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STIFFNESS EVALUATION OF SOME QUASI-ISOTROPIC FIBRE-REINFORCED COMPOSITE LAMINATES

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Abstract: *The paper presents the stiffness evaluation of various unidirectional quasi-isotropic fibre-reinforced composite laminates based on epoxy resin. The laminates are subjected to off-axis loading systems. The elastic constants have been determined. In order to obtain equal stiffness in all off-axis loading systems, a composite laminate have to approximates isotropy by orientation of plies in several or more directions in-plane. A comparison between the elastic properties of these quasi-isotropic laminates is presented. Tensile-shear interaction in a fibre-reinforced composite laminate occurs only if the off-axis loading system does not coincide with the main axes of a single lamina or if the laminate is not balanced.*
Keywords : *laminates, composites, fibres, stiffness, laminae*

1. INTRODUCTION

In order to obtain a composite laminate is necessary to bond together various unidirectional laminae. For a specific application, the laminae orientation is for a great importance since the laminate exhibit different properties for different fibres orientations [1-4]. In practice, the most encountered composite laminates are:

- Balanced angle-ply laminates;
- Symmetric angle-ply laminates;
- Anti-symmetric laminates;
- Cross-ply laminates;
- Cross ply, symmetric laminates;
- Quasi-isotropic laminates.

A quasi-isotropic laminate exhibits equal stiffness and approximates isotropy in all off-axis loading systems. Some authors relate that quasi-isotropic laminates are not completely isotropic showing that the laminate characteristics can differ perpendicular to the laminate [5]. It is well known that composite laminates with aligned reinforcement are very stiff along the fibres, but also very weak transverse to the fibres direction. This fact is more obvious in the case of advanced composite laminates reinforced with anisotropic carbon or aramid fibres but this is fair accurate for glass fibre-reinforced laminates also [6-10].

2. THEORETICAL BACKGROUND

A composite laminate (fig. 1) formed by a number of unidirectional reinforced laminae subjected regarding to the loading scheme presented in fig. 2 is considered. The elasticity law for a unidirectional lamina K can be written as following:

$$\begin{bmatrix} \sigma_{xx\,K} \\ \sigma_{yy\,K} \\ \tau_{xy\,K} \end{bmatrix} = \begin{bmatrix} r_{11\,K} & r_{12\,K} & r_{13\,K} \\ r_{12\,K} & r_{22\,K} & r_{23\,K} \\ r_{13\,K} & r_{23\,K} & r_{33\,K} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx\,K} \\ \varepsilon_{yy\,K} \\ \gamma_{xy\,K} \end{bmatrix}, \tag{1}$$

where r_{ijK} represent the transformed stiffness, σ_{xxK} , σ_{yyK} are the mean stresses of K lamina on x- respective y-axis and τ_{xyK} represent the mean shear stress of K lamina against the x-y coordinate system.

