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MECHANICAL APPLICATION OF THE THEORY OF DODECAHEDRON AND HEXAHEDRON

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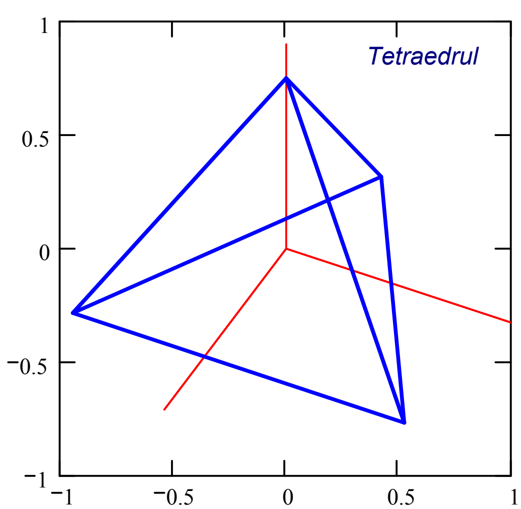
De mai bine de **2300 de ani reflectăm** la **elementele** lui **EUCLID / PLATON !,** găsind încă o fereastră deschisă către noi reflexii !

Tipul	f	v	m	$p = \frac{2m}{v}$	$q = \frac{2m}{f}$
Tetraedrul	**4**	**4**	**6**	**3**	**3**
Hexaedrul (cubul)	**6**	**8**	**12**	**3**	**4**
Octoedrul	**8**	**6**	**12**	**4**	**3**
Dodecaedrul	**12**	**20**	**20**	**2**	**3.333**
Icosaedrul	**20**	**12**	**30**	**5**	**3**

$ii := 1..5$

$$p_{ii-1} := \frac{2\cdot ge_{ii-1,2}}{ge_{ii-1,1}} \qquad q_{ii-1} := \frac{2\cdot ge_{ii-1,2}}{ge_{ii-1,0}}$$

$$ge := \begin{pmatrix} 4 & 4 & 6 \\ 6 & 8 & 12 \\ 8 & 6 & 12 \\ 12 & 20 & 20 \\ 20 & 12 & 30 \end{pmatrix} \qquad p = \begin{pmatrix} 3 \\ 3 \\ 4 \\ 2 \\ 5 \end{pmatrix} \qquad q = \begin{pmatrix} 3 \\ 4 \\ 3 \\ 3.333 \\ 3 \end{pmatrix}$$



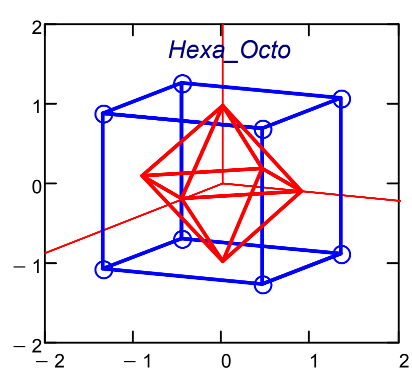
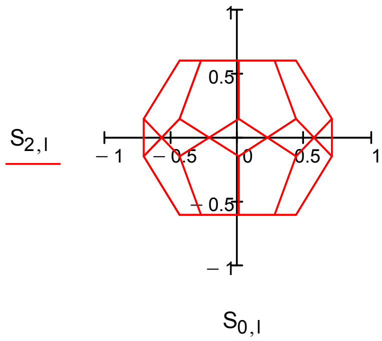
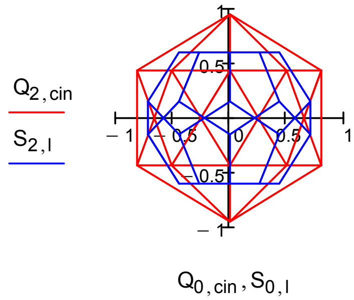


Fig. 1 Octo înscris în hexaedru

De astă dată, problema care ne-a frământat a fost ridicată de numărul segmentelor funcției poligonale antrenate la desenarea în MathCAD a poliedrelor, întru figurarea tuturor muchiilor. La început, în 99, desenarea cele **30** de muchii ale icosaedrului ne-a reclamat **65** de segmente, acum, după zece ani, revennd ca din întâmplare, de altfel ca şi prima experienţă, şi grupând muchiile în polare şi cele ale brîului icosaedral, am **redus acest număr la 41**, **dar încă departe de bornă !** $\mathrm{col}(Q) = 65$ **Fig.4, 8, 10** $\mathbf{PA} := \mathrm{EXT3}(\mathbf{PN}, \mathbf{PB}, \mathbf{PS})$ $\mathrm{col}(\mathbf{PA}) = 41$ **Fig.4, 6 !**





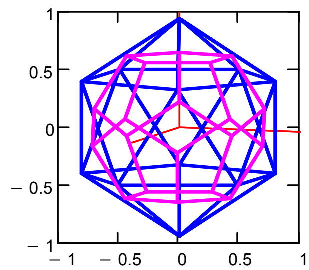


Fig. 2 Dodeca înscris în icosaedru

$\mathrm{col}(Q) = 65$ **Fig.4, 8, 10**

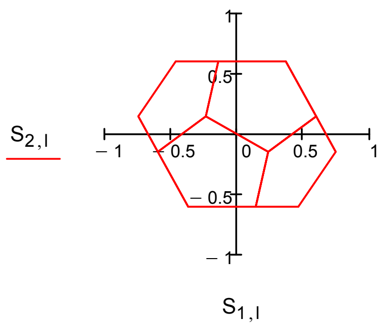
$\mathbf{PA} := \mathrm{EXT3}(\mathbf{PN}, \mathbf{PB}, \mathbf{PS})$

