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THE QUASI-PLANE MODEL FOR CALCULATING THE FRICTION DRAG ON A SHIP'S HULL

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Abstract: *The paper presents the contributions of the authors regarding the extension of boundary layer theory for a plane plate to a quasi-cylindrical surface. Then the new model is applied to a ship's hull in order to determine the friction drag.*
Keywords: *boundary layer, quasi-plane, friction drag.*

1. PRELIMINARY CONSIDERATIONS

Be T the draft ship full loaded. By cutting-off the hull with "m" equally spaced horizontal planes "Π_j", with the levels, $z_j = j\dfrac{T}{m}$, $j = 1,2,\ldots m$ we obtain the volumes V_j in Figure 1:

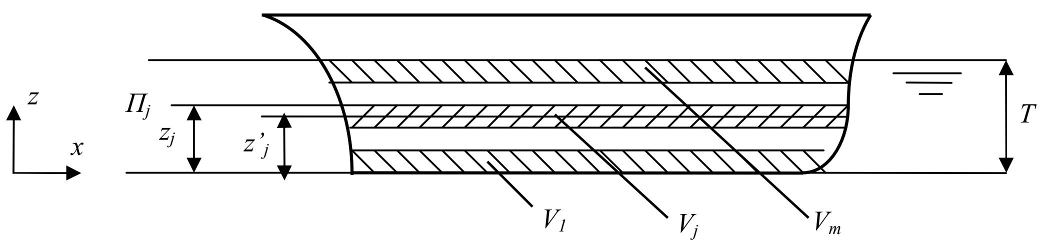

Figure 1

By cutting-off the volumes V_j with horizontal planes taken to the levels

$$z'_j = \frac{1}{2}\left[(j-1)\frac{T}{m} + j\frac{T}{m}\right] = (2j-1)\frac{T}{2m}\,,\ j = 1,2,\ldots,m$$

, we obtain the water lines in Figure 2.

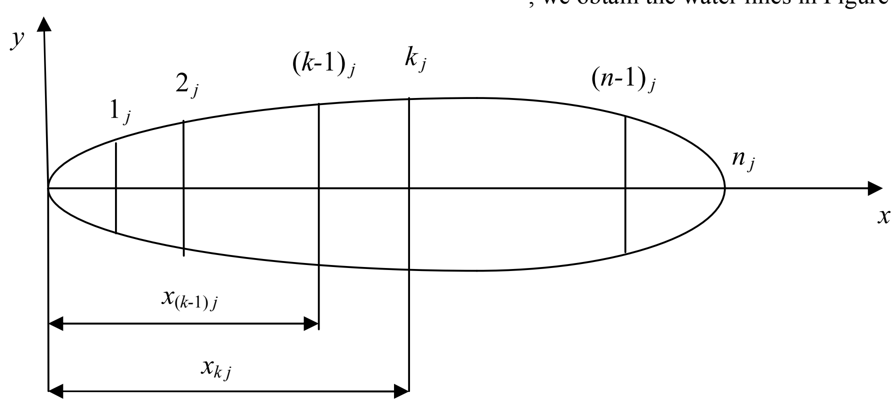

Figure 2

If the ratio $\frac{T}{m}$ is sufficiently small,then the bottom of the segments V_j are quasi-cylindrical, and the fluid flow around them is almost flat. Outlines the current line of L_j we choose the $1_j, 2_j, ..., (n\text{-}1)_j, n_j$. In the proximity of the arc of the curve $(k-1)_j - k_j$ the velocity of current potential is equal to the weighted average velocity of the fluid $(k-1)_j$, and k_j, calculated by a procedure of Kármán type. The arch of the curve $(k-1)_j - k_j$ is then approximated through the string $(k-1)_j - k_j$.

If $v_{(k-1)_j}$ and v_{kj} are the velocities in points $(k-1)_j$ and k_j, and 1, p_{kj} – is their weight, then the average weighted mead of fluid will be:

$$\bar{v}_{p_{kj}} = \frac{\bar{v}_{(k-1)_j} + p_{kj}\,\bar{v}_{kj}}{1+p_{kj}} \tag{1}$$

where $\bar{v}_{(k-1)_j}$ and $\bar{v}_{kj}$ is considered to be included in water line plane L_j.