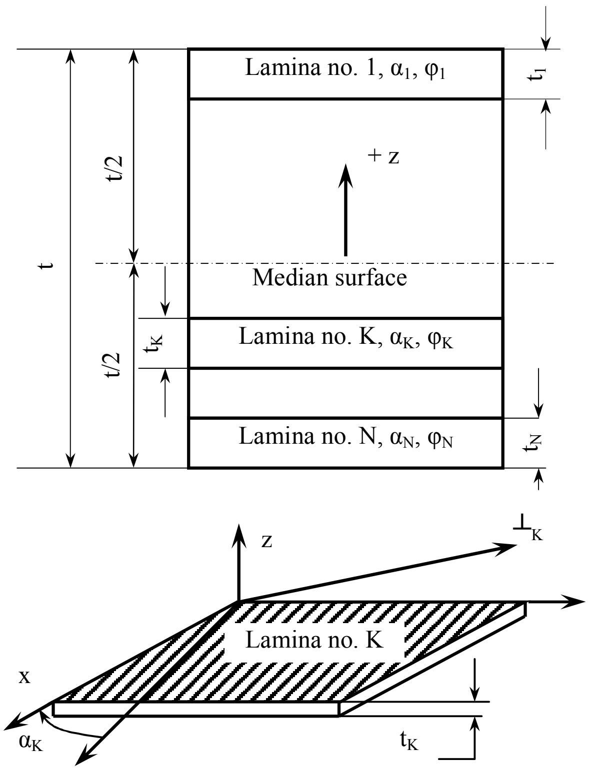

Figure 1: Constructive scheme of a composite laminate

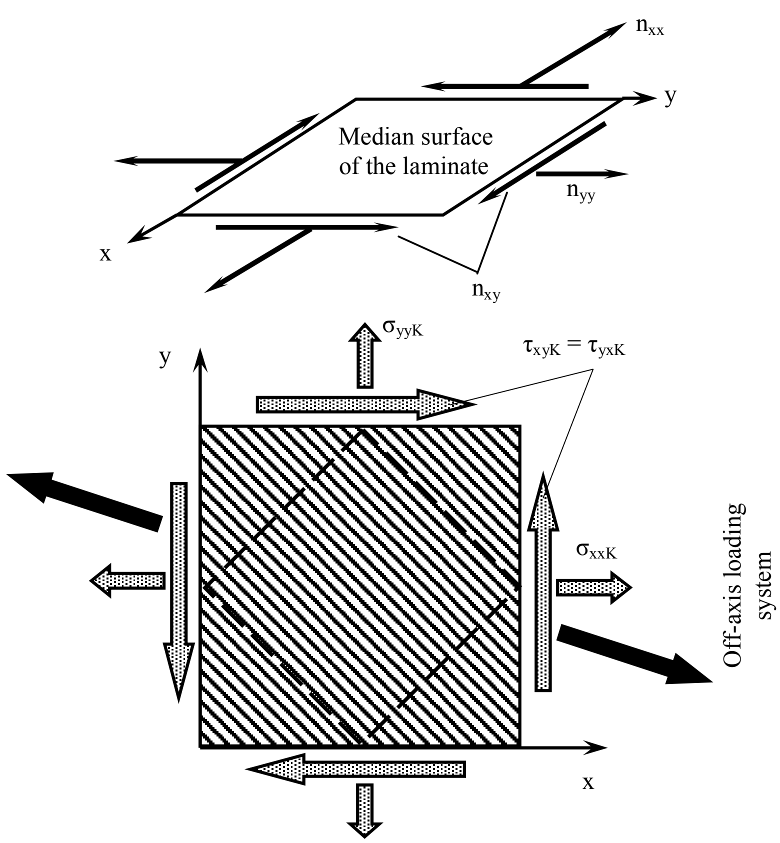

Figure 2: Off-axis loading scheme of a composite laminate

The balance equations of the laminate structure can be computed as following:

$$n_{xx} = \underline{\sigma}_{xx} \cdot t = \sum_{K=1}^{N} \left(\sigma_{xxK} \cdot t_K \right) = \sum_{K=1}^{N} n_{xxK} \,, \tag{2}$$

$$n_{yy} = \underline{\sigma}_{yy} \cdot t = \sum_{K=1}^{N} \left(\sigma_{yyK} \cdot t_K \right) = \sum_{K=1}^{N} n_{yyK} \,, \tag{3}$$

$$n_{xy} = \underline{\tau}_{xy} \cdot t = \sum_{K=1}^{N} \left(\tau_{xyK} \cdot t_K \right) = \sum_{K=1}^{N} n_{xyK} \,, \tag{4}$$

where n_{xx} , n_{yy} are the normal forces on the unit length of the laminate on x- respective y-axis and n_{xy} represents the shear force, in plane, on the unit length of the laminate against the x-y coordinate system. $\underline{\sigma}_{xx}$, $\underline{\sigma}_{yy}$ are the normal stresses on x- respective y-axis of the laminate, $\underline{\tau}_{xy}$ represent the shear stress of the laminate against the x-y coordinate system. t_K , t represent the thickness of the K lamina respective the laminate thickness, n_{xxK} , n_{yyK} are forces on the unit length of K lamina on x- respective y-axis directions and n_{xyK} is the shear force in plane, on the unit length of K lamina against the x-y coordinate system.
Beside the balance equations, the geometric conditions must be also determined, to compute the stresses. For composite laminates these conditions imply that all laminas are bonded together and withstand, in a specific point, the same strains ε_{xx}, ε_{yy}, γ_{xy} as well as for the entire laminate:

$$\begin{aligned} \varepsilon_{xxK} &= \varepsilon_{xx} \,, \\ \varepsilon_{yyK} &= \varepsilon_{yy} \,, \\ \gamma_{xyK} &= \gamma_{xy} \,. \end{aligned} \tag{5}$$

