\vartheta E_c}{\vartheta \dot{q}}\right\} - \left\{\frac{\vartheta E_c}{\vartheta q}\right\} = \{F\} + [B]^T \cdot \{\lambda\} \tag{5}$$

Forwards one uses the scalar notations:

- (x_C, y_C, z_C) the co-ordinates of point C in the system $Oxyz$
- $(v_x, v_y, v_z), (\omega_x, \omega_y, \omega_z)$ the components of the velocity of the point O respectively of the angular velocity in the system $Oxyz$

$(J_x, J_y, J_z), (J_{xy}, J_{xz}, J_{yz})$ moments of inertia relative to system $Oxyz$

and the matrix notations,

$$[m] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}; [S] = \begin{bmatrix} 0 & -mz_c & my_c \\ mz_c & 0 & -mx_c \\ -my_c & mx_c & 0 \end{bmatrix}; [J] = \begin{bmatrix} J_x & -J_{xy} & -J_{xz} \\ -J_{xy} & J_y & -J_{yz} \\ -J_{xz} & -J_{yz} & J_z \end{bmatrix}$$

$$[\psi] = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}; [\theta] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}; [\varphi] = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}; [\widetilde{\theta}] = \begin{bmatrix} 1 & \sin\theta & \cos\theta \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$[U_1] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}; [U_2] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}; [U_3] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; [\theta_P] = \begin{bmatrix} 0 & \cos\theta & -\sin\theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\{v\} = (v_x \quad v_y \quad v_z \quad \omega_x \quad \omega_y \quad \omega_z)^T \tag{7}$$

results in orders:

- the rotational matrix and its partial derivatives relative to ψ,θ,φ

$$[A]=[\psi]\cdot[\theta]\cdot[\varphi]\,;\quad [A_\psi]=[U_3]\cdot[A]\,;\quad [A_\theta]=[A]\cdot[\varphi]^T[U_1]\cdot[\varphi]\,;\quad [A_\varphi]=[A]\cdot[U_3] \tag{8}$$

- the matrix $[Q]$ and its partial derivatives relative to θ,φ

$$[Q]=[\tilde{\theta}]\cdot[\varphi];\quad [Q_\theta]=[\tilde{Q}_P]\cdot[\varphi];\quad [Q\varphi]=[Q]\cdot[U_3] \tag{9}$$

- the time derivatives

$$[\dot{A}]=\psi\cdot[A_\psi]+\dot{\theta}\cdot[A_\theta]+\dot{\varphi}\cdot[A_\varphi];\ [\dot{Q}]=\dot{\theta}\cdot[Q_\theta]+\dot{\varphi}\cdot[Q_\varphi] \tag{10}$$

- the matrices $[C]$ and its derivatives

$$[C]=\begin{bmatrix}[A] & [0]\\ [0] & [Q]\end{bmatrix};\ [C_\psi]=\begin{bmatrix}[A_\psi] & [0]\\ [0] & [0]\end{bmatrix};\ [C_\theta]=\begin{bmatrix}[A_\theta] & [0]\\ [0] & [Q_\theta]\end{bmatrix};\ [C_\varphi]=\begin{bmatrix}[A_\varphi] & [0]\\ [0] & [Q\varphi]\end{bmatrix} \tag{11}$$

- the matrices of inertia and its derivatives

$$\begin{gathered}[\tilde{M}]=\begin{bmatrix}[m] & [S]^T\\ [S] & [J]\end{bmatrix};\quad [M]=[C]\cdot[\tilde{M}]\cdot[C]^T\\ [M_\psi]=[C_\psi]\cdot[\tilde{M}]\cdot[C]^T+[C]\cdot[\tilde{M}]\cdot[C_\psi]^T;\quad [M_\theta]=[C_\theta]\cdot[\tilde{M}]\cdot[C]^T+[C]\cdot[\tilde{M}]\cdot[C_\theta]^T\\ [M_\varphi]=[C_\varphi]\cdot[\tilde{M}]\cdot[C]^T+[C]\cdot[\tilde{M}]\cdot[C_\varphi]^T;\quad [\dot{M}]=\dot{\psi}\cdot[M_\psi]+\dot{\theta}\cdot[M_\psi]+\dot{\varphi}\cdot[M_\theta]\end{gathered} \tag{12}$$

