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STRESS-STRAIN DIAGRAM FOR HIGH STRENGTH CONCRETE ELEMENTS IN FLEXURE

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Abstract: *This paper will present experimental results for C80/95 concrete grade beams reinforced with S500, in flexure. The influence of the longitudinal coefficient of reinforcement and the stress-strain diagram in compression on the behaviour in the Ultimate Limit State will be analysed based on the data collected in the fore mentioned study.*
Keywords: *High Strength Concrete, flexure, stress-strain, diagram, compression*

1. INTRODUCTION

High Strength Concrete is the next-to-be construction material whenever reinforced concrete is deemed mandatory. Although many pride examples may be brought to attention, the full potential of this material is yet to be locked until code provisions will provide the necessary guidance for engineers, architects or contractors. In order to answer the various needs that field construction may require, the authors of this paper intend to provide their findings on a C80/95 concrete grade tested in flexure.
This research is part of an extended experimental program on High Strength Concrete which will hopefully provide valuable data to be shared with the scientific community and the construction industry. Due to the compressed format for this article, the authors will only provide some of the highlights of the fore mentioned study.

2. RESEARCH PROGRAM

2.1. Member Casting

The results presented herein are possible due to the following research programs: [1] Grant A (1036/2004, Betoane de Înaltă Rezistență and Performanță...) and [2] TD (280/2007, Ductilitatea Betoanelor de Înaltă Rezistență and Performanță). Both studies aided at describing the behaviour of High Strength Concrete under various conditions by testing to rupture a number of 14 beams of $125 \cdot 250 \cdot 3200\ mm$ for two concrete grades, C60 (8 beams) and C80 (6 beams). The authors' opinion is based on the accompanying test specimens when discussing the stress-strain diagram and on the C80 concrete beams, when discussing other aspects (i.e. longitudinal coefficient of reinforcement).
The mix proportions have been established based on the previous experience of the research team at the Reinforced and Prestressed Concrete Department from the Faculty of Civil Engineering in Cluj-Napoca. The matrix consists basically of type CEM I Portland Cement provided by LAFARGE, silica fume provided by SIKA, and local crushed aggregates of 0-4 [mm], 4-8 [mm] and 8-16 [mm] provided by MORLACA quarry. W/C ratio is 0.266. A policarboxilate plasticizer under the commercial name of RAVENIT, at a dosage of 0.88 [%], has been added to improve mix properties. Table 1 lists in detail all of the above; unless otherwise specified, the units are in [kg/mc] and are omitted for clarity.
Each casted beam was accompany by a number of at least three cubes of 150 mm and at least three prisms of $100 \cdot 100 \cdot 300$ mm, cured under the same conditions as the beams, namely the following: under water (at

$20^{o}C \pm 2^{o}C$) and standard ($20^{o}C \pm 2^{o}C$ for RH $60\% \pm 5\%$). Compressive strength for concrete has been studied at 28 days and the age of testing for the beams (approximately 90 days).

Table 1: Mix proportions

Constituents	Type	C80 grade
Cement	CEM I 52.5 R	520
Crushed aggregates	8-16 [mm]	706
Crushed aggregates	4-8 [mm]	530
Sand	0-4 [mm]	530
Silica fume	SIKA	52
Super plasticizer	RAVENIT	13.5 [l/mc]
Water	plain	152 [l/mc]
Water/Cement (W/C)		0.29
Water/Binder (W/B)		0.27

The testing machine used in uniaxial compression is an Advantest 9 type, first class precision equipment with a maximum frame load of 3000 kN. The load has been set to a constant ratio of 1N/mm2. The testing machine used in flexure is a WPM 262/6-1977 type, first class precision equipment with a maximum piston load of 3000 kN. Each loading step was 1/10 of the calculated bending resistance moment according to the national code, [3] SR EN 1992-1-1:2004 (Eurocod 2: Proiectarea structurilor de beton…), and took about 10 to 15 minutes to complete (to allow for crack stabilisation and record of data).