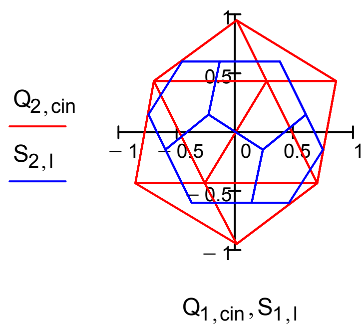
$\mathrm{col}(\mathbf{PA}) = 41$ **Fig.4, 6**

$ja := 0 .. 40$

$\mathbf{PA} := \mathrm{EXT3}(\mathbf{PN}, \mathbf{PB}, \mathbf{PS})$

$\mathrm{col}(\mathbf{PA}) = 41$





$cin := 0 .. pin_{2,19} + 5$ $\quad Pin_{u,a} := pin_{a,u\cdot 19}$

$Pin = \begin{pmatrix} 0 & 1 & 2 \\ 57 & 58 & 59 \end{pmatrix}$

$pin_{2,19} + 5 = 64$

$\mathrm{col}(Q) = 65$ $\quad \mathrm{col}(\mathbf{Q}) = 65$

Dodekaeder *l = 0 ... 61*
Ikosaeder *cin = 0 ... 64*

ICOSAEDRUL A .xmcd

$c(q) := \cos(q)$ $\quad s(q) := \sin(q)$ $\quad (u \;\; a \;\; i \;\; ax) := (0..1 \;\; 0..2 \;\; 0..5 \;\; 0..5)$ $\quad \mathrm{unu}(n) := \mathrm{identity}(n)$ $\quad g(q) := q\cdot \mathrm{deg}^{-1}$

$\mathrm{EXT}(A, B) := \mathrm{augment}(A, B)$ $\quad \mathrm{EXT3}(A, B, C) := \mathrm{augment3}(A, B, C)$ $\quad \mathrm{col}(A) := \mathrm{cols}(A)$ $\quad \mathrm{lin}(A) := \mathrm{rows}(A)$

$\alpha := \frac{\pi}{5}$ $\quad \chi := \mathrm{acos}\left[(2\cdot s(\alpha))^{-1}\right]$ $\quad Rs := (2\cdot s(\chi))^{-1}$ $\quad Rp := (2\cdot s(\alpha))^{-1}$ $\quad \beta := \mathrm{acos}(Rp)$ $\quad \beta - \chi = 0$ $\quad g(\alpha) = 36$

$(g(\beta) \;\; \chi - \beta) = (31.717 \;\; 0)$ $\quad H_0 := Rp\cdot \tan(\beta)$ $\quad H_1 := \sqrt{\frac{3}{4} - Rp^2\cdot(1 - c(\alpha))^2}$ $\quad RS := H_0 + H_1\cdot .5$

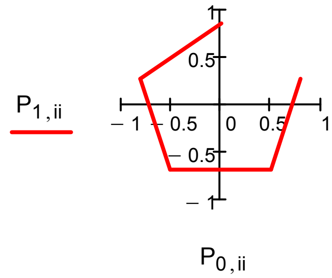
$Rs - RS = 0$ $\quad H^T = (0.526 \;\; 0.851)$ $\quad P^{\langle i+1\rangle} := \mathbf{RZ}(2\cdot\alpha\cdot i)\cdot \mathbf{RX}(-\chi)^{\langle 1\rangle} + \mathbf{E}^{\langle 2\rangle}\cdot Rs$ $\quad P^{\langle u\cdot 11\rangle} := \mathbf{E}^{\langle 2\rangle}\cdot Rs\cdot(-1)^u$

$P^{\langle i+6\rangle} := \mathbf{RZ}\left[2\cdot\alpha\cdot\left(i + \frac{1}{2}\right)\right]\cdot \mathbf{RX}(\chi)^{\langle 1\rangle} - \mathbf{E}^{\langle 2\rangle}\cdot Rs$ $\quad (g(\chi) \;\; g(\beta) \;\; g(\alpha)) = (31.717 \;\; 31.717 \;\; 36)$

$\begin{pmatrix} H_0 & Rs \\ H_1 & Rp \end{pmatrix} = \begin{pmatrix} 0.526 & 0.951 \\ 0.851 & 0.851 \end{pmatrix}$ $\quad ls := 0..11$ $\quad (i \;\; ip \;\; jb) := (0..4 \;\; 0, 2..4 \;\; 0..9)$ $\quad Nb_{u,i} := 1 + 5\cdot u + i$

$x := 0..5$ $\quad (i \;\; in \;\; jn) := (0..4 \;\; 0..19 \;\; 0..9)$ $\quad jb := 0..10$ $\quad jc := 0..22$ $\quad Nc_{u+2\cdot i} := Nb_{u,i}$ $\quad Nc_{10} := Nc_0$

$\mathrm{col}(P) = 12$ $\quad Pv^{\langle u\rangle} := P^{\langle u\cdot 11\rangle}$ $\quad Pv = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0.951 & -0.951 \end{pmatrix}$ $\quad \mathrm{EXT}\left(P^{\langle 0\rangle}, P^{\langle 11\rangle}\right) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0.951 & -0.951 \end{pmatrix}$



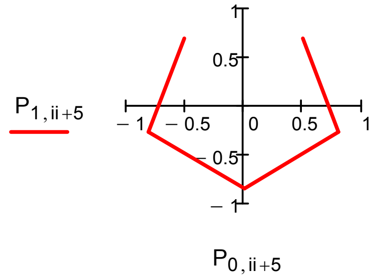
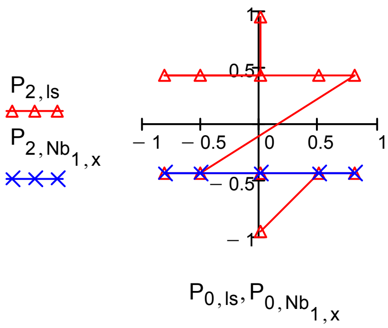
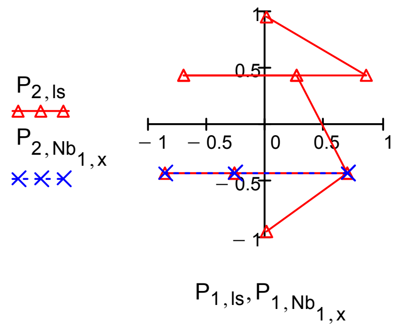


