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STUDYING THE VEHICLE'S DYNAMICS USING TENSOR ANALYSIS

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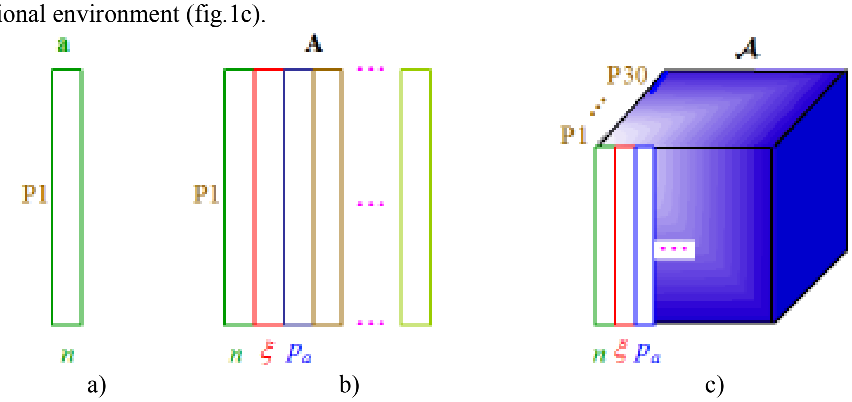
Abstract: *The paper highlights the main possibilities used when studying the vehicle's dynamics using elements from tensor calculus, presented as a superior step of the matrix calculus. The main aspects of theoretical tensor calculus are presented here especially those elements that can be applied when studying the vehicle's dynamic behavior.We are going to present how those algorithms, especially those algorithms that permit the study of vehicle's dynamics, will be applied using experimental data gathered from various vehicle tests. In the paper we used data collected from the onboard computer of those vehicles that have electronic control. We call on elements referring to tensor analysis, spectral analysis, modal analysis as well as multidimensional mathematical models for the vehicle's dynamics like PARAFAC, TUCKER, PARALIND etc. Based on what the paper presents, we can conclude that specific elements of tensor calculus can be used for studying the vehicle's dynamics and how their engine behave, as well as for studying their fuels saving parameters ant it's dynamics.*
Key words: *Tensor analysis, spectral analysis, modal analysis, tensor decomposition, vehicle dynamics*

1. GENERAL KNOWLEDGE. TENSORS

Tensor analysis, known also as multimodal analysis or multidimensional analysis, is an extension of matrix analysis, which in its turn is an extension of vector analysis; so vector analysis relies on vectors, multivariable analysis calls on matrix, and tensor analysis relies on tensors [7; 8].

Saying that, we can analyze the gathered experimental data and the derived data from it can be analyzed in three ways. If we look on only one functional parameter (engine's speed for example - n) taken during only one test (P1 for example), than there is a vector of values, so we deal with only one dimension (fig.1a). If we take into consideration two or more functional parameters (engine speed n, throttle's position ξ, intake air pressure p_a etc.) taken from only one experimental test (P1 for example), then there is a matrix of values, so we speak of a two dimensional environment (fig.1b). Finally, if we take into analysis several functional parameters taken from several experimental tests (P1, P2, …, P30 for example), then we confront a tensor, which describes a three dimensional environment (fig.1c).



a) b) c)

Figure 1

As we can see from figure 1, a vector is being noted with a bold small character (**a** for example), a matrix is noted with capital bold letters (**A** for example) and a tensor is being noted using Euclid mathematical characters, calligraphically written letters like A; for tensors, $\underline{\mathbf{A}}$ notation is also being used [1; 8]. So a tensor represents a multidimensional picture of data, and the order of the tensor is equal with the dimension of the space, in figure 1c we can see represented a 3rd degree tensor $N = 3$ (three dimensional space). Therefore a scalar (one number only/value) is a tensor of the degree zero, meaning $N = 0$. A vector is a first degree tensor ($N = 1$), and a matrix is a second degree tensor ($N = 2$).

Figure 2 shows us a tensor with three dimensions (a third degree tensor $N = 3$) when taking into account 30 experimental tests (marked with P1, P2, …, P30). As we can see from figure 2, in the front plane of the tensor there is a matrix which contains the values of the functional parameters which were recorded during the first experimental test P1; on the rows we have the values (at a given time t) for the engine speed n, throttle's position ξ etc. So as a consequence, a certain point B point from the 30th experimental test has its own given characteristics: $B(t_{B_{30}}, n_{B_{30}}, \xi_{B_{30}}, \ ....)$, and similarly we can write down the coordinates for the A point of the experimental test P1.

As we can see from figure 2, time can be considered continuous, meaning t [s] or discreet, meaning t_d [number of values]. We have to notice that the experimental data have a discreet behavior so the second way of expressing time is of much more helpful, especially for the computers which operate in discreet time.

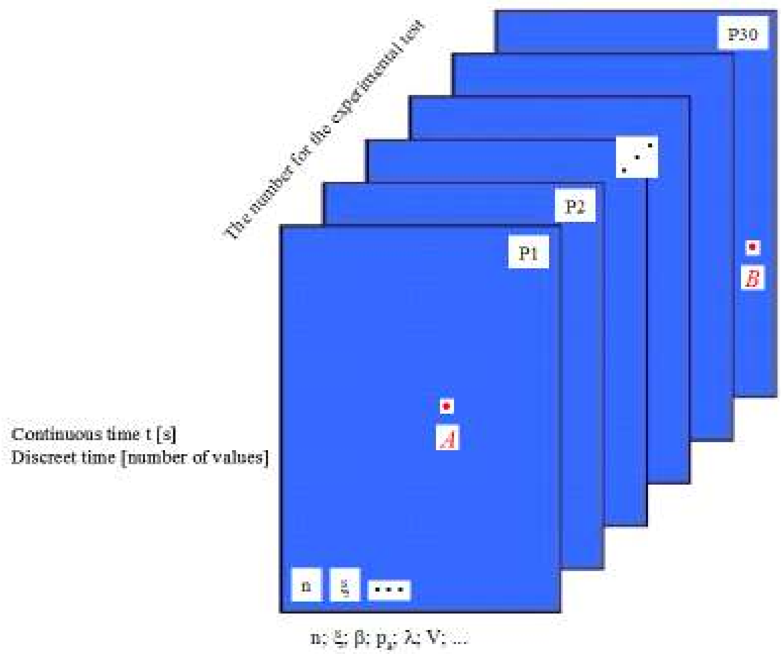


Figure 2

From what we have presented, we can see that in general a tensor has a degree of N, thus being represented in the space with N dimensions. The order of a tensor leads to the notion of mode; for example a third degree tensor has three modes (mode 1, mode 2 and mode 3). Just the same we can see that in general a tensor is being marked as $\mathrm{A}(i_1, i_2, \ldots, i_N)$ using small indices, just the same as matrix is; often you may come upon this notation $\mathrm{A}_{i_1, i_2, \ldots, i_N}$ $\mathbf{A}(i,:)$. To give you an example, in the matrix case, we mark with i the row, and with j the column $\mathbf{A}(i,:)$. For multiple dimensions, there is an extension in the notations [1; 4; 8].

