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PLOTTING EFFORT DIAGRAMS IN POLAR COORDINATES FOR CURVED CIRCULAR BEAMS LOADED WITH PERPENDICULAR-ON-PLANE UNIFORMLY DISTRIBUTED LOADS USING MATHCAD

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Abstract. *Plotting shear, bending and torsion efforts for the plane curved circular beam in polar coordinates using computer software means a significant improvement of the student's or engineer's activity to show the variation of those functions. This can be done using the Step Function Φ in Mathcad, which allows computation at the left and right of a force(or couple of forces) section. This function can limit a function's representation for a given interval. This actual work presents some numerical results for this type of computations, considering different uniformly distributed loads perpendicular on the beam's circular axis plane.*
Key Words: *effort diagrams, polar coordinates , step-function*

1. MATHEMATIC EXPRESSIONS OF THE SHEAR FORCE T, BENDING MOMENT M_I AND TORSION MOMENT M_T, FOR UNIFORMLY DISTRIBUTED PERPENDICULAR-ON-PLANE LOADS

One considers the curved beam with the geometric circular axis of radius R and $3\pi/2$ angle in the Oxy plane. The beam has a free end at right and a fixed one at left. The loads *p* are radial and uniformly distributed, perpendicular on beams circular axis plane, as it can be seen in figure 1.a.

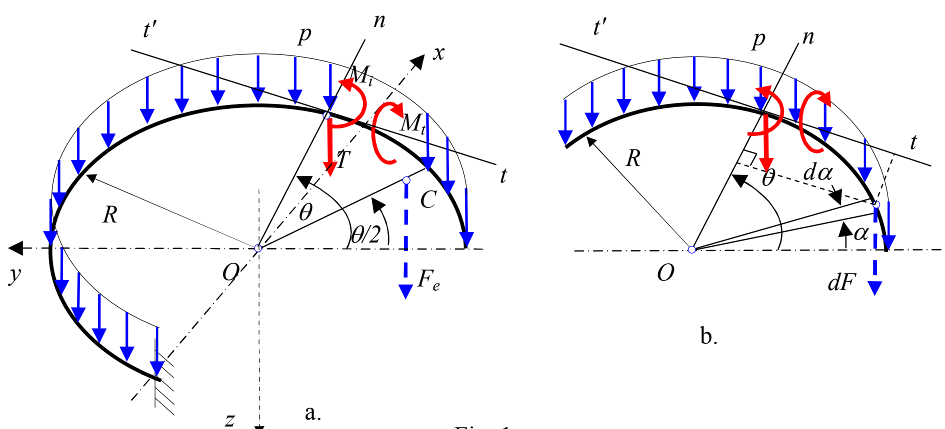

Fig. 1

The shear force T expressions are obtained by integrating the elementary force $dF = p \cdot ds$ along the entire length of the circle sector having the central angle θ, as shown in fig. 1.b.

The expression of the bending moment M_i and torsion moment M_i can be determined computing the elementary force $dF=p\cdot ds$ moment, with respect to the normal axis *On* and the tangent one *tt'* and integrating along the length of the circle sector as seen in fig. 1.b.
Taking into account the sectional efforts sign rule for straight bars [4], one obtains:

$$\begin{cases} T(\theta)=\int_0^\theta (pRd\alpha)=pR\theta; \\ M_i(\theta)=-\int_0^\theta (pRd\alpha)R\sin(\theta-\alpha)=-pR^2(1-\cos\theta); \\ M_t(\theta)=-\int_0^\theta (pRd\alpha)R[1-\cos(\theta-\alpha)]=-pR^2(\theta-\sin\theta). \end{cases} \quad (1)$$

One can obtain the same expressions of the efforts by computing the equivalent force F_e corresponding to the beam sector with central angle θ, applied on the direction of the θ angle bisector and with the application point in the gravity center C of the circular sector [4], as it can be seen in fig.1.a: $F_e=\int_0^\theta pds=pR\cdot\theta$ (2)

The bending and torsion moments will be obtained as the moment of the equivalent force F_e with respect to the normal axis *On* and the tangent one *tt'* as seen in fig. 1.a:

$$\begin{cases} T=F_e=pR\theta; \\ M_i=-F_e\left[\frac{2R}{\theta}\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\right]=-pR^2(1-\cos\theta); \\ M_t=-F_e\left[R-\frac{2R}{\theta}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right]=-pR^2(\theta-\sin\theta). \end{cases} \quad (3)$$

2. POLAR DIAGRAMS OF EFFORTS

The efforts diagrams in *polar coordinates* will be plotted on both sides of the beam's geometrical axis, by the same sign rules as in the case of straight beams: positive T and M_t are on the exterior side and positive M_i is on the interior side.
This can be done using the step function Φ from Mathcad 14, which allows:

- Limitation of a function representation for a given angular interval
- Representation of the diagram jumps corresponding to concentrated forces (or force couples) by computing efforts at left and right limit of the specific section.

