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MODIFICATIONS OF SPACECRAFT ORBITS

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Abstract : *The paper present a basic methods for the preliminary calculation of the parameters needed to modify the orbit of a spacecraft. The modifications and corrections of the orbit can be related to the orbit radius, eccentricity, speed and inclination. The methods presented make use of the classical space mechanics in order to address the important problems of optimization for fuel consumption, energy, time and costs.*
Keywords : *spacecraft, trajectory, transfer orbit, energy, optimization*

1. INTRODUCTION

This paper discusses a basic methods used to design transfer orbits. Various methods of minimum energy transfer orbits exists and are discussed at the present time: the two-point two-body boundary value problem, method of patched-conics, and other. Most of them are very time consuming and the final result is only with small fraction more exact. Therefore, as a convenient method to perform preliminary mission analysis, the following quick algorithm is proposed.
While most theory is typically applied to travel among different solar system planets, it can also be applied to travel between other celestial bodies such as moons or comets, or just to change the orbit around the same central body.

2. MINIMUM ENERGY PROBLEM

Often it is of interest to find a suitable transfer orbit that will connect two points in space. The following development will derive the concept of a minimum energy transfer orbit. As discussed in [1], this minimum energy orbit becomes a good initial guess at an orbit to start the numerical iteration to find the solution to the two-point boundary value problem. It is also convenient to find other specialized orbit transfers such as the Hohmann transfer.

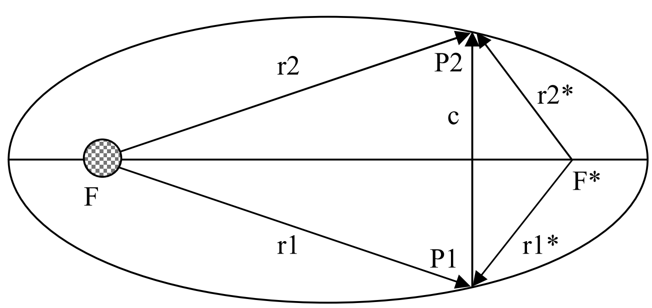

Figure 1: General elliptic orbit with position vectors

Consider the general elliptical orbit that connects points P1 and P2 shown in figure 1. Let r1 and r2 be the corresponding position vectors relative to the occupied focus F, and let r1* and r2* be the position vectors relative to the unoccupied focus F*. Noted with a is the semi-major axis of the ellipse. As a geometric property of the ellipse:

$$FP1 + F^*P1 = 2a \qquad r1 + r1^* = 2a \tag{1}$$

$$FP2 + F^*P2 = 2a \qquad r2 + r2^* = 2a \tag{2}$$

Summing equations (1) and (2) gives:

$$r1 + r2 + r1^* + r2^* = 4a \qquad (3)$$

Since the radial distances r1 and r2 are specified by the two point boundary value problem statement, only parameters r1*, r2* and a can be chosen. Note that the radial distances r1* and r2* to the unoccupied focus are related to the chord:

$$c \leq r1^* + r2^* \qquad (4)$$

The vis-viva energy equation of the elliptical orbit [2]:

$$E = - 1 / 2a \qquad (5)$$

where E is the total energy and v is the instantaneous speed. From the above equation it can be noticed that in order to minimize E, a transfer orbit with the minimum semi-major axis a is needed. Regarding equation (3), minimizing a means to minimize the sum of r1* and r2*. However, according to constraint in equation (4), the smallest possible value that this sum can achieve is the chord length c. Thus, the un-occupied focus of the minimum energy transfer orbit must lie on the chord vector c. The semi-major axis am of this minimum energy orbit is then given by [1]:

$$am = (r1 + r2 + c) / 4 \qquad (6)$$

Notice that (r1 + r2 + c) is the perimeter of the triangle F-P1-P2. Given the initial and the final position vectors r1 and r2, c can be compute:

$$c = |r2 - r1| \qquad (7)$$

If the true anomaly difference between the points P1 and P2 is known, then, using the law of cosines, c can be compute through the scalar values r1 and r2.

$$c = (r2^2 + r1^2 - 2\ r1\ r2\ \cos(t))^{0.5} \qquad (8)$$

Figure 2 presents an illustration of a Hohmann Minimum Energy transfer orbit (the ellipse between the initial circular orbit and the final circular orbit).

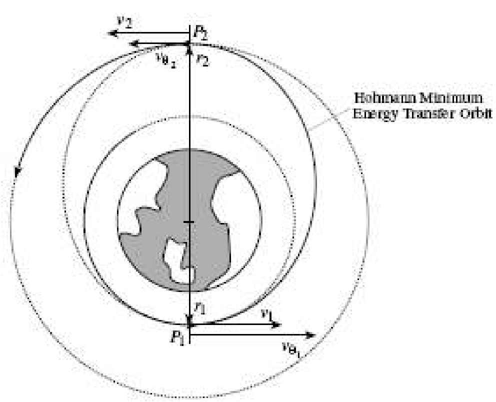

Figure 2: Illustration of a Hohmann Minimum Energy Transfer Orbit

3. CONCLUSION

The presented mathematical algorithm gives a quick and easy method to calculate the minimum energy trajectory needed to change the orbit. This preliminary calculation can be a more accurate starting point for the following more complex and accurate calculations that takes into account the factors that act upon the spacecraft, including the perturbing factors [3].

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