Like all the other tables of values, we can operate with tensors using mathematical calculus like adding, subtracting, multiplying or dividing etc. We can also perform mathematical operations between a vector and a tensor, between a matrix and a tensor and of course between two tensors etc.

To give an example, the external product between the tensor $\mathrm{A}(I_1 \times I_2 \times \ldots \times I_m)$ and the tensor $\mathrm{B}(J_1 \times J_2 \times \ldots \times J_N)$, noted with $\mathrm{A} \circ \mathrm{B}$, is the tensor C [7; 8]:

$$\mathrm{C}(I_1 \times \ldots \times I_m \times J_1 \times \ldots \times J_N) = \mathrm{A}(i_1, \ldots, i_M) \circ \mathrm{B}(j_1, \ldots, j_N) \quad \textbf{(1)}$$

…where "$\circ$" symbol is the Khatri-Rao symbol.

The internal product between two tensors requires that they must have equal dimensions. Therefore the tensor product between $\mathrm{A}(I_1 \times \ldots \times I_n)$ and $\mathrm{B}(I_1 \times \ldots \times I_n)$ tensors are given by the relation:

$$\mathrm{C} = \mathrm{A} \otimes \mathrm{B} = \sum_{i_1=1}^{I_1}\sum_{i_2=1}^{I_2}\cdots\sum_{i_N=1}^{I_n}\mathrm{A}(i_1,i_2,\ldots,i_N)\mathrm{B}(i_1,i_2,\ldots,i_N) \quad \textbf{(2)}$$

…where the "$\otimes$" symbol is the Kronecker symbol.

Using the internal tensorial symbol, the following relation represents the Frobenius tensor norm:

$$\|\mathrm{A}\|_F^2 = \mathrm{A} \otimes \mathrm{A} = \sum_{i_1=1}^{I_1}\sum_{i_2=1}^{I_2}\cdots\sum_{i_N=1}^{I_n}\mathrm{A}(i_1,i_2,i\ldots,i_N)^2 \quad \textbf{(3)}$$

also called Euclidian norm, that has the same significance as the case for vectors and matrix: Euclid's norm raised at the power of two (2nd norm) represents the energy and that is why it is being used as an appreciation criteria for the vehicle's dynamics or/and its fuel savings [2].

Many times, in order to make some operations, a tensor is being transformed into matrix, an operation which is called matricizing the tensor [1; 8]. Usually we deal with column matrix, so we consider $k = 1$ for exemplifying a tensor with three modes (meaning $N = 3$). The resulted columns may be arranged in any way as we see fit; most of the times two special arrangements are practiced and in the end we come up with the matrix noted with $\mathbf{A}_{(1)}, \mathbf{A}_{(2)}, \ldots, \mathbf{A}_{(n)}$.

As we can see, the advantage of matricizing a tensor is the fact that the operations that follow are made using matrix operations; so matricizing a tensor means to reduce its degree. Likewise, the simplification can continue some more eventually dealing with a vector, operation known as vectorizing a matrix, thus turning a matrix into a vector.

In the end, just as the case of matrix, the tensor's rank is established [5]; to this purpose we call on matricizing the tensor, operation which we have discussed a little bit earlier, and with the help of which we achieve the $\mathbf{A}_{(1)}, \mathbf{A}_{(2)}, \ldots, \mathbf{A}_{(n)}$ matrices. To this purpose we define the rank of a tensor as being:

$$\mathrm{rank}(\mathrm{A})_n = \mathrm{rank}(\mathbf{A}_{(n)}) \quad \textbf{(4)}$$

where $\mathbf{A}_{(n)}$ represents the given matrix of the n mode obtained through matricizing the A tensor.

Similar as the matrix, a tensor's rang is important in the spectral analysis and in data compression (reducing the volume of data). According to what we have just presented, a tensor's rank is equal with the minimum rank of the matrices obtained following the matricizing of that tensor.

2. STUDYING THE VEHICLE'S DYNAMICS USING SPECTRAL ANALYSIS

Just as the case of matrix, spectral analysis relies on the decomposition (factorizing) a tensor, thus obtaining its eigenvalues, eigenvectors and its singular values [1; 8].

So, it is being known that the singular values of a rectangular sized matrix of $\mathbf{A}(m \times n)$ shape are achieved by spectral decomposition, in such a way:

$$\mathbf{A} = \mathbf{USV}^T = \sum_{j=1}^{p}\tau_j u_j v_j^T + \cdots + \tau_p u_p v_p^T \quad \textbf{(5)}$$

…where $\mathbf{S}(m \times n)$ matrix, contains on its main diagonal all the singular values of **A** matrix, is the matrix of eigenvectors on the left, is eigenvectors on the right side of **A** matrix, and p is the number of singular values.

Similarly, tensors which contain experimental data are also rectangular (parallelepiped) and not square (cubic), for example the tensor from figure 2 is usually rectangular, because it is very unlikely that the number of values measured for the targeted parameter to be the same as the number of the experimental test carried out.

We know from matrix mathematics that the eigenvalues of the $\mathbf{A}$ matrix are the eigenvalues of $\mathbf{AA}^T$ matrix. Also in the case of tensors we will call on their eigenvalues and eigenvectors, as well as on to singular values; this time instead of applying SVD (Singular Value Decomposition), HOSVD (Higher Order Singular Value Decomposition) will be applied.