As the weight p_{kj}, it is determined from the condition that $\bar{v}_{pkj}$ to have a direction and purpose-oriented segment $(k-1)_j - k_j$ (see Figure 3).

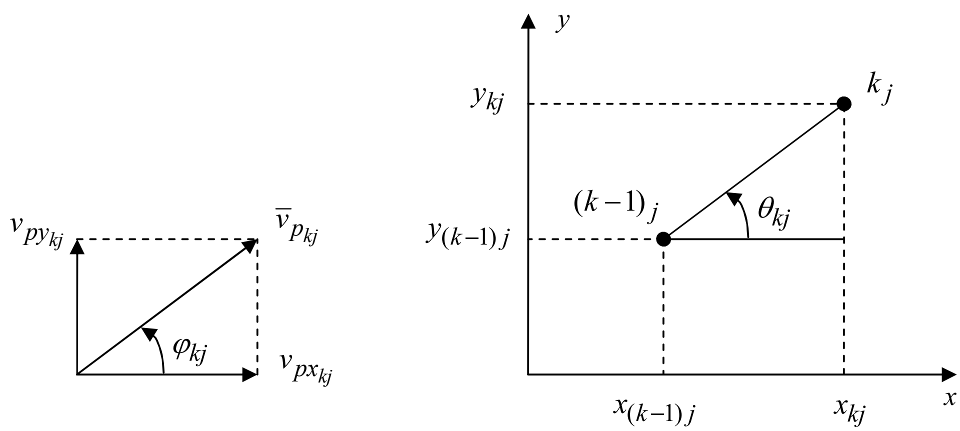

Figure 3

The above condition is equal to :

$$\tan(\varphi_{kj}) = \tan(\theta_{kj}) \tag{2}$$

yields:

$$\frac{v_{yp_{kj}}}{v_{xp_{kj}}} = \frac{y_{kj} - y_{(k-1)j}}{x_{kj} - x_{(k-1)j}} = m_{kj} \tag{3}$$

m_{kj} is noted as slope the segment defined by the points $(k-1)_j$ and k_j (string).

From the relation (1) we deduce that :

$$v_{yp_{kj}} = \frac{v_{y(k-1)j} + p_{kj}v_{y_{kj}}}{1+p_{kj}} \quad \text{and} \quad v_{yp_{kj}} = \frac{v_{y(k-1)j} + p_{kj}v_{y_{kj}}}{1+p_{kj}} \tag{4}$$

Thus, relation (1) becomes an equation with the independent variable „ p_{kj} ”:

$$\frac{v_{y(k-1)j} + p_{kj}v_{y_{kj}}}{v_{x(k-1)j} + p_{kj}v_{xkj}} = m_{kj} \tag{5}$$

the solution

$$p_{kj} = \frac{-v_{y(k-1)j} + m_{kj} v_{x(k-1)j}}{v_{ykj} - m_{kj} v_{xkj}} \tag{6}$$

by having the weight p_{kj} ,the relationship (1) is used to calculate the velocity $\bar{v}_{pkj}$ and its module:

$$\left|\bar{v}_{p_{kj}}\right| = v_{p_{kj}} = \sqrt{\frac{\left(v_{x(k-1)j} + p_{kj} v_{xkj}\right)^2 + \left(v_{y(k-1)j} + p_{kj} v_{ykj}\right)^2}{\left(1 + p_{kj}\right)^2}} \tag{7}$$

Remarks:

A numerical calculation carried out in accordance with the relationship (6) can lead to the following critical situations:

(a) $m_{kj} = \frac{v_{ykj}}{v_{xkj}} \neq \frac{v_{y(k-1)j}}{v_{x(k-1)j}}$ (the denominator= 0)

(b) $m_{kj} = \frac{v_{ykj}}{v_{xkj}} = \frac{v_{y(k-1)j}}{v_{x(k-1)j}}$ $\left(\frac{0}{0}\right)$

(c) $p_{kj} = -1$ (denominator=0 in the formulae (7))

