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ANALYTICAL MODEL OF A MATRIX HEAT EXCHANGER WITH LONGITUDINAL HEAT CONDUCTION IN THE MATRIX

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Abstract: *The paper presents an analytical method for the solution of the problem of a fixed matrix heat exchanger with axial heat conduction within the matrix. The small parameter method and Laplace transform have been applied. A general solution has been obtained for the unsteady state in the form of function series, using single and double convolutions of functions, as well as a particular solution for both the uniform and non-uniform initial temperature of the matrix and for an arbitrary function of the fluid temperature at the inlet. Particular solutions have been used in the study of the matrix dynamics in determining dynamic characteristics for the standard input signals in the form of: Dirac δ pulse, Heaviside function and the function of sinusoidal variable temperature of the fluid at the inlet. The results obtained both illustrate and enable the assessment of the effect of axial heat conduction in the matrix on the dynamic properties of the heat exchanger.*
Keywords: *mathematical model, matrix heat exchanger, longitudinal heat conduction*

1 INTRODUCTION

The development of the theory of mathematical modeling of matrix heat exchangers has progressed in two parallel directions, with the use of either numerical or analytical methods. The development of the numerical methods and computer technology, made it possible to solve even very complex mathematical models of matrix heat exchangers, including the non-linear ones.
Analytical methods, on the other hand, allow for more thorough physical interpretation of the process, are more useful in the control and automatic control theory, and the solutions they offer are usually more convenient in terms of application. A matrix heat exchanger used in air-conditioning systems is usually an element with the greatest inertia, and hence having a significant effect on the entire system dynamics.
Thorough investigation of the dynamic properties of the matrix heat exchanger makes it possible to find an analytical solution of its model based on an integer transform. The solution, in turn, makes it possible to determine dynamic characteristics, the order of inertia, time constant and operator transmittance-for the step change of the function of fluid temperature at the inlet and the spectrum transmittance-for periodically variable function of temperature of the fluid at the inlet. An effective analytical solution of a matrix heat exchanger is also most beneficial in a simulation model of an air-conditioning system using a heat exchanger of this type. This paper presents an alternative and more general analytical method of solving the matrix heat exchanger problem taking into account the effect of longitudinal heat conduction in the matrix for fixed exchanger, non-uniform initial temperature of the matrix and an arbitrary function of the temperature of fluid at the inlet. The Laplace transform and one of the small parameter methods have been used. The particular results obtained have been used to study the dynamics of the matrix type heat exchanger.

2. THE FORM OF THE PROBLEM

The fluid and the matrix of a heat exchanger form two separate thermodynamic systems, an open and a closed one, respectively. To each of these systems the general energy balance equation applies. For the open system it has the form:

$$\rho c_p\left(\frac{\partial t}{\partial \tau}+\vec{w}\nabla t\right)=\nabla(\lambda\nabla t)+\dot{q}_v+\frac{\partial p}{\partial \tau}+\vec{w}\nabla p+\mu\Phi_v \tag{1}$$

while for the closed system, it is assumed that $\vec{w}=0$.

As a geometrical model of the matrix the spatial regular Brenner's model has been adopted, in which the matrix is treated as a pseudohomogenous medium with a specified porosity ε, specific surface of heat exchange S and adequately defined physical constants.

Porosity of the matrix is determined by the following equation:

$$\varepsilon=\frac{dV_f}{dV_c} \tag{2}$$

and it denotes a proportion of the free space filled with fluid in the total matrix volume, while the specific surface of heat exchange is defined by the following relationship:

$$S=\frac{dA}{dV_c} \tag{3}$$

and it denotes the surface area of heat exchange per unit of the total matrix volume. Using the general balance of energy Eq. 1 and the adopted geometrical model of the matrix, and taking into account that for the matrix $w_s = 0$, $D_p/D_\tau=0$, and $\Phi_v=0$, one obtains the following general system of equations of the balance of energy for the fluid and the matrix:

$$\rho_f c_f \varepsilon\left(\frac{\partial t_f}{\partial \tau}+\vec{w}\nabla t_f\right)=\varepsilon\nabla(\lambda_f\nabla t_f)+\alpha S(t_s-t_f)$$
$$+\varepsilon\left(\frac{\partial p}{\partial \tau}+\vec{w}_f\nabla p+\mu\Phi_v\right) \tag{4}$$

$$\rho_s c_s(1-\varepsilon)\frac{\partial t_s}{\partial \tau}=(1-\varepsilon)\nabla(\lambda_s\nabla t_s)+\alpha S(t_f-t_s) \tag{5}$$

The equations are coupled through the term of convective heat exchange, which can be interpreted also as an internal heat source. In further considerations the computational model of the matrix heat exchanger depicted in Fig. 1 and the following assumptions have been adopted, [1], [3]:

(a) the matrix is treated as a homogenous medium with a specified porosity;
(b) both the fluid and matrix temperatures are only a function of the geometrical coordinate in the direction of the fluid flow and time;
(c) the heat exchange between the fluid and the matrix takes place by convection (influence of radiation may be taken into account through modification of the convective heat-transfer coefficient);
(d) coefficients and numbers characterizing the heat exchange process are the same in volume and constant in time;
(e) thermophysical properties of the fluid and the matrix are the same in volume and constant in time;
(f) the velocity of the fluid flow through the matrix is the same in the cross-section and longitudinally and constant in time;
(g) the process is isobaric;
(h) there are no heat losses to the environment;
(i) dissipation of kinetic energy in the fluid is considered negligible;
(j) accumulation and conduction of heat in the fluid are considered negligible (justified when fluid is, e.g., air).

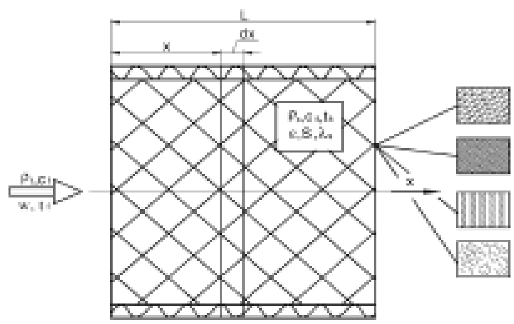

Figure1. Computational model of a matrix heat exchanger

Taking into account the above assumptions the system of the energy balance equations (4) and (5) for the fluid and matrix is simplified as follows [5], [7]:

$$\rho_f c_f \varepsilon w_f \frac{\partial t_f}{\partial x} = \alpha S\left(t_s - t_f\right) \tag{6}$$

$$\rho_s c_s (1-\varepsilon)\frac{\partial t_s}{\partial \tau} = (1-\varepsilon)\lambda_s \frac{\partial^2 t_s}{\partial x^2} + \alpha S\left(t_f - t_s\right) \tag{7}$$

Equations 6 and 7 are supplemented with the following initial and boundary conditions:

$$\tau = 0\,,\ t_s(x,0) = t_o(x), \tag{8}$$

x = 0

$$\rho_f c_f \varepsilon w_f \left[t_f(0,\tau) - t_i(\tau)\right] = \alpha_o (1-\varepsilon) t_s \left[(0,\tau) - t_i(\tau)\right], \tag{9}$$

$$\lambda_s \left.\frac{\partial t_s}{\partial x}\right|_{x=0} = \alpha_o \left[t_s(0,\tau) - t_i(\tau)\right] \tag{10}$$

x = L,

$$\rho_f c_f \varepsilon w_f \alpha_o \left[t_f(L,\tau) - t_f(L\tau)\right] = -\left[(1-\varepsilon)\alpha_o + \rho_f c_f \varepsilon w_f\right]\lambda_s \left.\frac{\partial t_s}{\partial x}\right|_{x=L} \quad , \tag{11}$$

where $t_i(\tau)$ denotes the function of temperature of the fluid at the inlet. The boundary condition (9) denotes the equality of the change of convective transfer stream on the boundary surface x=0 and the convective exchange stream, condition (10) is a type 3 boundary condition for the boundary surface x = 0, while relation (11) combines both conditions analogous to the aforementioned ones but for x = L and it has come to existence as a result of elimination of the unknown of fluid temperature at the inlet from these equations [6].
For generality of the solutions the following dimensionless parameters have been introduced:

– dimensionless coordinates:

$$\xi = \frac{x}{L},\ \eta = \frac{\tau}{\tau_o} \tag{12}$$

– dimensionless temperatures:

$$T_s(\xi,\eta) = \frac{t_s(\xi,\eta) - t_{f\min}}{t_{f\max} - t_{f\min}},\ T_f(\xi,\eta) = \frac{t_f(\xi,\eta) - t_{f\min}}{t_{f\max} - t_{f\min}} \tag{13}$$

where: t_{fmin}, t_{fmax} denote the minimum and maximum temperature of the fluid in the fluid–matrix system, respectively.
As a consequence of the introduction of dimensionless parameters the following dimensionless numbers are obtained:

$$\Lambda = \frac{\alpha S L}{\rho_f c_f \varepsilon w_f}, \tag{14}$$

$$\Pi = \frac{\alpha S \tau_o}{\rho_s c_s (1-\varepsilon)}, \tag{15}$$

$$K_s = \frac{(1-\varepsilon)\lambda_s \alpha S}{\left(\rho_f c_f \varepsilon w_f\right)^2}, \tag{16}$$

where parameter Λ denotes a dimensionless length of the matrix, while parameter K_s is a measure of the longitudinal heat conduction in the matrix.
Introduction of dimensionless parameters (12)–(16) to Eqs. 6, 7, 8, 9, 10 and 11 leads to the following dimensionless form of the problem:
– a system of the energy balance equations

$$\frac{1}{\Lambda}\frac{\partial T_f}{\partial \xi} = T_s - T_f \tag{17}$$

$$\frac{1}{\Pi}\frac{\partial T_s}{\partial \eta} = \frac{K_s}{\Lambda^2}\frac{\partial^2 T_s}{\partial \xi^2} + T_f - T_s, \tag{18}$$

– initial and boundary conditions

$$\eta = 0 \,,\; T_s(\xi,0) = T_o(\xi), \tag{19}$$
$$\xi = 0 \,,\; T_f(0,\eta) = HT_s(0,\eta) + (1-H)T_i(\eta), \tag{20}$$
$$\left.\frac{\partial T_s}{\partial \xi}\right|_{\xi=0} = Bi_o\left[T_s(0,\eta)T_i(0,\eta)\right], \tag{21}$$
$$\xi = 1 \,,\; \left.\frac{\partial T_s}{\partial \xi}\right|_{\xi=0} = \frac{Bi_o}{1+H}\left[T_f(1,\eta) - T_s(1,\eta)\right] \tag{22}$$

where:
Biot number
$$Bi_o = \frac{\alpha_o L}{\lambda_s} \tag{23}$$
$$H = \frac{\alpha_o(1-\varepsilon)}{\rho_f c_f \varepsilon w_f} \tag{24}$$
In a particular case, when heat penetration in boundary surfaces is neglected, which is admissible taking into account the fact that these surfaces constitute only an insignificant proportion of the total surface of heat exchange in the matrix, the boundary conditions (20)–(22) at $\alpha_0 = 0$, and hence $Bi_0 = 0$ and $H = 0$, become simplified to:
$$\xi = 0 \;\; T_f(0,\eta) = T_i(\eta), \tag{25}$$
$$\left.\frac{\partial T_s}{\partial \xi}\right|_{\xi=0} = 0\,, \tag{26}$$
$$\xi = 1 \,,\; \left.\frac{\partial T_s}{\partial \xi}\right|_{\xi=1} = 0\,, \tag{27}$$
The system of Eqs. 17 and 18 taking into account longitudinal heat conduction in the matrix, with the initial condition (19) and adiabatic boundary conditions (25)–(27) is a subject of the solution. It is a model describing a one-off process in which the matrix accumulates energy (charging) or undergoes a reverse process (discharging).