Similarly the spectral analysis of tensors is applied: in order to highlight the analogy between matrix and tensor, the 5th expression can be written;

$$\mathbf{A} = \mathbf{S} \times \mathbf{U}_{(1)} \times \mathbf{V}_{(2)} \quad \textbf{(6)}$$

considering the **A** matrix a second degree tensor.

So reasoning the same way, we can say that for a third degree tensor ($N = 3$) we have the following spectral decomposition:

$$A = S \times \mathbf{U}_{(1)} \times \mathbf{V}_{(2)} \times \mathbf{W}_{(3)} \quad \textbf{(7)}$$

…where the numbered indices marks the tensor's mode (1, 2 and 3).

And generalizing, we get the relation for an *N* degree tensor:

$$A \in R^{I_1 \times I_2 \times \cdots \times I_N} \quad \textbf{(8)}$$

We get:

$$A = S \times \mathbf{U}_{(1)} \times \mathbf{U}_{(2)} \times \ldots \times \mathbf{U}_{(N)} \quad \textbf{(9)}$$

…where Σ is called nucleus and is a tensor of the same dimensions that A tensor has; Σ is similar with the matrix that contains on its main diagonal the singular values of the targeted matrix that needs to be decomposed (SVD process from matrices).

Furthermore, in relation number (9) $\mathbf{U}_{(i)}$ are called modal matrixes (are specific for the modes 1, 2…, *N* of the tensor); these are orthogonal and contain the main vectors under the unit (that have the infinite norm equal to one) of $\mathbf{A}_{(n)}$ matrices obtained by matricizing the A tensor.

For example, for a three dimension tensor expressed by the 7th mathematical expression the obtained matrices by matricizing the tensor are given by these expressions:

$$\mathbf{A}_{(1)} = \mathbf{U}_{(1)}\mathbf{S}_{(1)}\mathbf{V}_{(1)}^T; \mathbf{A}_{(2)} = \mathbf{U}_{(2)}\mathbf{S}_{(2)}\mathbf{V}_{(2)}^T; \mathbf{A}_{(3)} = \mathbf{U}_{(3)}\mathbf{S}_{(3)}\mathbf{V}_{(3)}^T \quad \textbf{(10)}$$

…each of it having specific singular values.

The presented process, called HOSVD (Higher Order Singular Value Decomposition), specific for each tensor, is also called the *N*-mode SVD algorithm, thus highlighting the fact that it represents a generalization process extended from the matrices.

The presented relations allow us to calculate the eigenvalues of the A tensor:

$$\lambda_i = A \times \mathbf{U}_{(1)_i}^T \times \mathbf{U}_{(2)_i}^T \times \cdots \times \mathbf{U}_{(N)_i}^T \quad \textbf{(11)}$$

Consequently, for those *r* eigenvalues λ_i, we obtain the A tensor's estimation:

$$\hat{A} = \sum_{i=1}^{r} \lambda_i \mathbf{U}_{(1)_i} \circ \mathbf{U}_{(2)_i} \circ \cdots \circ \mathbf{U}_{(n)_i} \quad \textbf{(12)}$$

…case where the initial tensor represents a sum of *r* first degree tensors; in this case, *r* represents the tensor's rank.

The difference is also a tensor:

$$D_A \equiv A - \hat{A} \quad \textbf{(13)}$$

and it represents the estimation error of the the A tensor.

Similar to matrix, data compression for tensors means to establish the singular values that are the highest (biggest eigenvalues), that will ensure the estimation of the tensor with the imposed precision; in other words put, the sum expressed by (12) will keep only those singular values, from the total of *r*, that will ensure the imposed error.

For example, figure 3 presents how to establish the singular values for the A tensor which has the dimensions $A\,(256 \times 3 \times 50)$ in the case of three groups of three functional parameters mentioned on the graphs (each having 256 values) from all 50 experimental tests carried out with a Daewoo Tacuma vehicle; thus we obtain 3x50=150 singular values for each group, given in the figure 3.a. In order to establish each impact, figure 3a gives the relative singular values, meaning the current values divided by the maximum. To establish the dominant singular values (that has the biggest impact in the vehicle's dynamics), a precision is being set here of 3%, so as we can see from the figure 3.b where a detail is being presented.

