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THE SIMILITUDE THEORY APPLIED FOR THE STUDY OF THE HEAT TRANSMISSION IN CASE OF THE LASER DEPOSITION

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Abstract: *As the real phenomena of the heat transmission may intervene, in proportions which vary from a case to another, the all three modes of propagation, for the description of the global process can use a virtual model similar to the original. The similitude theory, establishing the criterion of the ascertainment of the phenomena of heat convection allows the attainment of some formulas for the determination of α, applicable to any similar case, and the performance of the experiments on models, allowing on a hand to the experiment to be possible, and on the other hand, to be economic.*
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The theory of the heat propagation or transmission treats the research of the phenomena and the measurement of the heat exchanges that take place in the material systems, whose components are there at different temperatures. The study of the heat propagation, examining the thermic phenomena that take place in time, follows the determination of the quantitative relationships which intervene in the deploy of this process.

In the study of the heat transfer processes, the analytical methods have limited possibilities, the differential equations analytical solving, in conditions of oneness imposed, having sometimes big difficulties.

For the practical resolving it is used in such cases, the experimentation on model, the results obtained like that, being then calculated again for the dimensions of the original phenomenon. But the generalization of the experimentation results on a model is possible only for the phenomena similar to the studied one, phenomena that fulfill certain conditions of similitude (resemblance).

The similitude theory, establishing the ascertainment criterion of the resemblance or similarity of the heat convection phenomena, allows the attainment of some formulas for the determination of α, applicable to any similar case, and the performance of the experiments on models, making one hand possible that the experiment to become possible, and on the other hand, economic.

The general solution of a physical process must be constituted so as to be independent from the system of measurement units used, condition fulfilled by an equation whose variables are without dimensions, variables called non - dimensional criteria invariants or specific numbers.

The object of the similitude theory consists in establishing of the liaison among the non – dimensional criteria, i.e. of the criteria relationships. The criteria principles have a constrained validity within certain limits, caused by the taking into account, in the researched study, only of some parameters that influence significantly the phenomenon.

The most important parameters that enter as criteria components, in the study of the heat pass, are : the thermic conductivity, the thermic giving up coefficient, the thermic diffusivity, the fluid movement speed, the specific heat, etc. The physical similitude is a generalization of the geometrical similitude. Two pipes in a shape of truncated cone a and b (fig 1) are geometrically alike, if :

$$\frac{d_1''}{d_1'} = \frac{d_2''}{d_2'} = \frac{l''}{l'} = k \qquad \textbf{(1)}$$

k being the proportionality coefficient or the similarity constant.

Similar to the geometrical similarity notion it can conceive the flow similarity of two fluids, the cinematic similarity, realized when in similar geometrical systems, the pressures and the speeds of a flowing fluid, taken as similar points and at corresponding moments, for similar elements, they are under the same proportion :

$$\frac{p_1''}{p_1'} = \frac{p_2''}{p_2'} = k_p \,; \quad \frac{w_1''}{w_1'} = \frac{w_2''}{w_2'} = k_w \,; \quad \frac{\rho_1''}{\rho_1'} = \frac{\rho_2''}{\rho_2'} = k_\rho \,; \quad \frac{\eta_1''}{\eta_1'} = \frac{\eta_2''}{\eta_2'} = k_\eta \,; \quad \frac{\tau''}{\tau'} = k_\tau \qquad \textbf{(2)}$$

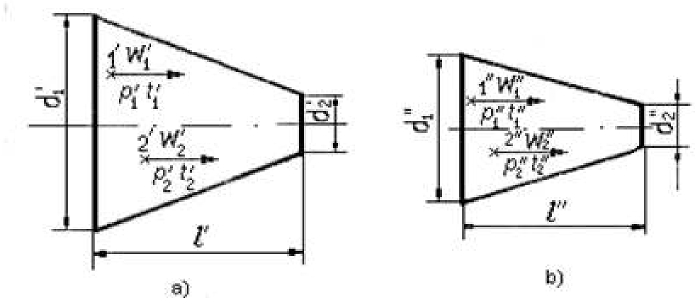

Figure 1. Similar elements in a shape of truncated cone.

The thermic similarity is performed when in similar geometrical systems, the considered temperatures in similar points and at corresponding time intervals, for similar elements, are under the same proportion :

$$\frac{t_1''}{t_1'} = \frac{t_2''}{t_2'} = k_t \,; \quad \frac{\tau''}{\tau'} = k_\tau \,; \quad \frac{\lambda_1''}{\lambda_1'} = \frac{\lambda_2''}{\lambda_2'} = k_\lambda \,; \quad \frac{\rho_1''}{\rho_1'} = \frac{\rho_2''}{\rho_2'} = k_\rho \quad etc. \qquad \textbf{(3)}$$

The notion of similarity is applicable only to the physical phenomena that are identical qualitatively (of the same nature) and which are represented analytically by identical equations (that differ only as values of the thermic numerical coefficients). The similar physical phenomena have to deploy in similar geometrical systems.