2.2. Equations used

The mathematical apparatus used to calculate the flexural capacity of the beams is presented here in (for a rectangular stress block):

$\lambda = \begin{vmatrix} 0,8 \ if \ f_{ck} \le 50MPa \\ 0,8-\left(f_{ck}-50\right)/400 \ \ if \ f_{ck} > 50MPa \end{vmatrix}$	coefficient for the reduction of the compressed height	**(1)**
$\eta = \begin{vmatrix} 1 \ if \ f_{ck} \le 50MPa \\ 1-\left(f_{ck}-50\right)/200 \ \ if \ f_{ck} > 50MPa \end{vmatrix}$	coefficient for the reduction of the effective compressive strength	**(2)**
$d = h - d_1$	the static depth	**(3)**
$F_s = A_{sl} \cdot f_{yd}$	the tensile force for the reinforcement	**(4)**
$F_c = \eta \cdot f_{cd} \cdot b \cdot \left(\lambda \cdot x\right)$	the compressive strength for the concrete	**(5)**
$\lambda \cdot x = \dfrac{F_s}{F_c}$	the effective compressed height	**(6)**
$x = \dfrac{\lambda \cdot x}{\lambda}$	the position of the neutral axis	**(7)**
$z = d - 0,5 \cdot \lambda \cdot x$	the level arm	**(8)**
$M_{Rd} = F_s \cdot z$	the resistive flexural moment	**(9)**

2.3. Beams data

The data recorded and calculated for the beams in the authors' study is presented in Table 2.

2.4. Specimens data

The data recorded for the accompanying specimens (numbered as such) has been graphed in Figure 1 as a family of stress-strain curves for which a mean curve has been also calculated. The units, in [MPa] for the vertical axis, and in [‰] for the horizontal axis, are omitted for clarity.

Table 2: Resistive flexural moment

BEAM	Reinforcement coefficient	Mechanical reinforcement coefficient	Effective compressive strength	Coefficient for: the effective compressed height	Coefficient for: the effective compressive strength	Static depth	Longitudinal reinforcement area	Effective tensile strength	Tensile force	Compressive force	The effective compressed height	Neutral axis position	The level arm	Resistive moment	Rupture moment	Experimental/ Resistive
	ρ	ω	$f_{cd,eff}$	λ	η	d	A_{sl}	f_{yd}	F_t	F_c	$\lambda \cdot x$	x	z	M_{Rd}	M_{Exp}	$\frac{M_{Exp}}{M_{Rd}}$
	[-]	[-]	[MPa]	[-]	[-]	[mm]	[mm²]	[MPa]	[kN]	$\lambda \cdot x$ [kN]	[mm]	[mm]	[mm]	[kNm]	[kNm]	[-]
BH 1-2	0.013	0.129	45.33	0.925	0.850	211.467	355.00	435.00	154.425	4.817	32.061	34.660	195.44	30.180	48.000	1.590
BH 1-1	0.013	0.129				211.467	355.00		154.425		32.061	34.660	195.44	30.180	49.000	1.624
BH 2-2	0.019	0.179				194.500	452.39		196.789		40.836	44.169	174.07	34.256	53.000	1.606
BH 2-1	0.019	0.179				194.500	452.39		196.789		40.836	44.169	174.07	34.256	54.500	1.591
BH 3-1	0.023	0.225				196.556	574.91		250.086		51.921	56.131	170.60	42.664	62.000	1.453
BH 3-2	0.023	0.225				196.556	574.91		250.086		51.921	56.131	170.60	42.664	63.000	1.524

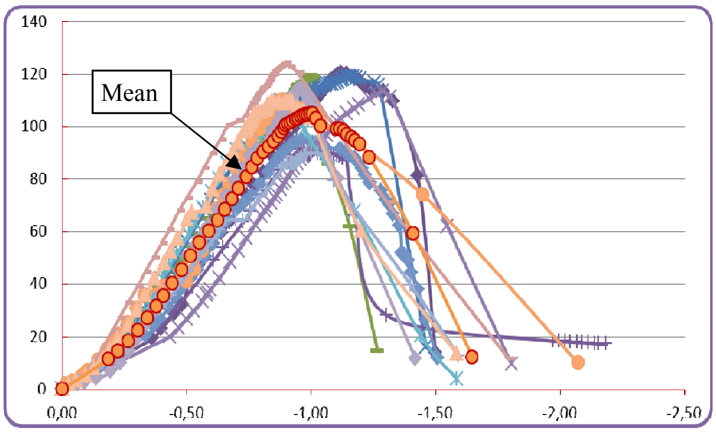

Figure 1: Stress-strain curves

3. DISCUSSION

First, the authors want to draw attention to Table 3, which presents several calculated ratios, mainly x/d and $\frac{M_{Exp}}{M_{Rd}}$ (as mean values) in view of the reinforcement coefficient, ρ, the mechanical reinforcement coefficient, ω, and the area of the longitudinal reinforcement, A_{sl}. The values in the half-right cells (even columns) represent the ratio of the first value (BH 1) with itself and each and all of the subsequent ones (BH 1/BH 1, BH 2/BH 1 and BH 3/BH 1).