3 THE SOLVING METHOD

In solving the problem (17)–(19), (25)–(27) it has been assumed that parameter K_s described by the formula (16), present in the energy balance equation for the matrix (18), and taking into account longitudinal heat conduction in the matrix is a small parameter. This assumption is correct for real matrix type heat exchangers used in air conditioning systems using air as fluid-then $K_s < 1$. Therefore, in order to solve the boundary problem under analysis one of the small parameter methods [5] has been utilized. A solution is sought to the system of Eqs. 17 and 18 in the form of the following series:
$$T_f(\xi,\eta) = \sum_{n=0}^{\infty} K_s^n T_{fn}(\xi,\eta), \tag{28}$$
$$T_s(\xi,\eta) = \sum_{n=0}^{\infty} K_s^n T_{sn}(\xi,\eta) \tag{29}$$
As a result, functions $T_{fn}(\xi,\eta)$ and $T_{sn}(\xi,\eta)$ are defined, solving successively the following systems of equations with initial and boundary conditions, obtained following the substitution of the series (28) and (29) into the Eqs. 17, 18, 19, 25, 26 and 27 and after comparing the terms at the successive powers K_s:
-for n=0
$$\frac{1}{\Lambda}\frac{\partial T_{f_o}}{\partial \xi} = T_{s_o} - T_{f_o} \tag{30}$$
$$\frac{1}{\Pi}\frac{\partial T_{s_o}}{\partial \eta} = T_{f_o} - T_{s_o} \tag{31}$$

$$T_{f_o}(0,\eta) = T_i(\eta),\tag{32}$$

$$T_{s_o}(\xi,0) = T_o(\xi),\tag{33}$$

-for n=1, 2, 3…

$$\frac{1}{\Lambda}\frac{\partial T_{fn}}{\partial \xi} = T_{sn} - T_{fn},\tag{34}$$

$$\frac{1}{\Pi}\frac{\partial T_{sn}}{\partial \eta} = \frac{1}{\Lambda^2}\frac{\partial^2 T_{s(n-1)}}{\partial \xi^2} + T_{fn} - T_{sn},\tag{35}$$

$$T_{fn}(0,\eta) = 0,\tag{36}$$

$$T_{sn}(\xi,0) = 0,\tag{37}$$

The solution is an iteration, since subsequent terms of the series (28) and (29) are determined one after another. Thus, the solution of the boundary value problem of the second order is reduced to solving a number of classical problems of the first order, and while for n = 0 the problem contains a heterogeneity in the boundary conditions, and the system of equations is homogenous, for n = 1,2,3,... the situation is reverse. At this point it needs to be said that-as a result of the method employed-the solution obtained neglects boundary conditions (26) and (27) related to the matrix temperature gradient as redundant for first-order differential equations, therefore the solution will not be accurate for $\xi \Rightarrow 0$ and $\eta \Rightarrow 0$.

The problem (30)–(33) is a classical problem, which is solved with the Laplace transform method, successively according to variables ξ and η.

The problem (34)–(37) is also solved by the Laplace transform method, successively according to variables ξ and η.

CONCLUSION

The method of the solution of a matrix heat exchanger taking into account the axial heat conduction in the matrix presented in this paper is highly accurate and requires only a short time for computations. The analytical nature of the solution enables a wide range of applications. This refers in particular to the studies of dynamic properties of matrix heat exchangers and the assessment of the effect of axial heat conduction in the matrix on the changes in the temperatures of the fluid and the matrix, which is of particular importance especially in the case of metal matrices. Further applications include the use of the solution presented as a constituent element of a mathematical model of a rotary energy regenerator and a cyclic regenerator, as well as for simulation of the operation of air conditioning systems with matrix type heat exchangers.

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