Taking into account of these, the kinetic energy is written

$$E_c=\frac{1}{2}\cdot\{v\}^T\cdot[\tilde{M}]\cdot\{v\} \tag{13}$$

or if one takes into account the dependence

$$\{v\}=\{C\}^T\cdot[\dot{q}] \tag{14}$$

$$E_c=\frac{1}{2}\cdot\{\dot{q}\}^T\cdot[M]\cdot\{\dot{q}\} \tag{15}$$

results

$$\frac{d}{dt}\left\{\frac{\vartheta E_c}{\vartheta\dot{q}}\right\}=[M]\cdot\{\ddot{q}\}+[\dot{M}]\cdot\{q\} \tag{16}$$

$$\frac{\vartheta E_c}{\vartheta\psi}=\frac{1}{2}\cdot\{\dot{q}\}^T\cdot[M_\psi]\cdot\{\dot{q}\};\ \frac{\vartheta E_c}{\vartheta\theta}=\frac{1}{2}\cdot\{\dot{q}\}^T\cdot[M_\theta]\cdot\{\dot{q}\};\ \frac{\vartheta E_c}{\vartheta\varphi}=\frac{1}{2}\cdot\{\dot{q}\}^T\cdot[M_\varphi]\cdot\{\dot{q}\} \tag{17}$$

$$\left\{\frac{\vartheta E_c}{\vartheta q}\right\}=\begin{pmatrix}0 & 0 & 0 & \frac{\vartheta E_c}{\vartheta\psi} & \frac{\vartheta E_c}{\vartheta\theta} & \frac{\vartheta E_c}{\vartheta\varphi}\end{pmatrix}^T \tag{18}$$

and if one notes

$$\{\tilde{F}\}=\{F\}-[\dot{M}]\cdot[\dot{q}]+\left\{\frac{\vartheta E_c}{\vartheta q}\right\} \tag{19}$$

is obtained from (5) the matrix equation of the motion

$$[M]\cdot[\ddot{q}]-[B]^T\{\lambda\}=\{\tilde{F}\} \tag{20}$$

to which is added the matrix equations of the contraints

$$[B]\{\dot{q}\}=\{0\} \tag{21}$$

If one derives in relation to the time the relation (21) then from (20), (21) is obtained the matrix equation

$$\begin{bmatrix}[M] & -[B]^T\\ [B] & [0]\end{bmatrix}\cdot\begin{bmatrix}\{\ddot{q}\}\\ \{\lambda\}\end{bmatrix}=\begin{bmatrix}\{\tilde{F}\}\\ -[\dot{B}]\cdot\{\dot{q}\}\end{bmatrix} \tag{22}$$

If the constraints matrix depends on other parameters than $X_0, Y_0, Z_0, \psi, \theta$ then the vector $\{q\}$ extends with the new parameters and the matrix $[M]$ extends with null rectangular matrices $[O_{mn}]$ with m rows and n columns.

So if the extended vector is

$$\{q\}=\begin{pmatrix}X_0 & Y_0 & Z_0 & \psi & \theta & \varphi & \alpha & \beta\end{pmatrix}^T \tag{23}$$

then the extend matrix is

$$[M^*]=\begin{bmatrix}[M] & [O_{62}]\\ [O_{26}] & [O_{22}]\end{bmatrix} \tag{24}$$

4. THE DETERMINING OF THE CONSTRAINTS MATRIX

A. The contact from point A

In point A (fig. 1) is made the contact between a mobile curve (the circle with center in O) and a fixed surface (the plane O_0XYZ). Generally noting with $x(\xi), y(\xi), z(\xi)$ the parametric co-ordinates of the curve and with $X(u,v), Y(u,v), Z(u,v)$ the parametric co-ordinates of the surface, the contact conditions are written

$$\begin{bmatrix} X(u,v) \\ Y(u,v) \\ Z(u,v) \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + [A] \cdot \begin{bmatrix} x(\xi) \\ y(\xi) \\ z(\xi) \end{bmatrix} \tag{25}$$