In terms of the decrease of the capacity in flexure (column 9) it can be seen that for a variation of the reinforcement coefficient, ρ, of less than approx. 45% (column 2) the effect is less than 1% (column 10) and that only an increase of about 75% (column 2) has a significant effect, higher that 5% (column 10). On the other hand, the variation in the mechanical reinforcement coefficient, ω, is very similar to that of the variation in the x/d ratio (column 4 vs. column 8) and has a much higher magnitude, of about 40% and of about 75%, respectively (column 4).

Table 3: Calculated values

Beam	Reinforcement coefficient, ρ		Mechanical reinforcement coefficient, ω		Longitudinal reinforcement area, A_{sl}		x/d		Mean $\frac{M_{Exp}}{M_{Rd}}$	
No.	1	2	3	4	5	6	7	8	9	10
BH 1	0.013	1.000	0.129	1.000	355 mm^2	1.000	0.164	1.000	1.6070	1.000
BH 2	0.019	1.462	0.179	1.387	452 mm^2	1.274	0.227	1.384	1.5985	0.995
BH 3	0.023	1.769	0.225	1.744	575 mm^2	1.619	0.286	1.744	1.4885	0.926

Second, although the authors' intention was to study the descending branch of the stress-strain curve, by using specially designed machinery and specific test setup, even then, in most cases, the values collected are few and

scars. Because of a higher brittle behaviour of High Strength Concrete, after reaching the maximum stress, failure shortly followed, making it impossible to achieve adequate results.
After studying a large range of the most used stress-strain diagrams, as presented in Table 4, and conducting tests on a period of one year on the accompanying specimens, the authors have observed a highly dependence on the testing procedure for the descending branch, as well as an increase in the brittleness of the specimens, with an increase in age.