Fig. 3 Pentagoanele din vecinătățile polilor





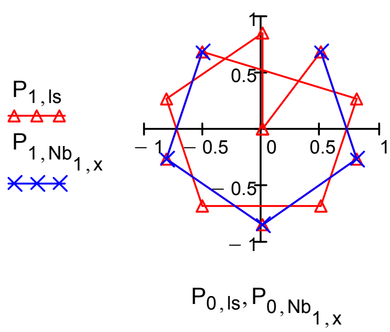


Fig. 4 $Pb^{\langle jb\rangle} := P^{\langle Nc_{jb}\rangle}$ $\quad \mathbf{P} := \mathbf{B}\cdot P$

$Nc_{u+2\cdot i} := Nb_{u,i}$ $\quad Nc_{10} := Nc_0$ $\quad jb := 0..10$ $\quad jv := 0..14$

$Nb = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \end{pmatrix}$ $\quad PO^{\langle jv+15\cdot u\rangle} := P^{\langle V_{u,jv}\rangle}$

$\mathbf{PO} := \mathbf{B}\cdot PO$ $\quad PN^{\langle jv\rangle} := PO^{\langle jv\rangle}$ $\quad PS^{\langle jv\rangle} := PO^{\langle jv+15\rangle}$

$PB^{\langle jb\rangle} := P^{\langle Nc_{jb}\rangle}$ $\quad \mathbf{PN} := \mathbf{B}\cdot PN$ $\quad \mathbf{PS} := \mathbf{B}\cdot PS$ $\quad \mathbf{PB} := \mathbf{B}\cdot PB$

$\mathbf{PA} := \mathrm{EXT3}(\mathbf{PN}, \mathbf{PB}, \mathbf{PS})$ $\quad \mathrm{col}(\mathbf{PA}) = 41$ $\quad ja := 0..40$

$\mathrm{col}(PO) = 30$

$(i\ \ in\ \ jn) := (0..4\ \ 0..19\ \ 0..9)$

$pin_{a,in} := 3\cdot in + a$ $\quad It^{\langle i\rangle} := (0\quad i+1)^T$

$It^{\langle i+15\rangle} := (10 - i\quad 11)^T$ $\quad Q^{\langle pin_{2\cdot u,in}\rangle} := P^{\langle It_{0,in}\rangle}$

$Q^{\langle pin_{1,in}\rangle} := P^{\langle It_{1,in}\rangle}$ $\quad Q^{\langle pin_{2,19}+1+js\rangle} := P^{\langle Ich_{js}\rangle}$

$\mathbf{Q} := \mathbf{B}\cdot Q$ $\quad \mathrm{col}(Q) = 65$

$Ich := (6\quad 1\quad 5\quad 10\quad 6)^T$ $\quad (js\ \ g) := (0..4\ \ 0..11)$

$It^{\langle 2\cdot i+5+u\rangle} := (i+1\quad i+5+u)^T$ $\quad It_{1,5} := 10$

$cin := 0..pin_{2,19}+5$ $\quad Pin_{u,a} := pin_{a,u\cdot 19}$

$Pin = \begin{pmatrix} 0 & 1 & 2 \\ 57 & 58 & 59 \end{pmatrix}$ $\quad pin_{2,19} + 5 = 64$

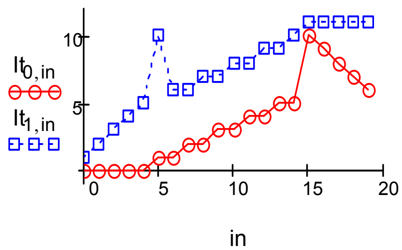
It =

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	0	0	0	0	0	1	1	2	2	3	3	4	4	5	5	10	9	8	7	6
1	1	2	3	4	5	10	6	6	7	7	8	8	9	9	10	11	11	11	11	11

$L_{in} := \left| P^{\langle It_{1,in}\rangle} - P^{\langle It_{0,in}\rangle} \right| - 1$ $\quad Ip^{\langle i\rangle} := (i+1\quad i+2)^T$ $\quad Ip_{1,4} := 1$ $\quad Ip^{\langle i+5\rangle} := (i+6\quad i+7)^T$ $\quad Ip_{1,9} := 6$

Ip =

	0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	1	7	8	9	10	6



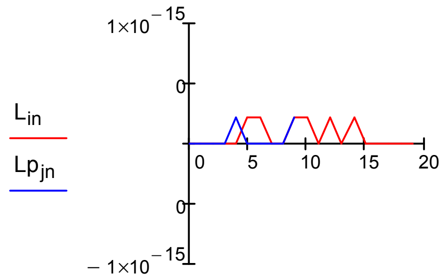


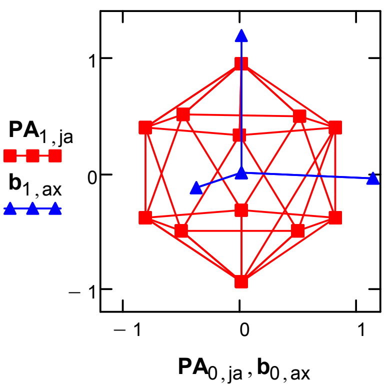
Fig. 5

$$Lp_{jn} := \left| P^{\langle lp_{1,jn} \rangle} - P^{\langle lp_{0,jn} \rangle} \right|^{in,jn} - 1$$