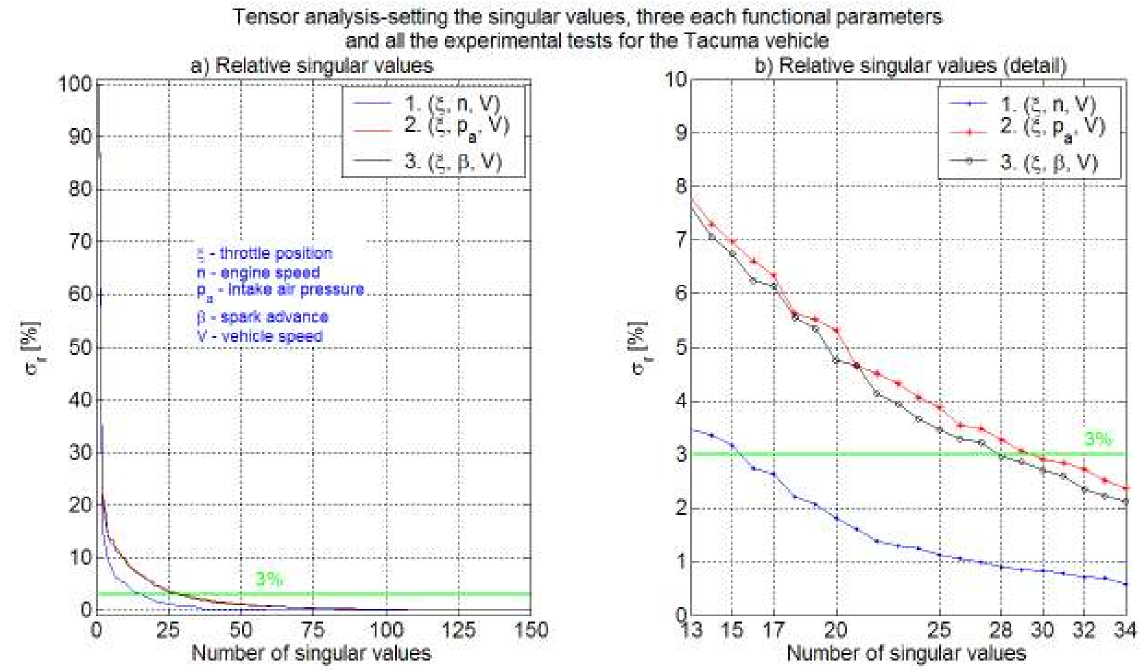


Figure 3

The graphs from figure 3 reveals different values of the singular values for divers groups of functional parameters; so it is confirmed that various parameters contributed to the vehicle's dynamic. More than that, a 3% horizontal is represented for an imposed error of 3%. As we can see from figure 3b, if we adopt a 3% error of calculation, than from the first group of functional parameters (ξ, n, V) we can only retain 16 singular values relevant to the vehicle's dynamics, when establishing the mathematical model. In comparison if we look at the second group (ξ, p_a, V) or at the third group (ξ, β, V) of parameters, we have to keep 28, respectively 30 singular values, meaning that the number of singular values is higher. In other words put in order to achieve calculus errors under 3% we should keep more singular values when we refer to the experimental dynamic series (estimating it through a mathematical pattern).

What we have presented here suggests the fact that the engine's speed is what it defines the best way the vehicle's dynamic behavior, a fact which is also revealed from figure 4 that looks at all 10 functional parameters and at all 50 experimental test, meaning that at all data that we've collected and had at our disposal. We can see from graph 4b that if we take into account the engine's speed, than the first 16 singular values can define the vehicle's dynamic behavior. But if it were not to take into consideration the engine's speed when establishing the experimental tensor, we can see that we need 26 singular values in order to appreciate the vehicle's dynamic behavior and maintaining a calculus error under 3%; in other words put, the number of relevant singular values is higher.

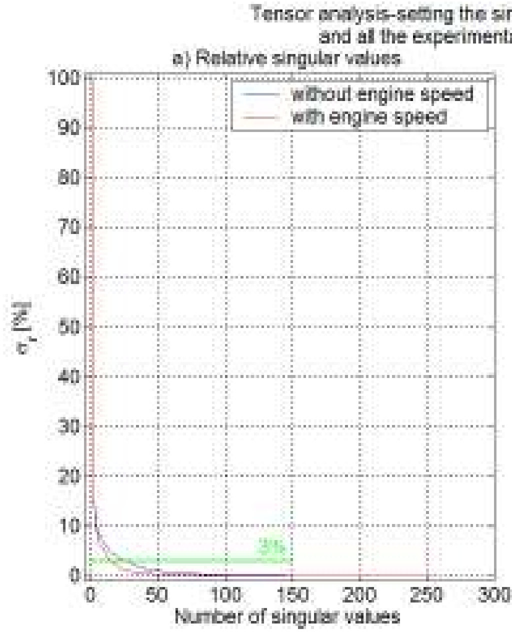


Figure 4

Figure 5 presents the three dimensional tensor A(256×10×50) which contains the mode 1, the discreet time t_d [number of values], 10 functional parameters as mode 2, and fifty experimental tests carried out on the Tacuma vehicle as mode 3.

To the tensor represented in figure 5, figure 6 represents the nucleus tensor that is calculated with the help of the already presented expressions.