The normal to the surface has the controlling parameters A, B, C given by the functional determinants

$$A = \frac{D(Y,Z)}{D(u,v)} \;\; ; \;\; B = \frac{D(Z,X)}{D(u,v)} \;\; ; \;\; C = \frac{D(X,Y)}{D(u,v)} \tag{26}$$

and then the tangent condition is

$$(A \quad B \quad C) \cdot [A] \cdot \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = 0 \;\; sau \;\; (x_p \quad y_p \quad z_p) \cdot [A] \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix} = 0 \tag{27}$$

where x_p, y_p, z_p are the derivatives in relation to parameter ξ.
For the contact from point A result

$$\begin{aligned} &X = u \; ; \; Y = v ; \; Z = 0 \; ; \; x = r\cos\xi \; ; \; y = r\sin\xi \; ; z = 0 \; ; \\ &A = 0 ; B = 0 \; ; \; C = 0 ; \; x_p = -r\sin\xi \; ; y_p = r\cos\xi \; ; z_p = 0 \end{aligned} \tag{28}$$

and the second relation (28) is written

$$\sin\theta \cos(\varphi + \xi) = 0 \tag{29}$$

wherefrom result $\varphi + \xi = -\frac{\pi}{2}$, the co-ordinates of point A $x_A = r\sin\varphi \; ; \; y_A = r\cos\varphi \; ; \; z_A = 0$ and from the last scalar equation of relation (26) is deducted the expression

$$Z_0 - r\sin\theta = 0 \tag{30}$$

B. The contact from point B

In point B is made the contact like the contact from point A and result

$$\begin{aligned} &X = R\cos u_B \; ; \; Y = R\sin u_B \; ; \; Z = v \; ; \; x = r\cos\xi_B \; ; y = r\sin\xi_B \; ; z = 0 \\ &A = \cos u_B \; ; B = \sin u_B \; ; \; C = 0 ; \; x_p = -r\sin\xi_B \; ; y_p = r\cos\xi_B \; ; z_p = 0 \end{aligned} \tag{31}$$

from the first relation (22) is obtained the expression

$$\cos(\psi - u_B)\sin(\varphi + \xi_B) + \sin(\psi - u_B)\cos\theta\cos(\varphi + \xi_B) = 0 \tag{32}$$

and from the first relations (25) are deducted the equality

$$\begin{aligned} &R\cos(\psi - u_B) - X_0\cos\psi - Y_0\sin\psi - r\cos(\varphi + \xi_B) = 0 \\ &R\sin(\psi - u_B) - X_0\sin\psi + Y_0\cos\psi + r \cdot \cos\theta \cdot \cos(\varphi + \xi_B) = 0 \end{aligned} \tag{33}$$

C. The contact from point D

In point D is made the contact between a mobile surface (the cylindrical surface of the pin) and a fixed curve (the circle of radius R what limit higher the bearing).
Generally, noting with $X(\xi), Y(\xi), Z(\xi)$ the parametric co-ordinates of the mobile curve, the conatct conditions are written

$$\begin{bmatrix} X(\xi) \\ Y(\xi) \\ Z(\xi) \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + [A] \cdot \begin{bmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{bmatrix} \tag{34}$$

The normals to the surface have the controlling parameters

$$a = \frac{D(y,z)}{D(u,v)} \;\; ; \;\; b = \frac{D(z,x)}{D(u,v)} \;\; ; \;\; c = \frac{D(x,y)}{D(u,v)} \tag{35}$$

and the tangent condition is

$$(a \quad b \quad c)\cdot[A]^T \cdot \begin{bmatrix} X_p \\ Y_p \\ Y_p \end{bmatrix} = 0 \ \ sau \ \ (X_p \quad Y_p \quad Z_p)\cdot[A]\cdot\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0 \tag{36}$$

where X_p, Y_p, Z_p are the derivatives in relation to the parameter ξ.