$Nc^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	1	6	2	7	3	8	4	9	5	10	1

$$V := \begin{pmatrix} 0 & 1 & 2 & 0 & 2 & 3 & 0 & 3 & 4 & 0 & 4 & 5 & 0 & 5 & 1 \\ 11 & 6 & 7 & 11 & 7 & 8 & 11 & 8 & 9 & 11 & 9 & 10 & 11 & 10 & 6 \end{pmatrix} \quad col(V) = 15$$



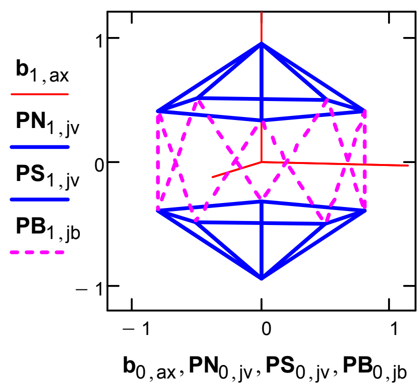


Fig. 6 Reformularea traseului

$PA := EXT3(PN, PB, PS) \quad col(PA) = 41$

$ku_{u,i} := 2 \cdot i + u \quad ku = \begin{pmatrix} 0 & 2 & 4 & 6 & 8 \\ 1 & 3 & 5 & 7 & 9 \end{pmatrix} \quad \kappa u := ku \quad \kappa u_{u,ip} := 2 \cdot ip + 1 - u \quad \kappa u = \begin{pmatrix} 1 & 2 & 5 & 6 & 9 \\ 0 & 3 & 4 & 7 & 8 \end{pmatrix}$

$pin_{a,in} := 3 \cdot in + a \quad cin := 0 .. pin_{2,19} + 5$

$pin_{2,19} + 5 = 64 \quad Q^{\langle pin_{2 \cdot u, in} \rangle} := P^{\langle It_{0,in} \rangle}$

$Q^{\langle pin_{1,in} \rangle} := P^{\langle It_{1,in} \rangle} \quad Q^{\langle pin_{2,19+1+i} \rangle} := P^{\langle Ich_i \rangle}$

$\mathbf{Q} := \mathbf{B} \cdot Q \quad \mathbf{P} := \mathbf{B} \cdot P$

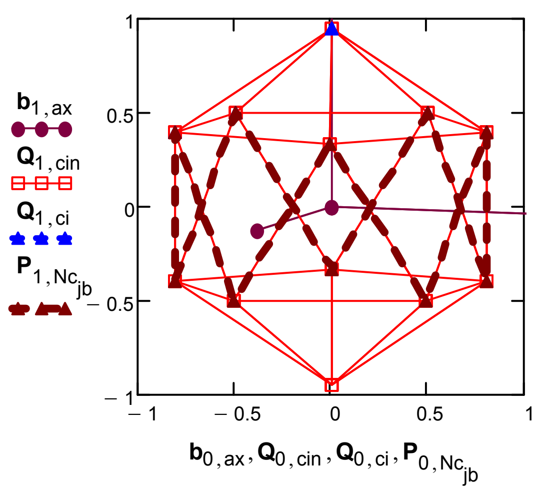


Fig. 7 Brîul **PB**
Polii PN & PS !

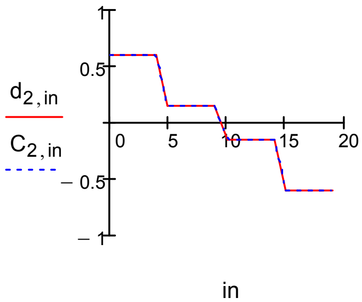
$$f := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 7 & 8 & 9 & 10 & 6 & 7 & 8 & 9 & 10 & 6 \\ 2 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 5 & 1 & 11 & 11 & 11 & 11 & 11 \end{pmatrix}$$

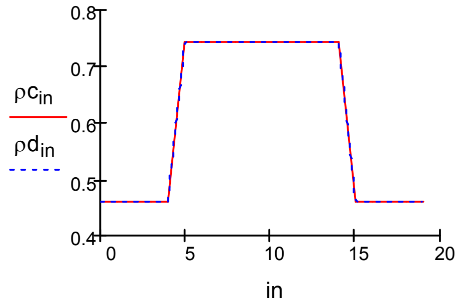
$$in := 0 .. 19 \quad C^{\langle in \rangle} := \frac{1}{3} \cdot \left(\sum_a P^{\langle f_{a,in} \rangle} \right) \quad \mathbf{C} := \mathbf{B} \cdot C \quad \mathbf{P} := \mathbf{B} \cdot P \quad r^{\langle i \rangle} := \mathbf{RZ}(i \cdot 2 \cdot \alpha)^{\langle 0 \rangle} \cdot Rp$$

$$d^{\langle i \rangle} := \mathbf{RZ}\left(\alpha + \frac{\pi}{2}\right) \cdot \left[\frac{2}{3} \cdot c(\alpha) \cdot r^{\langle i \rangle} + \left(Rs - \frac{2 \cdot H_0}{3} \right) \cdot \mathbf{E}^{\langle 2 \rangle} \right] \quad d^{\langle i+5 \rangle} := \mathbf{RZ}\left(\alpha + \frac{\pi}{2}\right) \cdot \left[\left(\frac{c(\alpha) \cdot 2}{3} + \frac{1}{3} \right) \cdot r^{\langle i \rangle} + \left(Rs - H_0 - \frac{H_1}{3} \right) \cdot \mathbf{E}^{\langle 2 \rangle} \right]$$