According to equations (1)-(5), the elasticity law for the whole laminate can be computed [1]:

$$\begin{bmatrix} \underline{\sigma}_{xx} \\ \underline{\sigma}_{yy} \\ \underline{\tau}_{xy} \end{bmatrix} = \begin{bmatrix} \sum\limits_{K=1}^{N}\left(r_{11K} \cdot \frac{t_K}{t} \right) & \sum\limits_{K=1}^{N}\left(r_{12K} \cdot \frac{t_K}{t} \right) & \sum\limits_{K=1}^{N}\left(r_{13K} \cdot \frac{t_K}{t} \right) \\ \sum\limits_{K=1}^{N}\left(r_{12K} \cdot \frac{t_K}{t} \right) & \sum\limits_{K=1}^{N}\left(r_{22K} \cdot \frac{t_K}{t} \right) & \sum\limits_{K=1}^{N}\left(r_{23K} \cdot \frac{t_K}{t} \right) \\ \sum\limits_{K=1}^{N}\left(r_{13K} \cdot \frac{t_K}{t} \right) & \sum\limits_{K=1}^{N}\left(r_{23K} \cdot \frac{t_K}{t} \right) & \sum\limits_{K=1}^{N}\left(r_{33K} \cdot \frac{t_K}{t} \right) \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}, \tag{6}$$

where the laminate stiffness $\underline{r}_{ij}$ are:

$$\underline{r}_{ij} = \sum_{K=1}^{N} \left(r_{ijK} \cdot \frac{t_K}{t} \right). \tag{7}$$

In other words, the laminate elasticity law becomes:

$$\begin{bmatrix} \underline{\sigma}_{xx} \\ \underline{\sigma}_{yy} \\ \underline{\tau}_{xy} \end{bmatrix} = \begin{bmatrix} \underline{r}_{11} & \underline{r}_{12} & \underline{r}_{13} \\ \underline{r}_{12} & \underline{r}_{22} & \underline{r}_{23} \\ \underline{r}_{13} & \underline{r}_{23} & \underline{r}_{33} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}, \tag{8}$$

Computing the laminate strains as a function of stresses, the expressions (8) become:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \underline{c}_{11} & \underline{c}_{12} & \underline{c}_{13} \\ \underline{c}_{12} & \underline{c}_{22} & \underline{c}_{23} \\ \underline{c}_{13} & \underline{c}_{23} & \underline{c}_{33} \end{bmatrix} \cdot \begin{bmatrix} \underline{\sigma}_{xx} \\ \underline{\sigma}_{yy} \\ \underline{\tau}_{xy} \end{bmatrix}, \tag{9}$$

where $\underline{c}_{ij}$ represents the laminate transformed compliance tensor. This tensor can be computed as a function of elastic constants. Thus [6, 10]:

$$E_x = \frac{1}{\underline{c}_{11}}; \quad G_{xy} = \frac{1}{\underline{c}_{33}}; \quad \upsilon_{xy} = -E_x \cdot \underline{c}_{12}. \tag{10}$$

It is obvious that the laminate will exhibit different elastic constants if the loading system is applied at a randomly angle, Φ, to the x-y coordinate system. The compounds of the transformed compliance tensor can be determined in the following way [1, 3, 11]:

$$\underline{c}_{11} = \frac{cos^4 \alpha}{E_{II}} + \frac{sin^4 \alpha}{E_{\perp}} + \frac{1}{4} \cdot \left(\frac{1}{G_{II\,\perp}} - \frac{2 \cdot \upsilon_{\perp\,II}}{E_{II}} \right) \cdot sin^2 2\alpha \,, \tag{11}$$

$$\underline{c}_{22} = \frac{sin^4 \alpha}{E_{II}} + \frac{cos^4 \alpha}{E_{\perp}} + \frac{1}{4} \cdot \left(\frac{1}{G_{II\,\perp}} - \frac{2 \cdot \upsilon_{\perp\,II}}{E_{II}} \right) \cdot sin^2 2\alpha \,, \tag{12}$$

$$\underline{c}_{33} = \frac{\cos^2 2\alpha}{G_{II\perp}} + \left(\frac{1}{E_{II}} + \frac{1}{E_{\perp}} + \frac{2 \cdot \upsilon_{\perp II}}{E_{II}} \right) \cdot \sin^2 2\alpha \,, \tag{13}$$