The similarity of two physical phenomena suppose the similarity of all values φ specific to those phenomena, i.e. :

$$\Phi'' = k_\varphi \times \varphi' \qquad \textbf{(4)}$$

Each physical value has its similarity constant, among the similarity constants being certain relationships. Therefore, in the case of two streams of fluid flow, that are alike, from the expressing of those speeds and by their reporting, it results :

$$w' = \frac{l'}{\tau'} \quad \text{şi} \quad w'' = \frac{l''}{\tau''} \cdot \frac{w''}{w'} = \frac{l''}{l'} \cdot \frac{\tau'}{\tau''} \;. \qquad \textbf{(5)}$$

As the phenomena are considered similar :

$$\frac{w''}{w'} = k_w; \quad \frac{l''}{l'} = k_l; \quad \frac{\tau''}{\tau'} = k_\tau \qquad \textbf{(6)}$$

Replacing in the relationships the formulas (5) and (6), it is obtained the relationship that unifies the similarity constants:

$$k_w = \frac{k_l}{k_\tau}, \quad sau \quad \frac{k_w \cdot k_\tau}{k_l} = 1 \qquad \textbf{(7)}$$

If in the formula (7) are replaced the values of the similarity constants from the formulas (6) , it results:

$$\frac{w'\tau'}{l'} = \frac{w''\tau''}{l''} = \frac{w\tau}{l} = const., \qquad \textbf{(8)}$$

the relationship or the formula that express the basic principle of the similar considered phenomena deploy, revealing the existence of some similarity criteria or invariants, non-dimensional values, having same value for all the similar among them phenomena.

The similitude theory is based on two principles and three theorems (laws), the principles of the similitude theory being, as follows :

- any physical phenomenon can be expressed by an analytical experimental relationship (formula);
- the invariants, deducted from the equations of a given phenomenon, for an element of the domain where the phenomenon is deployed, are valid for this whole domain.

The establishment of the relationships among the similarity constants K φ and the determination of the invariants (of the similarity criteria), form the object of the first theorem of the similitude theory, enunciated by Newton : *for similar among them phenomena, the similarity criteria have same value.*

This theorem allows the extension of the obtained results for a phenomenon, in the interior of the process within this one is contained.

The possibility of representing the integral solution of the differential equation as a function of invariants, is expressed as the second theorem , enunciated by Federman (1911) and by Buckingham (1914) and completed by M. Kirpicev : *the general solution of a system of equations, corresponding to a group of similar phenomena, may be expressed with the help of the respective similitude criteria. The particular solutions may be expressed with the help of the same criteria, of some special criteria and of some simple proportions of the variables that enter the respective relationships and of some particular values of these variables.*

This principle allows the replacement of the dependence among the variables specific to the deploy of the physical phenomenon, with a relationship among the invariants, called equation of invariants or criteria relationship (formula) :

$$f(K1, K2…., Kn) = 0 \qquad \textbf{(9)}$$

The invariants equation is valid for all similar phenomena, as for all these phenomena the invariants have the same value and criteria relationships (formulas) (9) are identical among them.

The third principle, enounced by M Kirpicev and Guhman (1931), specifies the conditions that are sufficient for two phenomena to be alike : *There are considered alike such phenomena whose conditions are alike, and the invariants have the same numerical values.*

In this enunciation, the univocity conditions are those particular conditions or the contour conditions, which limit the studied problem, i.e. separate, from the infinite number of processes, the examined one and determine it well defined (mathematically completely determined).

The criteria equations are expressed generally under the form of some multiplication results of the powers of the different criteria :

$$K_i = C_1 K_1^{m_1} \cdot K_2^{m_2} \dots K_n^{m_n} = C_i \sum_{i=1}^{n} K_i^{m_i} \qquad \textbf{(10)}$$

The similitude criteria for certain physical phenomena, are obtained by writing the differential equation for the original phenomenon, whose physical values are affected by the prime index (φ'), and then, taking into consideration the similarity constants which intervene (K φ) of each physical inputs (values), is expressed the differential equation of the phenomenon that is alike with the first one, i.e. to the model : (φ " = K φ x φ') .

As the two equations, written for the similar phenomena to be equal, it is imposed the equality condition among the proportions of the similarity coefficients that intervene in the formulas (relationships), resulting in this way a formula or a relationship among the similarity coefficients of the physical inputs .

By replacing, in these formulas (relationships) the similarity coefficients as proportions of the corresponding physical inputs (φ " =K φ x φ') it is obtained the similarity criterion.

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