In figures 2, 3 and 4 one plotted the shear force $T(\theta)$, torsion moment $M_t(\theta)$ and bending moment $M_i(\theta)$ diagrams in polar coordinates ρ - θ, using Mathcad 14.
The function representation limitation for the angular interval $(0, 3\pi/2)$ was done using: $L(\theta)=\Phi(\theta)-\Phi(\theta-3\pi/2)$
The term $4pR$ added to the shear force, $4pR^2$ for the bending moments and $9pR^2$ for the torsion moments (fig. 2, 3, 4), given by the relations (3), allowed the displacement of the diagrams origin in the point (4R,0) and their representation on both sides of the beam's geometrical axis [4].

$$p := 1$$

$$R := 1$$

$$Axis(\theta) := 4R\left(\Phi(\theta) - \Phi\left(\theta - 3\frac{\pi}{2}\right)\right)$$

$$T(\theta) := (4 \cdot p \cdot R + p \cdot R \cdot \theta) \cdot \left(\Phi(\theta) - \Phi\left(\theta - 3 \cdot \frac{\pi}{2}\right)\right)$$

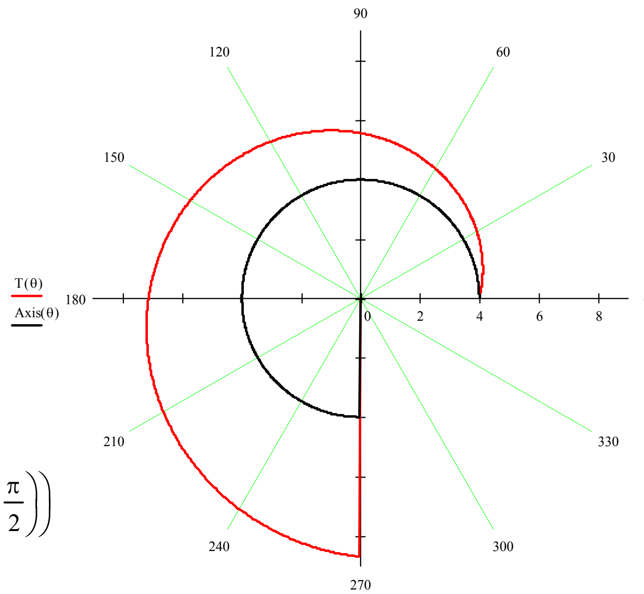

Fig. 2. Shear force diagram in polar coordinates

$$Axis(\theta) := 4R\left(\Phi(\theta) - \Phi\left(\theta - 3\frac{\pi}{2}\right)\right)$$

$$Mi(\theta) := \left[4 \cdot p \cdot R^2 - p \cdot R^2 \cdot (1 - \cos(\theta))\right] \cdot \left(\Phi(\theta) - \Phi\left(\theta - 3 \cdot \frac{\pi}{2}\right)\right)$$

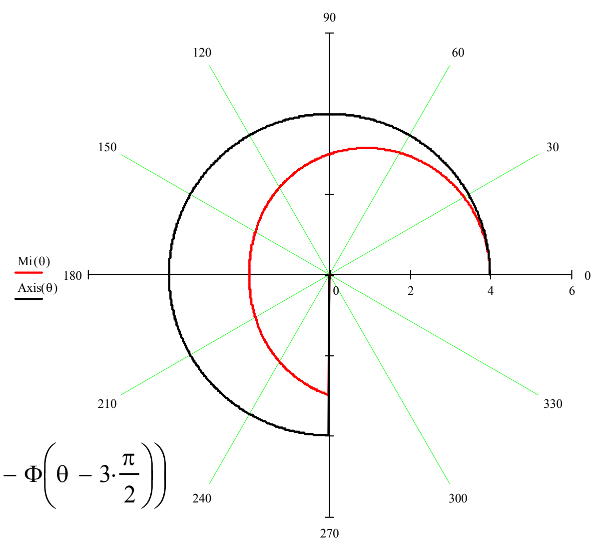

Fig. 3. Bending moment diagram in polar coordinates

$$Axis(\theta) := 9R\left(\Phi(\theta) - \Phi\left(\theta - 3\frac{\pi}{2}\right)\right)$$

$$Mt(\theta) := \left[9p\,R^2 - p \cdot R^2 \cdot (\theta - \sin(\theta))\right] \cdot \left(\Phi(\theta) - \Phi\left(\theta - 3 \cdot \frac{\pi}{2}\right)\right)$$

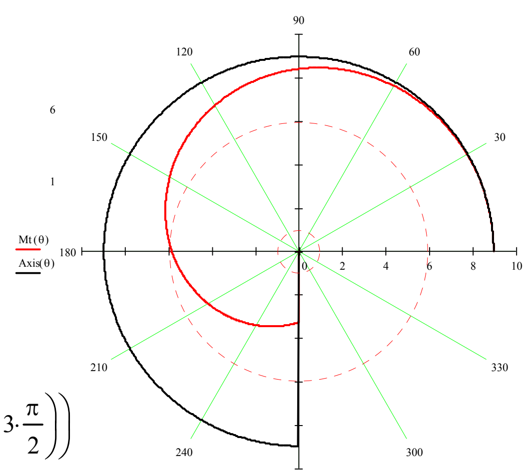

Fig. 4. Torsion moment diagram in polar coordinates

3. CONCLUSIONS

- In order to obtain the polar diagram for the efforts *T*, M_i and M_t one has to introduce a constant term $4pR$, $4pR^2$, $9pR^2$ (fig. 2, 3, 4), so that the diagram will be moved from the coordinate system origin (0,0) to the origin of the beam;
- In order to plot the efforts diagram one used the analytical expressions determined for each case, multiplied with the factor $L(\theta)=\Phi(\theta)-\Phi(\theta-3\pi/2)$, which actually limits the function representation in the interval $[0, 3\pi/2]$;
- In order to plot the circular axis, one used the constant function $Axis(\theta)$, multiplied with the same factoring function $L(\theta)=\Phi(\theta)-\Phi(\theta-3\pi/2)$, which actually limits the function representation in the interval $[0, 3\pi/2]$;
- The above presented method is really easy to approach and allows the plotting of the efforts diagrams for uniformly distributed loads and for concentrated ones as well.

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