$$\underline{c}_{12} = \frac{1}{4} \cdot \left(\frac{1}{E_{II}} + \frac{1}{E_{\perp}} - \frac{1}{G_{II\perp}} \right) \cdot \sin^2 2\alpha - \frac{\upsilon_{\perp II}}{E_{II}} \cdot \left(\sin^4 \alpha + \cos^4 \alpha \right), \tag{14}$$

$$\underline{c}_{13} = \left(\frac{2}{E_{\perp}} + \frac{2 \cdot \upsilon_{\perp II}}{E_{II}} - \frac{1}{G_{II\perp}} \right) \cdot \sin^3 \alpha \cdot \cos \alpha - \left(\frac{2}{E_{II}} + \frac{2 \cdot \upsilon_{\perp II}}{E_{II}} - \frac{1}{G_{II\perp}} \right) \cdot \cos^3 \alpha \cdot \sin \alpha \,, \tag{15}$$

$$\underline{c}_{23} = \left(\frac{2}{E_{\perp}} + \frac{2 \cdot \upsilon_{\perp II}}{E_{II}} - \frac{1}{G_{II\perp}} \right) \cdot \cos^3 \alpha \cdot \sin \alpha - \left(\frac{2}{E_{II}} + \frac{2 \cdot \upsilon_{\perp II}}{E_{II}} - \frac{1}{G_{II\perp}} \right) \cdot \sin^3 \alpha \cdot \cos \alpha \,. \tag{16}$$

3. RESULTS

The output data have been generated using the software developed by Hull and Clyne from Materials Science Department at Cambridge University, UK [6]. The quasi-isotropic laminates taken into account in the numerical analysis present following plies sequence: [30/-30/90], [0/45/-45/90] and [0/18/36/54/72/90/-18/-36/-54/-72]. General input data are: fibres volume fraction $\varphi = 0.5$ in all cases, plies thickness $t = 0.125$ mm and off-axis loading systems varies between 0° and 90°. For the glass fibre-reinforced laminates, following data have been used [12]:

- $E_M = 3.9$ GPa;
- $E_F = 73$ GPa;
- $\upsilon_M = 0.38$;
- $\upsilon_F = 0.25$;
- $G_M < 10$ GPa;
- $G_F < 25$ GPa.

For HM carbon fibres-reinforced laminates, following data have been used [12]:

- $E_M = 3.9$ GPa;
- $E_{\parallel} > 300$ GPa;
- $E_{\perp} < 100$ GPa;
- $\upsilon_M < 0.5$;
- $\upsilon_F < 0.4$;
- $G_M < 25$ GPa;
- $G_F < 50$ GPa.

The elastic constants E_{xx}, E_{yy}, G_{xy} as well as the Poisson ratio υ_{xy} are presented in figs. 3 – 5.

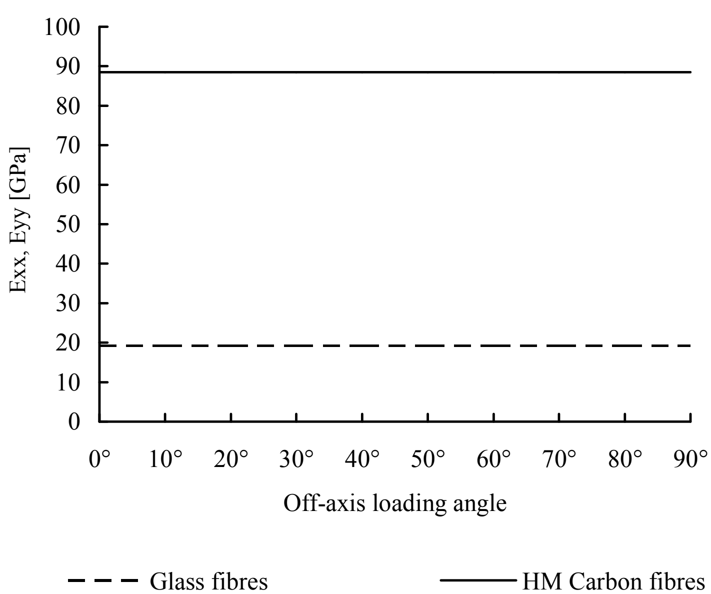

Figure 3: Distribution of Young moduli E_{xx} and E_{yy} of three epoxy based fibre-reinforced quasi-isotropic composite laminates: [30/-30/90], [0/45/-45/90] and [0/18/36/54/72/90/-18/-36/-54/-72]