In the situation (a) the weight p_{kj} becomes infinite, and the relation (7) equals:

$$v_{p_{kj}} = \sqrt{\lim_{p_{kj} \to +\infty} \frac{\left(v_{x(k-1)j} + p_{kj} v_{xkj}\right)^2 + \left(v_{y(k-1)j} + p_{kj} v_{ykj}\right)^2}{\left(1 + p_{kj}\right)^2}} = \\ = \sqrt{v_{xkj}^2 + v_{ykj}^2} \tag{8}$$

In situation (b) the relationship (6) presents an indetermination, type $\left(\frac{0}{0}\right)$, which amounts by applying the rule of l'Hôspital in relation to the variable m_{kj} :

$$p_{kj} = -\frac{v_{x(k-1)j}}{v_{xkj}} \tag{9}$$

If the above value is different from "-1", the relationship (7) obviously leads to a finite amount of it.
If, however, p_{kj} from the relation (9) has the "-1"value,it will be taken into account the fact that v_{pkj} velocity must be finite, so the relationship (7) will present an indetermination type $\left(\frac{0}{0}\right)$ compared to the variable p_{kj} . Applying the rule of the l'Hôspital, it issuccessively obtained:

$$v_{p_{kj}} = \sqrt{\lim_{p_{kj} \to -1} \frac{\left(v_{x(k-1)j} + p_{kj} v_{xkj}\right)^2 + \left(v_{y(k-1)j} + p_{kj} v_{ykj}\right)^2}{\left(1 + p_{kj}\right)^2}} = \\ = \sqrt{\lim_{p_{kj} \to -1} \frac{\left(v_{x(k-1)j} + p_{kj} v_{xkj}\right) v_{xkj} + \left(v_{y(k-1)j} + p_{kj} v_{ykj}\right) v_{ykj}}{\left(1 + p_{kj}\right)^2}} = \\ = \sqrt{v_{xkj}^2 + v_{ykj}^2} \tag{10}$$

the same as in the relation (8).

Therefore, we can determine the velocity v_{pkj} (k=1,2,...,n) in all situations.

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Furthermore, the problem is reduced to the calculation of the tangential effort of friction developed on a plate plan by a current which is parallel to it,and having in the immediate vicinity of the boundary layer the velocity v_{pkj}.

2. THE CALCULATION OF THE TANGENTIAL EFFORT OF FRICTION AND OF THE FRICTION DRAG ON A SHIP

We consider the turbulent boundary layer created on a flat plate by a stream of fluid parallel to the plate,with the velocity „ v_0 ”. Kármán’s integral equation associated with the exponent law „1/*n*” of the velocity distribution, within the boundary layer is reduced to a simple differential equation of the form [2]:

$$\rho v_0^2 \frac{d\delta}{dx} \cdot \frac{n}{(n+1)(n+2)} = \tau_0 , \qquad (11)$$

where:

ρ - fluid density;
x - the direction of current fluid flow;
δ – the thickness of the boundary layer corresponding to an abscise (x);
τ_0 - Tangential friction effort on plate item built on the abscise "x";
n - a natural number $n \in \{7, 9, 10\}$, dependent on the degree of turbulence of the flow.

As regarding the effort of friction „ τ_0 ”, it is determined by the relationship (2):

$$\tau_0 = 0,028 \left(\frac{n}{n+1} \right)^{1,75} \left(\frac{\nu}{v_0 \delta} \right)^{0,25} \rho v_0^2 \qquad (12)$$

where υ is their kinematics viscosity of the fluid.
Substituting „ τ_0 ” in the above differential equation (11) and then integrating it, it is obtained:

$$\delta^{1,25} = 0,028(n+2) \left(\frac{n}{n+1} \right)^{0,75} \left(\frac{\nu}{v_0} \right) x + C \qquad (13)$$

The constant integration „*C*’ is determined in the condition:
- for x=0, $\delta=\delta_0$ and so

$$\delta^{1,25} = 0,028(n+2) \left(\frac{n}{n+1} \right)^{0,75} \left(\frac{\nu}{v_0} \right)^{0,25} + \delta_0^{1,25} \qquad (14)$$