For the contact from point D result

$X = R\cos\xi_D \ ; \ Y = R\sin\xi_D \ ; \ Z = h \ ; \ x = r\cos u_D \ ; y = r\sin u_D \ ; z = v_D$

$a = \cos u_D \ ; b = \sin u_D \ ; \ c = 0; \ x_p = -r\sin\xi_D \ ; y_p = r\cos\xi_D \ ; z_p = 0$

and from relation (36) and from the first equations (34) are obtained the expressions

$$\begin{aligned} &\cos(\varphi + u_D)\cdot\sin(\psi - \xi_D) - \cos\theta\cdot\sin(\varphi + u_D)\cdot\cos(\psi - \xi_D) = 0 \\ &R\cos(\varphi - u_D) - r\cos\theta\sin(\psi - \xi_D) + X_0\cos\psi + Y_0\sin\psi = 0 \\ &R\sin(\varphi + u_D) + r\cos(\psi - \xi_D) - X_0\cos\theta\cdot\cos\psi + Y_0\cos\theta\cdot\cos\psi + Z_0\sin\theta - h\sin\theta = 0 \end{aligned} \tag{37}$$

D. The expression of the constraints matrix

Were obtained in all seven equations of constraints (30), (32), (33), (37) and the extended vector $\{q\}$ is

$$\{q\} = (X_0 \quad Y_0 \quad Z_0 \quad \psi \quad \theta \quad \varphi \quad \xi_B \quad u_B \quad \xi_D \quad u_D) \tag{38}$$

By derivation with respect to time of these equations is obtained the constraints matrix with seven rows and ten columns

$$[B] = \lfloor B_{ij} \rfloor \tag{39}$$

where the components B_{ij} are null except the following relations

$$\begin{aligned} &B_{13} = 1 \ ; \ B_{15} = -r\cos\theta \ ; \\ &B_{24} = -B_{28} = -\sin(\psi - u_B)\sin(\varphi + \xi_B) + \cos(\psi - u_B)\cos\theta\sin(\varphi + \xi_B) \\ &B_{25} = -\sin(\psi - u_B)\sin\theta\cos(\varphi + \xi_B) \ ; \\ &B_{26} = B_{27} = \cos(\psi - u_B)\cos(\varphi + \xi_B) - \sin(\psi - u_B)\cos\theta\sin(\varphi + \xi_B) \ ; \\ &B_{31} = -\cos\psi \ ; \ B_{32} = -\sin\psi \\ &B_{34} = -B_{38} = -R\sin(\psi - u_B) \ ; \\ &B_{36} = B_{37} = -r\sin(\varphi + \xi_B) \ ; \ B_{41} = -\sin\psi \ ; \ B_{42} = -\cos\psi \ ; \\ &B_{44} = -B_{48} = R\cos(\psi - u_B) \\ &B_{45} = r\sin\theta\sin(\varphi + \xi_B) \ ; \ B_{46} = B_{47} = r\cos\theta\cos(\varphi + \xi_B) \\ &B_{55} = -B_{59} = \cos(\varphi + u_D)\cos(\psi - \xi_D) + \cos\theta\cos(\varphi + u_D)\sin(\psi - \xi_D) \\ &B_{54} = \sin\theta\sin(\varphi + u_D)\cos(\psi - \xi_D) \\ &B_{56} = B_{5,10} = -\sin(\varphi + u_D)\sin(\psi - \xi_D) - \cos\theta\cos(\varphi + u_D)\cos(\psi - \xi_D) \\ &B_{61} = \cos\psi \ ; \ B_{62} = \sin\psi \ ; \\ &B_{64} = -B_{69} = r\sin(\psi - \xi_D) \\ &B_{66} = B_{6,10} = -R\sin(\varphi + u_D) \\ &B_{71} = -\cos\theta\sin\psi \ ; \ B_{72} = \cos\theta\cos\psi \ ; \ B_{73} = \sin\theta \\ &B_{74} = -B_{79} = r\cos\theta\cos(\psi - \xi_D) \\ &B_{75} = -[r\sin(\psi - \xi_D) - X_0\sin\psi + Y_0\cos\psi]\sin\theta + (Z_0 - h)\cos\theta \ ; \\ &B_{76} = B_{7,10} = R\cos(\varphi + u_D) \end{aligned} \tag{40}$$