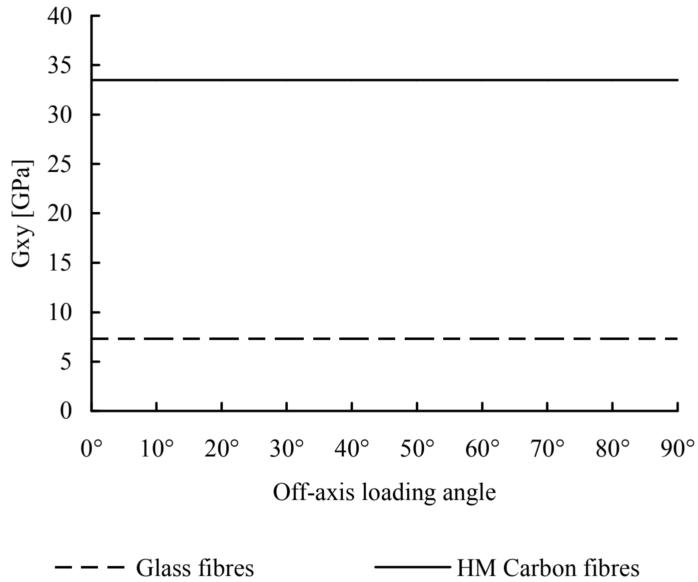

Figure 4: Distribution of shear modulus G_{xy} of three epoxy based fibre-reinforced quasi-isotropic composite laminates: [30/-30/90], [0/45/-45/90] and [0/18/36/54/72/90/-18/-36/-54/-72]

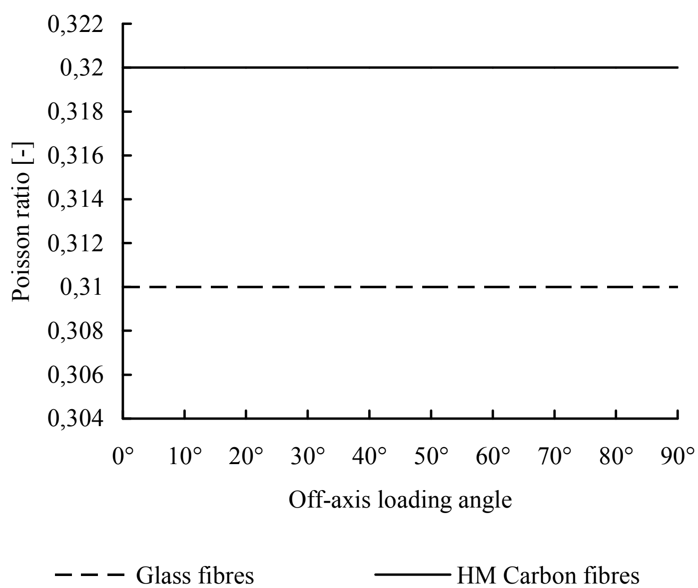

Figure 5: Distribution of Poisson ratio υ_{xy} of three epoxy based fibre-reinforced quasi-isotropic composite laminates: [30/-30/90], [0/45/-45/90] and [0/18/36/54/72/90/-18/-36/-54/-72]

4. CONCLUSIONS

Under off-axis loading, normal stresses produce shear strains and of course normal strains. Shear stresses produce normal strains as well as shear strains. Usually, tensile-shear interaction is also present in common laminates but does not occur if the loading system is applied along the main axes of a single lamina or if a laminate is a quasi-isotropic one. From the stiffness point of view there is no difference if a quasi-isotropic composite laminate is designed with three layers (for instance [30/-30/90]), four layers (e.g. [0/45/-45/90]) or ten layers (for instance [0/18/36/54/72/90/-18/-36/-54/-72]).

For the strength point of view the quasi-isotropic composite laminate with plies sequence [0/18/36/54/72/90/-18/-36/-54/-72] can withstand at increased loadings than the quasi-isotropic composite laminate with plies sequence [30/-30/90]. The values of Young moduli E_{xx} and E_{yy} as well as the shear modulus G_{xy} for HM carbon fibres laminates are more than four times greater than the Young moduli and shear modulus of the glass fibres ones. The distribution of Poisson ratio υ_{xy} are close in both cases of reinforcement.

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