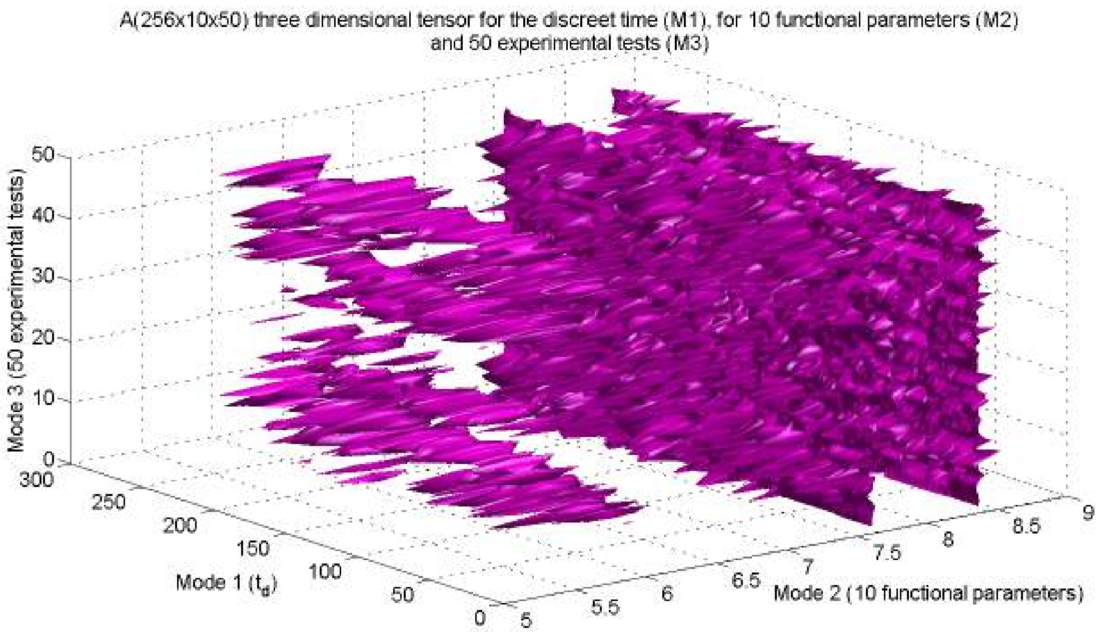


Figure 5

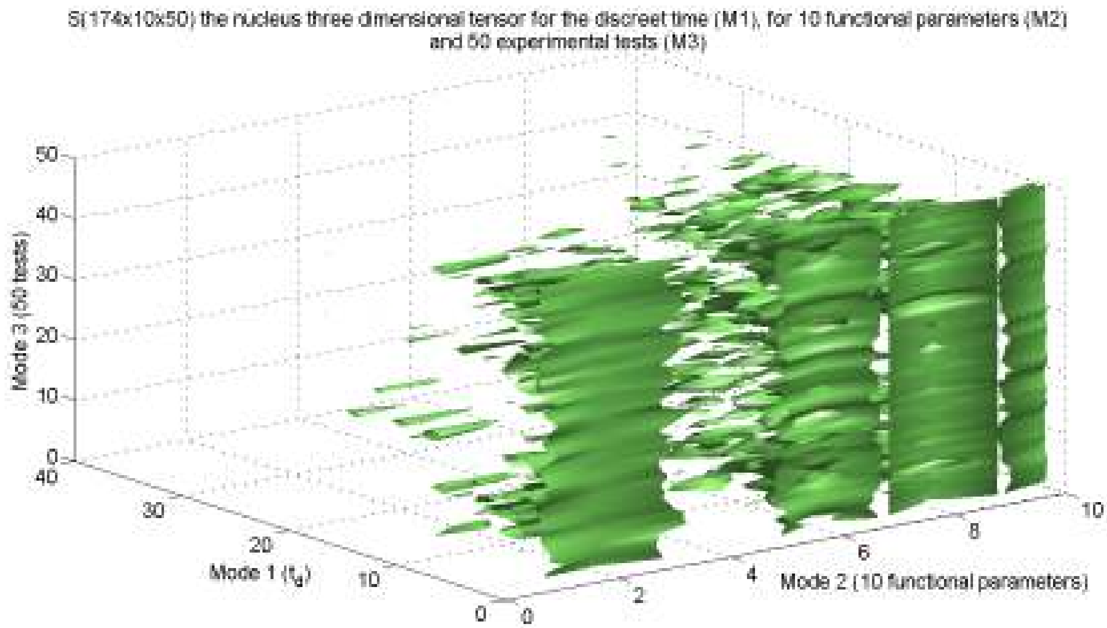


Figure 6

As we can see from figure 6, the nucleus tensor Σ is structured by (174×10×50), and it means that the experimental tensor A of (256x10x50) from figure 5 has the rank equal to 174. Similarly we can present the modal matrices $\mathbf{U}_{(\cdot)}$ from the previous expressions.

In this case, a three dimensional tensor, there are three modal matrices $\mathbf{U}_1$, $\mathbf{U}_2$ and $\mathbf{U}_3$ presented in figure 7.

As we can see from the three graphs, modal matrices have the resulted dimensions from the targeted experimental tensor's dimensions, in this case A (256×10×50).

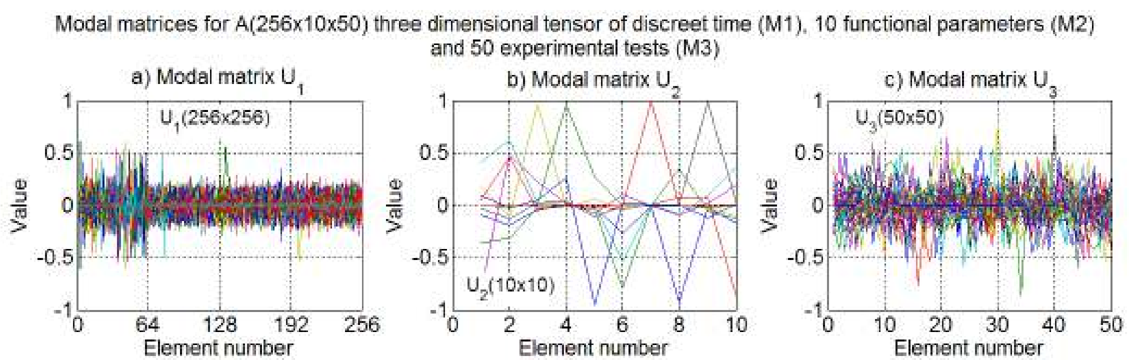


Figure 7

For the same example, figure 8 presents the reconstruction based on the singular values of the A(256×10×50) tensor, and its estimation through matricizing.

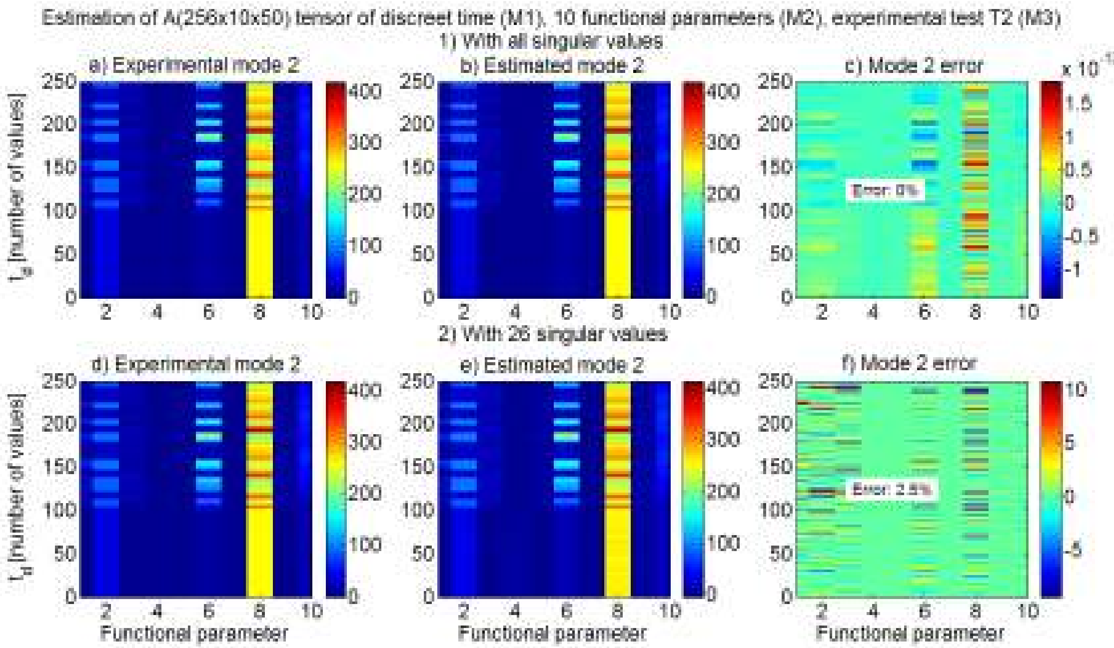


Figure 8

In the upper graphs all singular values are used, and in the lower graphs only 26 relevant singular values are used, those that have the most relevant contribution to the vehicle's dynamic behavior (according to figure 4b, amongst the 10 functional parameters, the engine's speed isn't one of them).