E. The time derivative of the constraints matrix

$$[\dot{B}] = [\dot{B}_{ij}] \tag{41}$$

where $\dot{B}_{ij} = 0$ except the following relations

$$
\begin{aligned}
&\dot{B}_{15} = r\dot{\theta}\,sin\,\theta\ ;\\
&\dot{B}_{24} = -\dot{B}_{28} = -(\dot{\psi} - \dot{u}_B)[cos(\psi - u_B)sin(\varphi + \xi_B) + sin(\psi - u_B)cos\,\theta\,cos(\varphi + \xi_B)] -\\
&-\dot{\theta}\,cos(\psi - u_B)sin\,\theta\,cos(\varphi + \xi_B) -\\
&-(\dot{\varphi} + \dot{\xi}_B)[sin(\psi - u_B)cos(\varphi + \xi_B) + cos(\psi - u_B)cos\,\theta\,sin(\varphi + \xi_B)]\\
&\dot{B}_{25} = -(\dot{\psi} - \dot{u}_B)cos(\psi - u_B)sin\,\theta\,cos(\varphi + \xi_B) - \dot{\theta}\,sin(\psi - u_B)cos\,\theta\,cos(\varphi + \xi_B) +\\
&+(\dot{\varphi} + \dot{\xi}_B)sin(\psi - u_B)sin(\varphi + \xi_B)\\
&\dot{B}_{26} = \dot{B}_{27} = (\dot{\psi} - \dot{u}_B)[-sin(\psi - u_B)cos(\varphi + \xi_B) - cos(\psi - u_B)cos\,\theta\,sin(\varphi + \xi_B)] +\\
&\dot{\theta}\,sin(\psi - u_B)sin\,\theta\,sin(\varphi + \xi_B) +\\
&+(\dot{\varphi} + \dot{\xi}_B)[-cos(\psi - u_B)sin(\varphi + \xi_B) - sin(\psi - u_B)cos\,\theta\,sin(\varphi + \xi_B)]\\
&\dot{B}_{31} = -\dot{\psi}\,sin\,\psi\ ;\ \dot{B}_{32} = -\dot{\psi}\,cos\,\psi\ ;\ \dot{B}_{34} = -\dot{B}_{38} = -R(\dot{\psi} - \dot{u}_B)cos(\psi - u_B);\\
&\dot{B}_{36} = \dot{B}_{37} = r(\dot{\varphi} + \dot{\xi}_B)cos(\varphi + \xi_B);\ B_{41} = -\dot{\psi}\,cos\,\psi\ ;\ \dot{B}_{42} = -\dot{\psi}\,sin\,\psi\\
&\dot{B}_{44} = -\dot{B}_{48} = -R(\dot{\psi} - \dot{u}_B)sin(\psi - u_B);\\
&\dot{B}_{45} = r\left[\dot{\theta}\,cos\,\theta\,sin(\varphi + \xi_B) + (\dot{\varphi} + \dot{\xi}_B)sin\,\theta\,cos(\varphi + \xi_B)\right]\\
&\dot{B}_{46} = \dot{B}_{47} = r\left[-\dot{\theta}\,sin\,\theta\,cos(\varphi + \xi_B) - (\dot{\varphi} + \dot{\xi}_B)cos\,\theta\,sin(\varphi + \xi_B)\right]\\
&\dot{B}_{54} = -\dot{B}_{59} = (\dot{\psi} - \dot{\xi}_D)[cos(\varphi + u_D)sin(\psi - \xi_D) + cos\,\theta\,sin(\varphi + u_D)cos(\psi - \xi_D)] -\\
&\dot{\theta}\,sin\,\theta\,sin(\varphi + u_D)sin(\psi - \xi_D) +\\
&+(\dot{\varphi} + \dot{u}_D)[-sin(\varphi + u_D)cos(\psi - \xi_D) + cos\,\theta\,cos(\varphi + u_D)sin(\psi - \xi_D)]\\
&\dot{B}_{55} = -(\dot{\varphi} + \dot{\xi}_B)sin\,\theta\,sin(\varphi + u_D)sin(\psi - \xi_D) + \dot{\theta}\,cos\,\theta\,sin(\varphi + u_D)cos(\psi - \xi_D) +\\
&+(\dot{\varphi} + \dot{u}_D)sin\,\theta\,cos(\varphi + u_D)cos(\psi - \xi_D);\\
&\dot{B}_{56} = \dot{B}_{5,10} = (\dot{\psi} - \dot{\xi}_D)[-sin(\varphi + u_D)cos(\psi - \xi_D) + cos\,\theta\,cos(\varphi + u_D)sin(\psi - \xi_D)] +\\
&+\dot{\theta}\,sin\,\theta\,cos(\varphi + u_D)cos(\psi - \xi_D) +\\
&+(\dot{\varphi} + \dot{u}_D)[-cos(\varphi + u_D)sin(\psi - \xi_D) + cos\,\theta\,sin(\varphi + u_D)cos(\psi - u_D)]\\
&\dot{B}_{61} = -\dot{\psi}\,sin\,\psi\ ;\ \dot{B}_{62} = \dot{\psi}\,cos\,\psi\ ;\ \dot{B}_{64} = \dot{B}_{69} = r(\dot{\psi} - \dot{\xi}_D)cos(\psi - \xi_D);\\
&\dot{B}_{66} = \dot{B}_{6,10} = -R(\dot{\varphi} + \dot{u}_D)cos(\varphi + u_D)\\
&\dot{B}_{71} = \dot{\theta}\,sin\,\theta\,sin\,\psi - \dot{\psi}\,cos\,\theta\,cos\,\psi\ ;\ \dot{B}_{72} = -\dot{\theta}\,sin\,\theta\,cos\,\varphi - \dot{\psi}\,cos\,\theta\,sin\,\psi\ ;\ \dot{B}_{73} = \dot{\theta}\,cos\,\theta\ ;\\
&\dot{B}_{74} = -\dot{B}_{79} = r\left[-\dot{\theta}\,sin\,\theta\,cos(\psi - \xi_D) - (\dot{\psi} - \dot{\xi}_D)cos\,\theta\,sin(\psi - \xi_D)\right];\\
&\dot{B}_{75} = -\dot{\theta}\,cos\,\theta\left[r\,sin(\psi - \xi_D) - X_0\,sin\,\psi + Y_0\,cos\,\psi\right] - (Z_0 - h)\dot{\theta}\,sin\,\theta +\\
&+\dot{Z}_0\,cos\,\theta - r(\dot{\psi} - \dot{\xi}_D)cos(\psi - \xi_D)sin\,\theta -\\
&-(\dot{X}_0\,sin\,\psi + \dot{\psi}X_0\,cos\,\psi)sin\,\theta + (\dot{Y}_0\,cos\,\psi - \dot{\psi}Y_0\,sin\,\psi)sin\,\theta
\end{aligned}
\tag{42}
$$