$$d^{\langle i+10 \rangle} := \mathbf{RZ}\left(\alpha \cdot 2 + \frac{\pi}{2}\right) \cdot \left[\left(\frac{c(\alpha) \cdot 2}{3} + \frac{1}{3} \right) \cdot r^{\langle i \rangle} + \left(-Rs + H_0 + \frac{H_1}{3} \right) \cdot \mathbf{E}^{\langle 2 \rangle} \right] \quad d^{\langle i+15 \rangle} := \mathbf{RZ}\left(\alpha \cdot \frac{4}{2} + \frac{\pi}{2}\right) \cdot \left[\frac{2}{3} \cdot c(\alpha) \cdot r^{\langle i \rangle} - \left(Rs - \frac{2 \cdot H_0}{3} \right) \cdot \mathbf{E}^{\langle 2 \rangle} \right]$$

$$\mathbf{d} := \mathbf{B} \cdot d \quad \rho c_{in} := \left| Pr \cdot C^{\langle in \rangle} \right| \quad \rho d_{in} := \left| Pr \cdot d^{\langle in \rangle} \right|$$





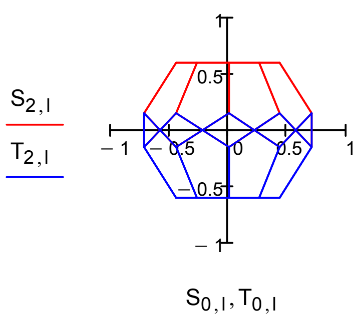
$$F := \begin{pmatrix} 0 & 0 & 1 & 2 & 3 & 4 & 15 & 15 & 16 & 17 & 18 & 19 \\ 1 & 5 & 6 & 7 & 8 & 9 & 16 & 10 & 11 & 12 & 13 & 14 \\ 2 & 10 & 11 & 12 & 13 & 14 & 17 & 6 & 7 & 8 & 9 & 5 \\ 3 & 6 & 7 & 8 & 9 & 5 & 18 & 11 & 12 & 13 & 14 & 10 \\ 4 & 1 & 2 & 3 & 4 & 0 & 19 & 16 & 17 & 18 & 19 & 15 \end{pmatrix}$$

$$h := 0 .. 5 \quad k_{i,h} := 5 \cdot h + i \quad ll := k_{4,5} + 1$$

$$l := 0 .. ll - 1 \quad S^{\langle k_{i,h} \rangle} := C^{\langle F_{i,h} \rangle} \quad T^{\langle k_{i,h} \rangle} := C^{\langle F_{i,h+6} \rangle}$$

$$S^{\langle ll+u \rangle} := C^{\langle 5 \cdot (u+1) \rangle} \quad S^{\langle ll+2+l \rangle} := T^{\langle l \rangle} \quad cols(S) = 62$$

$$\mathbf{S} := \mathbf{B} \cdot S \quad \mathbf{T} := \mathbf{B} \cdot T \quad l := 0 .. cols(S) - 1$$



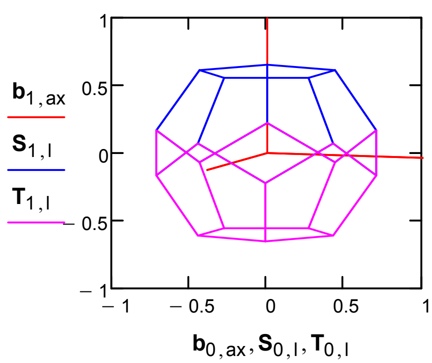


Fig. 8 Dodecaedrul

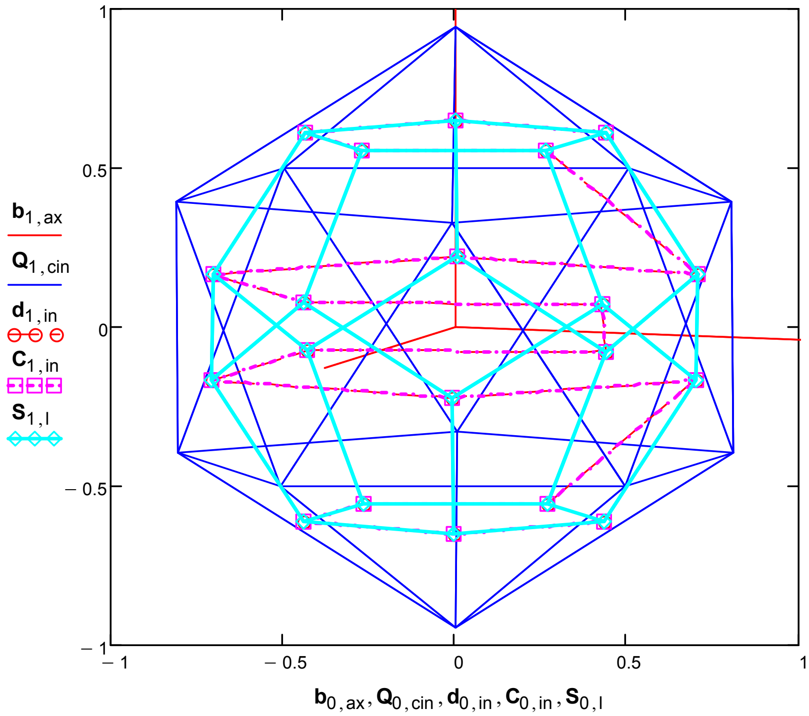


Fig. 9 Înscrierea dodeca în icosaedru

$$Cb^{\langle 5\rangle} := Cb^{\langle 0\rangle} \quad Cb^{\langle 11\rangle} := Cb^{\langle 6\rangle} \quad hx := 0 .. 12$$

$$Cb^{\langle hx+12\rangle} := Cb^{\langle Tc_{hx}\rangle} \quad cx := 12 .. 24$$