From the superior graphs we can see that the reconstruction error is practically null, when considering all singular values (there are extremely low values for the difference between the experimental values and the ones calculated based on the singular values). Also we can see from the lower graphs that when taking into consideration only the 26 relevant singular values, the estimation error is acceptable, only 2,5%; as we can see from figure 8, these verifications are made for mode 2 (the 10 functional parameters) and in the case of the experimental test T2. So, in this case the volume of experimental data was reduced from 128000 values to 66560, which means a reduction of 48%.

3. STUDYING THE VEHCILE'S DYNAMIC BEHAVIOR AND IT'S FUEL SAVINGS

Tensor analysis can be applied when studying the dynamic behavior and the vehicle's fuels savings; frequently, the appreciation criteria calls on vector analysis because it uses vectors of values of certain functional parameters.

In the classic technical literature, in order to appreciate a vehicle's dynamic behavior usually it uses the medium value for its acceleration [2]. To this extent, we have to mention that it is much fair to adopt as dynamic criteria the second norm of the acceleration; it is much fair to use it because the second norm raised at the power of 2 represents the energy.

Besides, in the case of vehicles that have electronically controlled engines, in order to appreciate the dynamic behavior we can adopt as appreciation criteria the second norm of the engine's speed, throttle's position, engine power etc. Just the same, quite frequently in order to appreciate the vehicle's fuel savings, we adopt [2] the medium value/ second norm for the fuel consumption of the engine when travelling 100 km, horary consumption of fuel, actual specific consumption of fuel etc.

To the extent of what we have previously presented, it results that applying tensor analysis is much closer to the reality when appreciating the fuel savings and dynamic behavior, because this way we have a much more general picture of the targeted matter. And indeed other are the results if we analyze for example, the second norm values for the tensor that contains more functional parameters from several more experimental tests.

In order to make comparisons to this purpose, figure 9a presents the classical case, frequently used to appreciate the vehicles dynamic behavior based on the average value for its acceleration during 10 experimental tests carried out on the Daewoo Tacuma vehicle. As we can see from the graph, the decreasing order of the dynamics is test T8 (with an average value for the acceleration of 0.18 m/s), followed by tests T9, T10, T7 etc.

If we look at figure 9b, where the second norm for the acceleration is given we can see a different order for the dynamic behavior: test T10 followed by T8, T7, T9 etc. As we have mentioned a little bit earlier this classification is much fair to refer to than the previous one.

In the upper graphs from figure 10 tensor analysis is being applied and in the lower graphs, vector analysis (the one from figure 9) was applied when studying the vehicle's dynamic behavior during the 10 experimental tests carried out; the lower graphs refer to the three mentioned functional parameters.

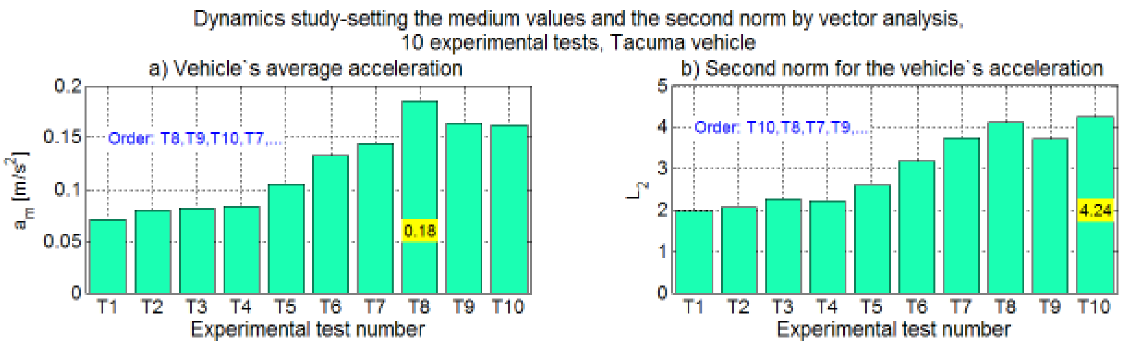


Figure 9

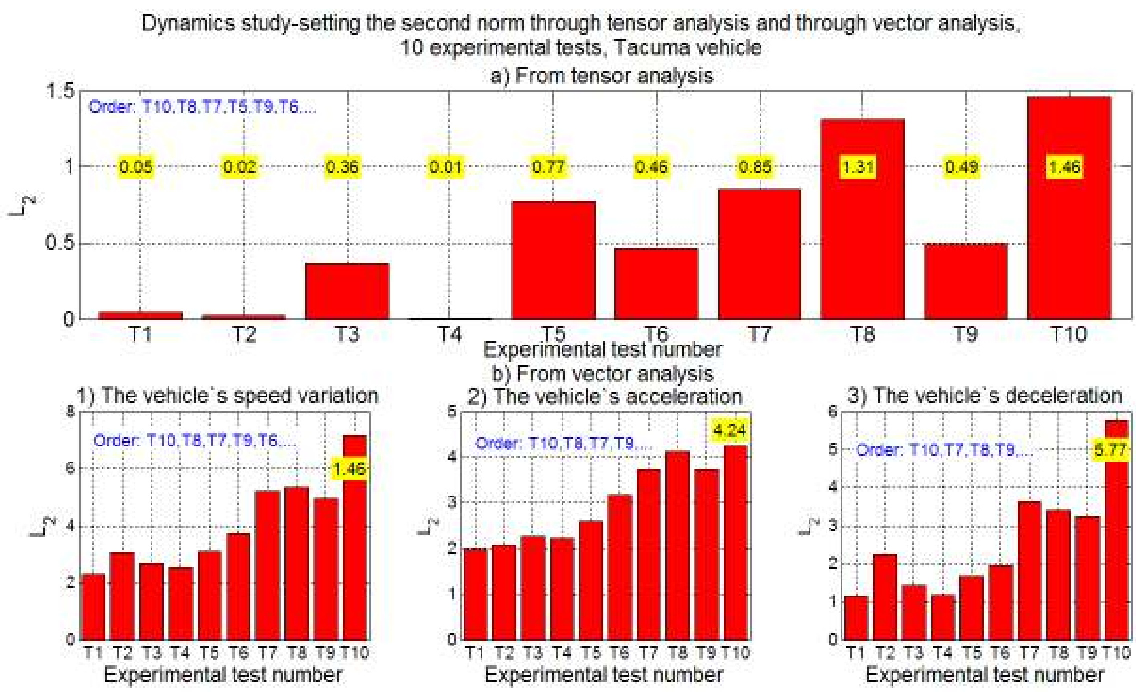


Figure 10

As we can see from figure 10a, tensor analysis offers a different classification for the vehicle's dynamics; this classification is the correct one, decreasing the dynamics (T10, T8, T7, T5, T9, T6, ...), because it takes into consideration at the same time all three parameters.