Table 4: Stress-strain models addressed in this study

Model (authors)	Main Equations
JENSEN (1943), [4] (Ultimate strength of reinforced...)	$f_c = \begin{cases} f_c' \cdot (\varepsilon / \varepsilon_o) \ if \ \varepsilon_c \le \varepsilon_o \\ f_c' \ if \ \varepsilon_o < \varepsilon_c \le \varepsilon_{cu} \end{cases}$
HOGNESTAD (1951), [5] (A study of combined bending...)	$f_c = \begin{cases} k_3 \cdot f_c' \cdot \left[2 \cdot (\varepsilon / \varepsilon_o) - (\varepsilon / \varepsilon_o)^2 \right] \ if \ \varepsilon_c \le \varepsilon_o \\ k_3 \cdot f_c' - \left[\left(0{,}1275 \cdot f_c' \right) \cdot \left(\dfrac{\varepsilon - \varepsilon_o}{\varepsilon_{cu} - \varepsilon_o} \right) \right] \ if \ \varepsilon_o \le \varepsilon_c \le \varepsilon_{cu} \end{cases}$
DESAYI and KRISHNAN (1964), [6] (Equation for the Stress-Strain…)	$f = \dfrac{E \cdot \varepsilon}{1 + (\varepsilon / \varepsilon_o)^2}$
SARGIN and HANDA (1968), [7] (Structural Concrete and...)	$Y = \dfrac{A \cdot X + (B-1) \cdot X^2}{1 + (A-2) \cdot X + B \cdot X^2}$
POPOVICS (1973), [8] (A numerical approach to the...)	$f_c = f_c' \cdot \dfrac{\varepsilon_c}{\varepsilon_{co}} \cdot \dfrac{n}{n - 1 + (\varepsilon_c / \varepsilon_{co})^n}$
WANG et al. (1978), [9](Stress-strain curves of normal...)	$Y = \dfrac{A \cdot X + B \cdot X^2}{1 + C \cdot X + D \cdot X^2}$
CARREIRA and CHU (1985), [10] (Stress-strain relationship for plain…)	$f_c = f_c' \cdot \dfrac{\beta \cdot (\varepsilon / \varepsilon_o)}{\beta - 1 + (\varepsilon / \varepsilon_o)^\beta}$
THORENFELDT et al. (1987), [11] (Mechanical properties of High-Strength...)	$f_c = f_c' \cdot \dfrac{\varepsilon_c}{\varepsilon_{co}} \cdot \dfrac{n}{n - 1 + (\varepsilon_c / \varepsilon_{co})^{n \cdot k}}$
Comité Euro-International du Béton (CEB) (1990), [12] (Design Code...)	$f_c = \begin{cases} f_c' \cdot \dfrac{(E_{it}/E_o) \cdot (\varepsilon / \varepsilon_o) - (\varepsilon / \varepsilon_o)^2}{1 + (E_{it}/E_o - 2) \cdot (\varepsilon / \varepsilon_o)} \ if \ 0 \le \varepsilon \le \varepsilon_{max} \\ \dfrac{f_c'}{\left[\dfrac{\zeta}{\varepsilon_{max}/\varepsilon_o} - \dfrac{2}{\varepsilon_{max}/\varepsilon_o{}^2} \right] \cdot (\varepsilon / \varepsilon_o)^2 + \left[\dfrac{4}{\varepsilon_{max}/\varepsilon_o} - \zeta \right] \cdot (\varepsilon / \varepsilon_o)} \ if \ \varepsilon > \varepsilon_{max} \end{cases}$
CEB (1994), [13] (High Performance Concrete)	$f_c = \begin{cases} f_c' \cdot \dfrac{(E_{it}/E_o) \cdot (\varepsilon / \varepsilon_o) - (\varepsilon / \varepsilon_o)^2}{1 + (E_{it}/E_o - 2) \cdot (\varepsilon / \varepsilon_o)} \ if \ 0 \le \varepsilon \le \varepsilon_{max} \\ \dfrac{f_c'}{1 + \left[(\varepsilon / \varepsilon_o - 1) / (\eta - 1) \right]^2} \ if \ \varepsilon > \varepsilon_{max} \end{cases}$
fib (2009), [14] (Bulletin no. 42, Constitutive modelling...)	$\sigma_c = f_c \cdot \dfrac{k \cdot \eta - \eta^2}{1 + (k-2) \cdot \eta} \ if \ 0 \le \lvert \varepsilon_c \rvert \le \lvert \varepsilon_{c,lim} \rvert$
LOOV (1991), [15] (A General Stress-Strain Curve for Concrete…)	$f_c = f_c' \cdot \dfrac{A \cdot (\varepsilon / \varepsilon_o)}{1 + B \cdot (\varepsilon / \varepsilon_o) + C \cdot (\varepsilon / \varepsilon_o)^n}$