$$Cb^{\langle ax\rangle} := C^{\langle ax+5\rangle} \quad Cb^{\langle ax+6\rangle} := C^{\langle ax+10\rangle}$$

$$Tc := (1\ \ 7\ \ 2\ \ 8\ \ 3\ \ 9\ \ 4\ \ 10\ \ 5\ \ 11\ \ 0\ \ 6\ \ 1)^T$$

$$(\mathrm{lin}(Tc)\ \ \mathrm{col}(Cb)) = (13\ \ 25)$$

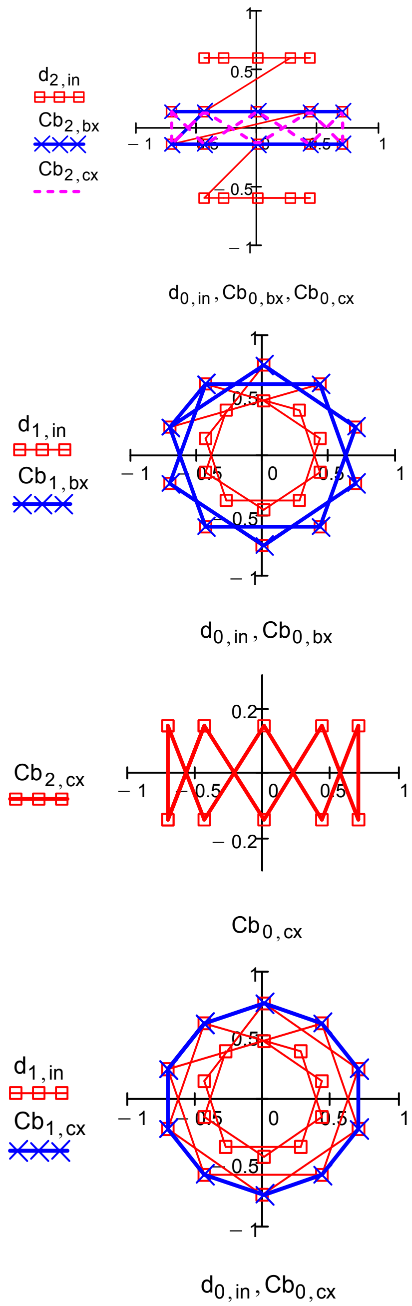
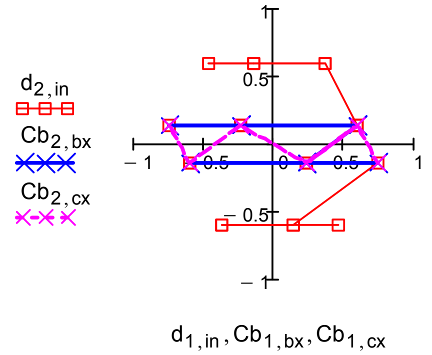


Fig. 10



$$F := \begin{pmatrix} 0 & 0 & 1 & 2 & 3 & 4 & 15 & 15 & 16 & 17 & 18 & 19 \\ 1 & 5 & 6 & 7 & 8 & 9 & 16 & 10 & 11 & 12 & 13 & 14 \\ 2 & 10 & 11 & 12 & 13 & 14 & 17 & 6 & 7 & 8 & 9 & 5 \\ 3 & 6 & 7 & 8 & 9 & 5 & 18 & 11 & 12 & 13 & 14 & 10 \\ 4 & 1 & 2 & 3 & 4 & 0 & 19 & 16 & 17 & 18 & 19 & 15 \end{pmatrix}$$

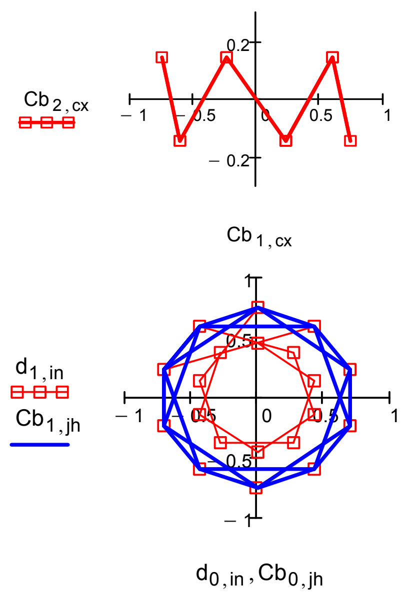
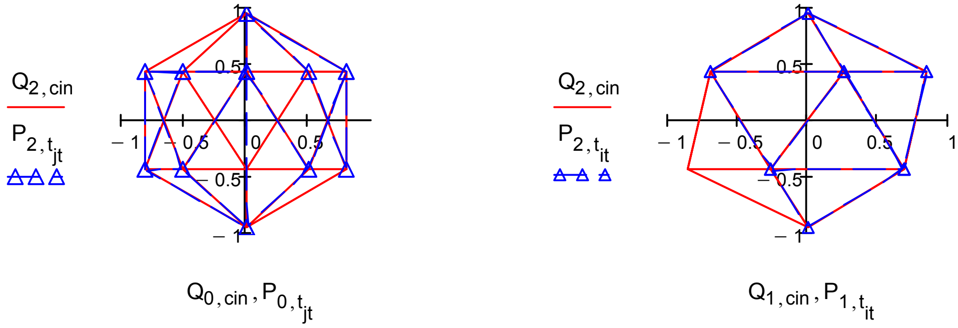


Fig. 11

Tentativa construirii cu **migală în Word**, a unui traseu mai "scurt" a rămasa fără răspuns în MathCAD, rețeaua muchiilor reclamând prin penta - incidențele polare, multiple reveniri / supratrasări a unora dintre ultimele 10 muchii!