In order to correctly confirm the appreciation on the vehicle's dynamic behavior using tensor analysis, figure 11 takes into consideration other 3 parameters: throttle's position, engine speed and it's power output. The reasons for which this is done are: the throttle's position represents the engine load, the engine's speed represents a relevant variable when studying the dynamic behavior (figure 4), and the power reserve is used for acceleration [2].

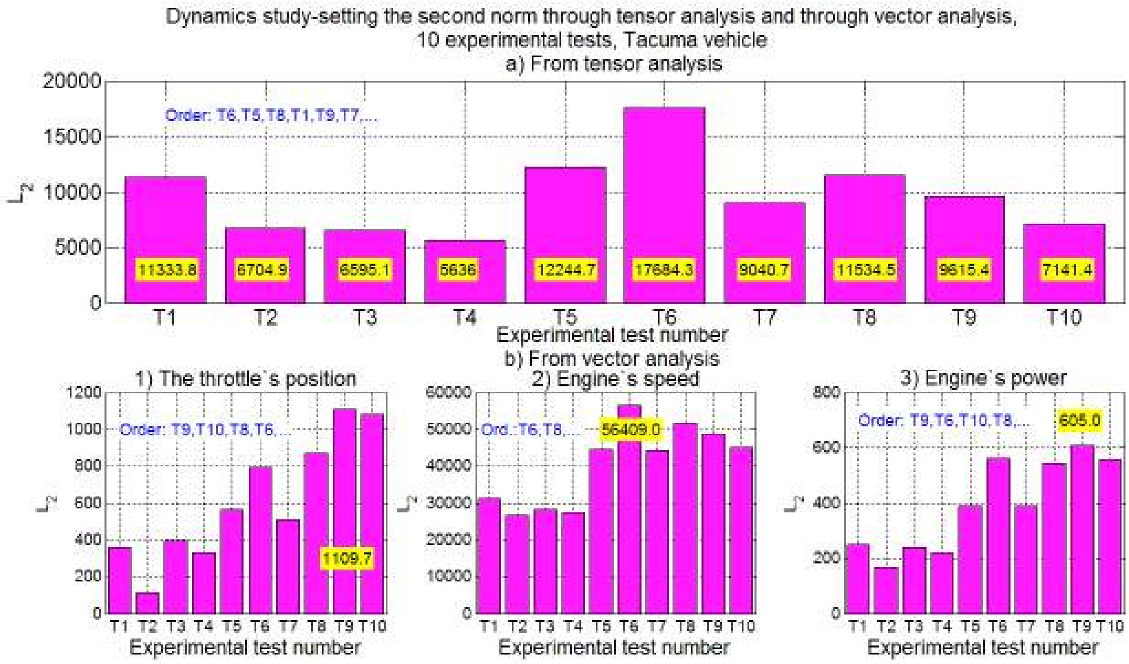


Figure 11

As we can see from figure 10 and figure 11, various parameters offer different classifications for the vehicles' dynamic behavior. So in order to make a correct appreciation we have to take into account all parameters and all experimental tests, and this is possible only by using tensor analysis.

4. MULTIDIMENSIONAL MATHEMATICAL MODELS FOR THE VEHICLE'S DINAMICS

Spectral analysis previously presented, based on decomposition (factorizing) and which relies on eigenvalues, eigenvectors and singular values, was extended and improved, thus obtaining multidimensional mathematical models (or modal models).

So in the initial phase, three models were proposed, which rely themselves still on decomposition [1; 3]:

- L.R. Tucker proposed in the year of 1966 the model known by the name of TUCKER;
- J.D. Carroll and J. Chang elaborated in 1970 the model called CANDECOMP(**CAN**onical **DECOMP**osition);
- R.A. Harshman proposed in 1970 the model known by the name of PARAFAC (**PARA**llel **FAC**tors).

We have to mention that the last two models were joint together and this way we came to know the model called CP (**C**ANDECOMP-**P**ARAFAC).

Later on other variants of these models came to life, as well as other mathematical models from which the most used ones are:

- the models from the family of PARAFAC: PARAFAC2 (in the year of 1972), S-PARAFAC (in the year of 2003), PARAFAC3W (in the year of 2004), c–PARAFAC (in 2006), PARALIND (**PARA**llel factors with **Lin**ear **D**ependency) in 2005, PARATUCK (combination between PARAFAC and TUCKER);
- the models from the family of TUCKER: TUCKER1 (in 1980), TUCKER2 (in 1984), TUCKER3 (in 1992).

Being so many it is hard to make an extended presentation of all of them, and this is why we are going to present only a few basic elements of the PARAFAC model; in the technical literature we can find detailed approaches for the previous presented models [1, 3, 6].

PARAFAC model is a factorizing method (decomposition) which is a general representation of PCA (principal component analysis) from the matrix analysis; for example in the three dimensional analysis (third degree tensor) decomposition leads us to threeliniar components. Thus, in this example PARAFAC model has the structural form for Ξ tensor with its elements x_{ijk} :

$$x_{ijk} = \sum_{f=1}^{F} a_{if} b_{jf} c_{kf} + e_{ijk} \quad \textbf{(14)}$$

…where F represents the number of factors (equal to the Ξ tensor's rank), a_{if}, b_{jf} şi c_{kf} are the elements of the gravity matrices **A, B** and **C**, and e_{ijk} represents the modeling error (the residue).

In matrix form, PARAFAC model is:

$$\underline{\mathbf{X}} = \mathbf{ABC} + \underline{\mathbf{E}} \quad \textbf{(15)}$$

…where $\underline{\mathbf{E}}$ represents the residual tensor (modeling error, meaning the difference between the initial vector and the one obtained by the model); we previously mentioned that a tensor may be noted in two ways, here we can see $\underline{\mathbf{X}}$ or X .