5. The final form of the matrix differential equation of the motion

Because the extended vector $\{q\} = (X_0\ \ Y_0\ \ Z_0\ \ \psi\ \ \theta\ \ \varphi\ \ \xi_B\ \ u_B\ \ \xi_D\ \ u_D)^T$ has ten components and the constraints matrix has seven rows finally are obtained in order the expressions

$$\{\lambda\} = (\lambda_1\ \ \lambda_2\ \ \lambda_3\ \ \lambda_4\ \ \lambda_5\ \ \lambda_6\ \ \lambda_7)^T \tag{43}$$

$$\left[M^*\right] = \begin{bmatrix} [M] & [O_{64}] \\ [O_{46}] & [O_{77}] \end{bmatrix} \tag{44}$$

and the final matrix equation becomes

$$\begin{bmatrix} [M^*] & -[B]^T \\ [B] & [O_{77}] \end{bmatrix} \cdot \begin{bmatrix} \{\ddot{q}\} \\ \{\lambda\} \end{bmatrix} = \begin{bmatrix} \{\widetilde{F}\} \\ \{O_{14}\} \\ -[\dot{B}] \cdot \{\dot{q}\} \end{bmatrix} \qquad \textbf{(45)}$$

If the initial column matrices ($t = 0$), $\{q\}$, $\{\dot{q}\}$ are known from equation (45) are determined the vectors $\{\ddot{q}\}$, $\{\lambda\}$ and then the solution with function of time.

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