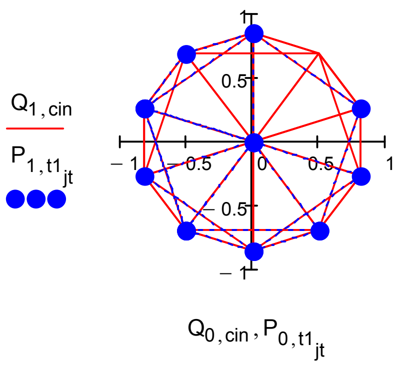


Fig. 12
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/ 16 : 36 ! → p. 8, Fig. 12

Am continuat suita reflexiei asaupra poliedrelor cu " începuturile seriei nemărginite" a însriptilităților, generate de simetria sferică !

DODECA şi înscrierea în ICOSAEDRU

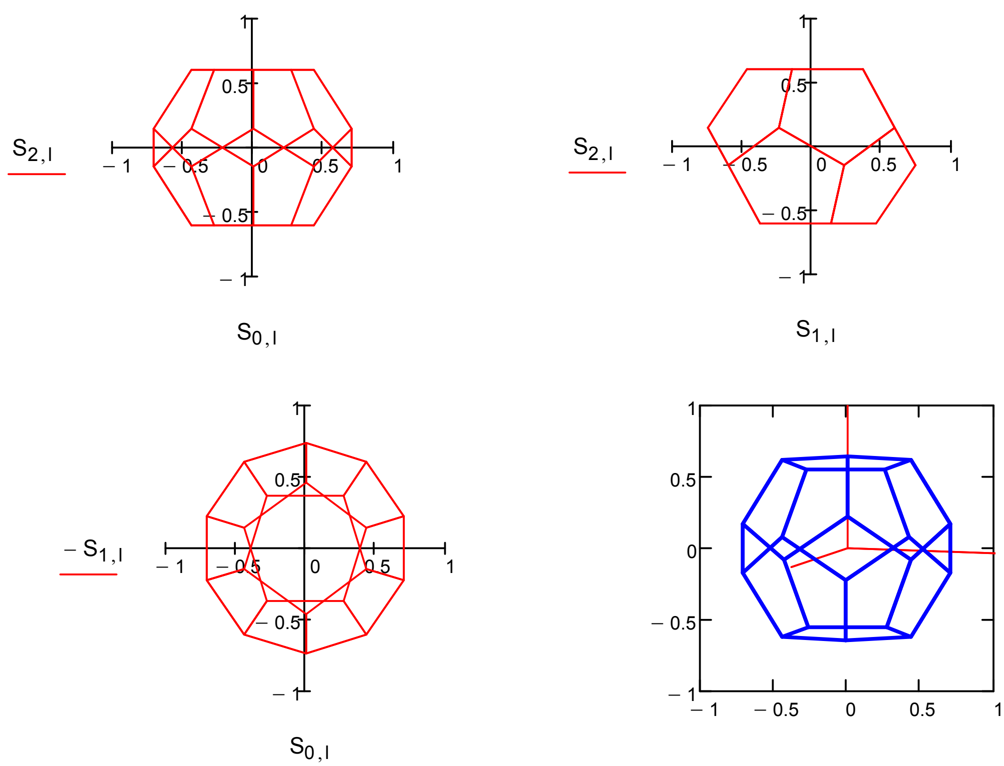
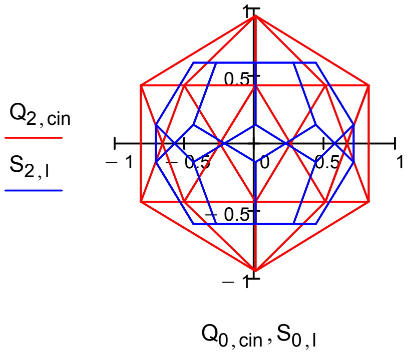
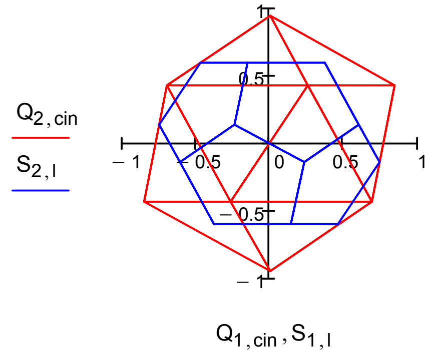


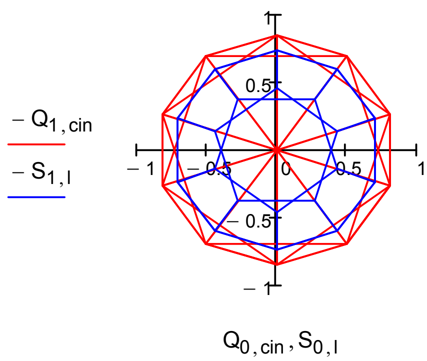
Fig. 13 Dodekaeder



Dodecaedrul l = 0 ... 61



Icosaedrul cin = 0 ... 64



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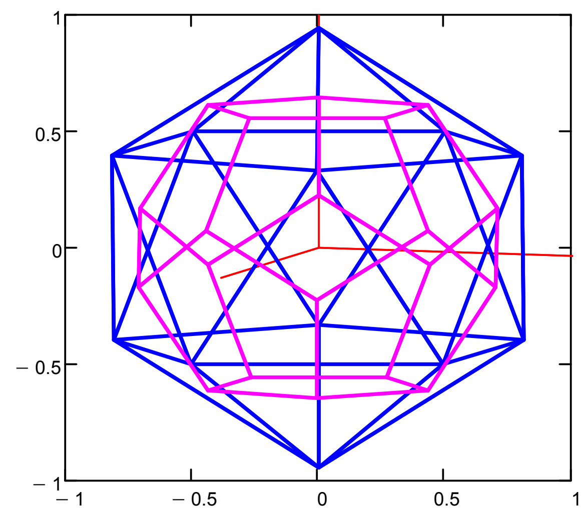
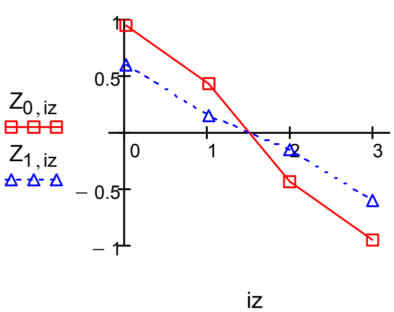