Hence:

$$\delta^{0,25} = \left(\delta^{1,25} \right)^{0,2} = \\ = \left[0,028(n+2) \left(\frac{n}{n+1} \right)^{0,75} \left(\frac{\nu}{v_0} \right)^{0,25} + \delta_0^{1,25} \right]^{0,2} \qquad (15)$$

The effort tangential of friction „ τ_0 ” becomes:

$$\tau_0 = \frac{0,028 \left(\frac{n}{n+1} \right)^{1,75} \left(\frac{\nu}{v_0} \right)^{0,25} \rho v_0^2}{\left[0,028(n+2) \left(\frac{n}{n+1} \right)^{0,75} \left(\frac{\nu}{v_0} \right)^{0,25} x + \delta_0^{1,25} \right]^{0,2}} \qquad (16)$$

In the following we will expose the arguments that allow the extension of the validity of the relation (16) in the case of the boundary layer on a surface curve (the surface of the nautical careen):

• as resulting from chapter1 the surface of the careen was replaced by a union of flat panels with the height $\frac{T}{m}$ and length (measured in horizontal plane) equal to the length of strings $(k-1)_j - k_j$ (Figure 2);

• along such a panel the thickness of the boundary layer is measured in the plane of the water line L_j, in a normal direction on the string $(k-1)_j - k_j$ (obviously, if the water line L_j is located in the area of high plating, then the normal line is rigorously included in the plan line L_j);

• the direction „x" in the relationship (16) equals, in the case of the studied panel, (x_*) the segment-oriented direction $(k-1)_j - k_j$;

• the velocity v_0 becomes v_{pkj} ;

• $\delta = \delta_{(k-1)j}$; $\delta_{0j} = 0, (\forall j = 1,2,...,m)$

Substituting the tangential effort of friction τ_0 with the symbol „τ_{kj}", the relationship (16) becomes:

$$\tau_{kj} = \frac{0,028\left(\frac{n}{n+1}\right)^{1,75}\left(\frac{\nu}{v_{pkj}}\right)^{0,25}\rho v_{pkj}^2}{\left[0,028(n+2)\left(\frac{n}{n+1}\right)^{0,75}\left(\frac{\nu}{v_{pkj}}\right)^{0,25}x_x + \delta_{(k-1)j}^{1,25}\right]^{0,2}} = \frac{\alpha_{kj}}{(a_{kj}x_x + b_{kj})^{0,2}} \tag{17}$$

(the corect α_{kj} , a_{kj} , b_{kj} is obvious).

Norming l_{kj} the length of the string $\overline{(k-1)j-kj}$, the average amount of the tangential effort of friction $\overline{\tau}_{kj}$, becomes:

$$\overline{\tau}_{kj} = \frac{1}{l_{kj}}\int_0^{l_{kj}} \frac{\alpha_{kj}}{(a_{kj}x_x + b_{kj})^{0,2}}dx_x = \frac{\alpha_{kj}}{l_{kj}a_{kj}}\int_{b_{kj}}^{a_{kj}l_{kj}+b_{kj}} \frac{du}{u^{0,2}}$$
$$= 1,25\frac{\alpha_{kj}}{l_{kj}a_{kj}}\left[\left(a_{kj}l_{kj} + b_{kj}\right)^{0,8} - l_{kj}^{\;0,8}\right] \tag{18}$$

The force of friction on the studied panel is :

$$F_{f_{kj}} = \overline{\tau}_{kj} A_{kj} \tag{19}$$

The projection of the force in the transverse of the ship will represent the friction drag developed by the panel „k_j":

$$R_{f_{kj}} = F_{f_{kj}} \cos\theta_{kj}$$
$$\cos\theta_{kj} = \frac{x_{kj}}{l_{kj}} \tag{20}$$

(see fig. 2, 3).

If we sum the friction drag both from the port and starboard side panel which are adjacent to the water line L_j, then we obtain :

$$R_{f_j} = 2\sum_{k=1}^{n} R_{f_{kj}} , \tag{21}$$

By adding the relationship (21) compared to „j", it results the for calculus equation of the friction drag with the Appendix:

$$R_f = \sum_{j=1}^{m} R_{f_j} \tag{22}$$

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