In what follows now we are going to present how to establish only one multidimensional PARAFAC model useful when studying the vehicle's dynamics using experimental data. In figure 12 are given the values for the modal matrices coefficients (14), matrices **A**, **B** and **C** from the expression number (15) of the PARAFAC model for a three dimensional tensor Ξ (256x4x10) which contains the throttle's position noted with ξ, the intake air pressure p_a, horary air consumption C_a and the vehicle's speed V (so we have 4 functional parameters, each having 256 values) measured during 10 experimental tests carried out on the Tacuma vehicle. As we can see from figure 12, F = 32 factors were adopted from the total of 40 ($4 \cdot 10 = 40$) and therefore the data volume was reduced with 20%.

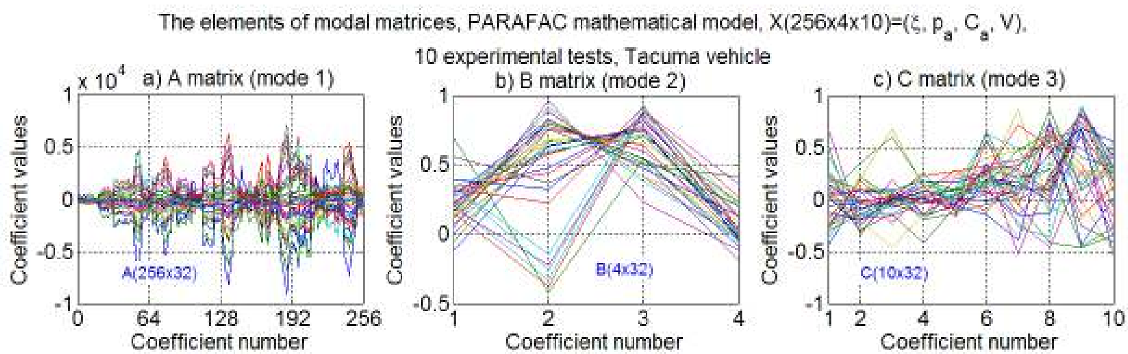


Figure 12

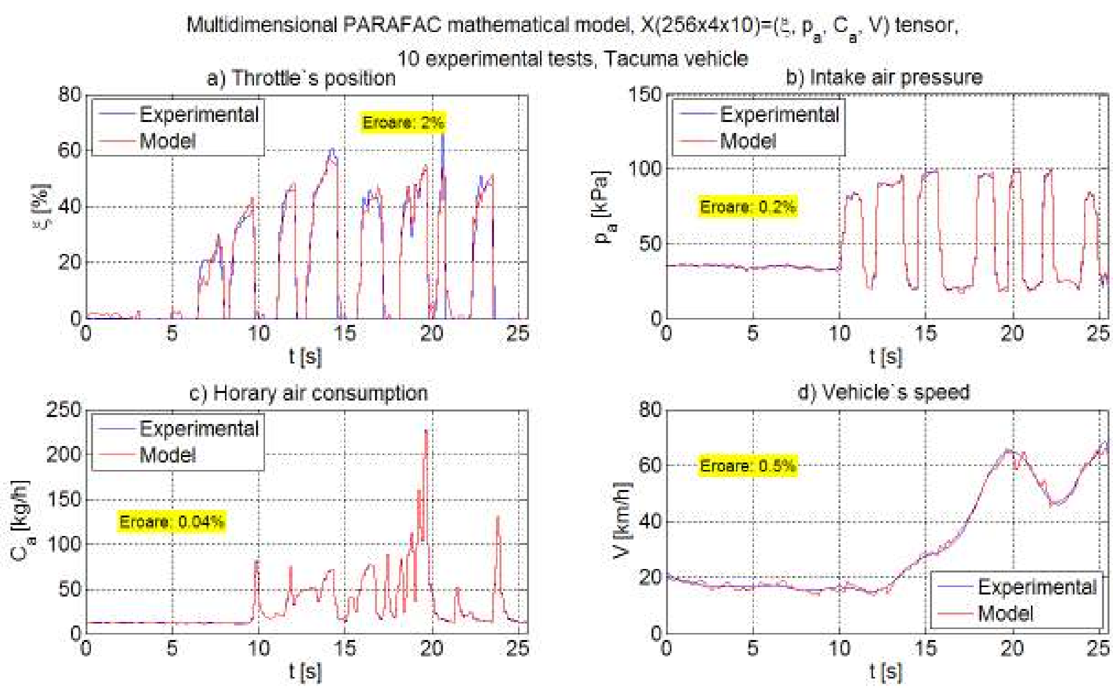


Figure 13

We can see in figure 13 the tensor's estimation (reconstruction) that the obtained errors of the multidimensional model are acceptable; if all factors would have been taken into account the errors would have been null.

Similarly we can set other multidimensional mathematical models for the vehicle's dynamic behavior or only for how he engine operates. As we have mentioned before these models can only ensure estimating precisions extremely high if the component number is equal with the experimental tensors rank.

5. CONCLUSIONS

Tensor analysis used to study the vehicle's dynamic behavior achieves results which are much closer to reality and are believable, because it can take into account at the same time all experimental data that you have in hand, meaning it can operate with tensors that have values; this aspect is very important to the vehicles that have electronic control, where there are inherent interactions, thus the functional parameters depend one on another. By eliminating some parameters you may end up drawing the wrong conclusions. We have to say that tensor approach for the vehicle's dynamics allows us to generally analyze it, not as the classic case where we only analyze the moving speed.

Just the same, tensor analysis can separate the factors that have the biggest impact on the vehicle's dynamics, thus highlighting the core trend for the phenomenon's evolution and reducing the volume of data used to operate. This last issue is at most importance for the electronically controlled vehicles, where the computer deals with large series of data and that is what it is most difficult to process and interpret.

Based on what we have presented in the paper, we can conclude in the end that specific algorithms for tensor calculus may be used for studying the vehicle's dynamic behavior and how their engine work, as well as for studying the vehicle's fuel savings and dynamics, thus the achieved results being much more trustfully and much closer to the reality facts, than it has been before.

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