MUGURUMA et al. (1991), [16] (Stress-Strain curve model for concrete…)	$f_c = \begin{cases} E_i \cdot \varepsilon_c + \dfrac{f_c^{'} - E_i \cdot \varepsilon_m}{\varepsilon_m^2} \cdot \varepsilon_c^2 & if \;\; 0 \le \varepsilon_c \le \varepsilon_m \\ \dfrac{f_c^{'}}{\varepsilon_m - 0,004} \cdot \left(\varepsilon_c - 0,004\right) & if \;\; \varepsilon_m \le \varepsilon_c \le 0.004 \end{cases}$
HSU and HSU (1994), [17] (Complete stress-strain behaviour of…)	$f_c = \begin{cases} f_c^{'} \cdot \dfrac{n \cdot \beta \cdot \left(\varepsilon / \varepsilon_o\right)}{n \cdot \beta - 1 + \left(\varepsilon / \varepsilon_o\right)^{n \cdot \beta}} & if \;\; 0 \le \varepsilon_c \le \varepsilon_{\max} \\ f_c^{'} \cdot \eta_d^{\left[-k_d \cdot \left(\varepsilon / \varepsilon_o - \varepsilon_{max} / \varepsilon_o\right)^a\right]} & if \;\; \varepsilon_c > \varepsilon_{\max} \end{cases}$
WEE et al. (1996), [18] (Stress-Strain Relationship of…)	$f_c = \begin{cases} f_c^{'} \cdot \dfrac{\beta \cdot \left(\varepsilon / \varepsilon_o\right)}{\beta - 1 + \left(\varepsilon / \varepsilon_o\right)^{\beta}} & if \;\; 0 \le \varepsilon_c \le \varepsilon_o \\ f_c^{'} \cdot \dfrac{k_1 \cdot \beta \cdot \left(\varepsilon / \varepsilon_o\right)}{k_1 \cdot \beta - 1 + \left(\varepsilon / \varepsilon_o\right)^{k_2 \cdot \beta}} & if \;\; \varepsilon_o < \varepsilon_c \le \varepsilon_{\max} \end{cases}$
Van GYSEL and TAERWE (1996), [19] (Analytical formulation of the…)	$f_c = \begin{cases} f_c^{'} \cdot \dfrac{\left(E_{co} / E_{c1}\right) \cdot \left(\varepsilon_c / \varepsilon_{c1}\right) - \left(\varepsilon_c / \varepsilon_{c1}\right)^2}{1 + \left(E_{co} / E_{c1} - 2\right) \cdot \left(\varepsilon_c / \varepsilon_{c1}\right)} & if \;\; 0 \le \varepsilon_c \le \varepsilon_{c1} \\ f_c^{'} \cdot \dfrac{1}{1 + \left(\dfrac{\varepsilon_c / \varepsilon_{c1} - 1}{\varepsilon_{c1} + t / \varepsilon_{c1}} - 2\right)} & if \;\; \varepsilon_c > \varepsilon_{c1} \end{cases}$
ATTARD and SETUNGE (1996), [20] (Stress-Strain relationship of confined...)	$f_c = f_c^{'} \cdot \dfrac{\left[\dfrac{f_{ic}}{\varepsilon_{co} \cdot \varepsilon_{ic}} \cdot \dfrac{\left(\varepsilon_{ic} - \varepsilon_{co}\right)^2}{f_c^{'} - f_{ic}}\right] \cdot \left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)}{1 + \left[\dfrac{f_{ic}}{\varepsilon_{co} \cdot \varepsilon_{ic}} \cdot \dfrac{\left(\varepsilon_{ic} - \varepsilon_{co}\right)^2}{f_c^{'} - f_{ic}} - 2\right] \cdot \left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right) + \left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)^2}$
OZTEKIN et al. (2003), [21] (Determination of rectangular stress block…)	$f_c = f_c^{'} \cdot \left[k \cdot \left(\varepsilon_c / \varepsilon_{cu}\right) - \left(k - 1\right) \cdot \left(\varepsilon_c / \varepsilon_{cu}\right)^2\right]$

4. CONCLUSION

The reader should be advised that the following conclusions are based on elements with the same cross section and various reinforcement schemes (placing and number of rebars).

As previously presented, in terms of the increase of the reinforcement coefficient, ρ, in the case of High Strength Concrete members, the decrease of the flexural capacity proves to be very conservative, for a significant effect (higher that 5%) was recorded for an increase in ρ of about 75%. Those values are specific for two different reinforcement schemes, the first with the reinforcement placed on a single row and the second with the reinforcement placed on two rows. Therefore, changing the placing of the reinforcement from one row to multiple rows (i.e., two) does not impair the capacity of the member. Moreover, a significant increase in the area of the reinforcement of about 27% generated the fore mentioned effect (i.e., a decrease in the flexural capacity higher than 5%). In those conditions, it is save to asses that High Strength Concrete member are little influenced by the changes in reinforcement, both as placing and as quantity. It is the authors' recommendation that, if necessary, any changes on a member to better adjust a particular loading case is to be operated on the cross section of the element (as dimensions or type) instead on the reinforcement (which is usually the case for Normal Strength Concrete).

On the other hand, the variation in the mechanical reinforcement coefficient, ω, is very similar to that of the variation in the x/d ratio, and had a much higher magnitude, of about 40% and of about 75%, respectively.

A warning is given also that the descending branch of the stress-strain curve highly depends on the testing procedure and its formulation should be addressed with caution. Furthermore, the authors recommend that no descending branch is to be considered when designing a High Strength Concrete element, because of the increase in brittleness with the increase in age.

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