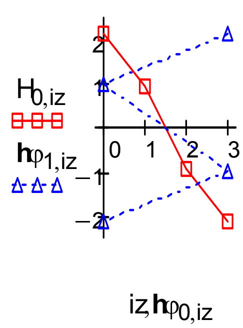
Fig. 14 Dodecaedrul însris în Icosaedru

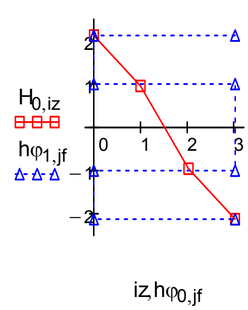
$\varphi := (1+\sqrt{5})\cdot 2^{-1}$ $\quad Z^{\langle iz \rangle} := \left(P_{2,Nz_{0,iz}} \quad d_{2,Nz_{1,iz}} \right)^T$ $\quad Z = \begin{pmatrix} 0.951 & 0.425 & -0.425 & -0.951 \\ 0.601 & 0.142 & -0.142 & -0.601 \end{pmatrix}$ $\quad jf := 0 .. 7$

$ka = \begin{pmatrix} 0 & 1 & 4 & 6 \\ 1 & 4 & 5 & 7 \end{pmatrix}$ $\quad H_{0,iz} := Z_{0,iz} \cdot (Z_{0,0})^{-1} \cdot (.5+\varphi)$ $\quad H = (2.118 \quad 0.947 \quad -0.947 \quad -2.118)$

$Jh = \begin{bmatrix} 1 & 0 \\ 0 & 2\cdot(1+2\cdot\varphi)^{-1} \end{bmatrix}$ $\quad h\varphi = \begin{pmatrix} 2.118 \\ 1 \\ -1 \\ -2.118 \end{pmatrix}$ $\quad h\varphi := Jh \cdot h\varphi$







$h\varphi := (1+2\cdot\varphi \quad 2)^T\, 2^{-1}$ $\quad h\varphi_{2+u} := -h\varphi_{1-u}$

$\mathbf{h}\varphi^{\langle iz \rangle} := (3 \quad h\varphi_{iz})^T$ $\quad h\varphi^{\langle 2\cdot iz \rangle} := \mathbf{h}\varphi^{\langle iz \rangle}$

$\mathbf{h}\varphi^{\langle 2\cdot u+1 \rangle} := \left[0 \quad h\varphi_{(2\cdot u+1)} \right]^T$ $\quad h\varphi_{u+iz\cdot 2} := \mathbf{h}\varphi_{u,iz}$

$h\varphi^{\langle 1 \rangle} := (0 \quad \mathbf{h}\varphi_{1,0})^T$ $\quad h\varphi^{\langle 3+2\cdot u \rangle} := \left[3\cdot(1-u) \quad (-1)^u \right]^T$

$h\varphi^{\langle 7\rangle} := \left(3 \quad \mathbf{h}\varphi_{1,3}\right)^T \quad Jh := \begin{bmatrix} 1 & 0 \\ 0 & 2\cdot(1+2\cdot\varphi)^{-1} \end{bmatrix}$

$\mathbf{h}\varphi = \begin{pmatrix} 3 & 0 & 3 & 0 \\ 2.118 & 1 & -1 & -2.118 \end{pmatrix}$

$h\varphi = \begin{pmatrix} 3 & 0 & 0 & 3 & 3 & 0 & 0 & 3 \\ 2.1 & 2.1 & 1 & 1 & -1 & -1 & -2.1 & -2.1 \end{pmatrix}$

$h\varphi := Jh\cdot h\varphi$

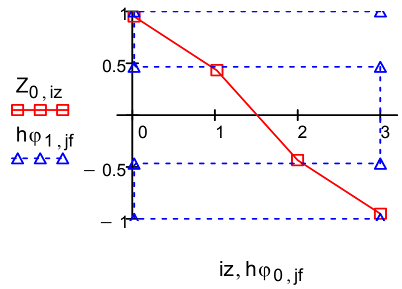


Fig. 15 Organizarea verticală a fețelor penta ale dodecaedrului !

PENTA - DECAGONUL - Grig p. 63, 64, incidențe ale numărului de aur □

I / Penta-Decagon . xmcd**,** ***Ma, 6 Oct.*** '09 / 21 : 07, ***Grig*** $\varphi := \left(1+\sqrt{5}\right)\cdot 2^{-1}$ $\left(1+\sqrt{5}\right)\cdot 2^{-1} = 1.618034$

$P := \begin{pmatrix} -1 & 1 & 1 & -1 & -1 & 0 \\ 0 & 0 & 2 & 2 & 0 & 0 \end{pmatrix}\cdot 2^{-1} \quad rs^{\langle 0\rangle} := (0 \quad \varphi)^T \quad (n \quad i) := (36 \quad 0..n) \quad R := \left(\varphi \quad 2^{-1} + \varphi^{-1}\right)^T$

$\alpha := \mathrm{acos}\left[(2\cdot\varphi)^{-1}\right] \quad p := 0..4 \quad \mathbf{e}(q) := (c(q) \quad s(q))^T \quad k_{a,i} := a\cdot(n+1) + i \quad KL_{u,a} := k_{a,u\cdot n}$

$KL = \begin{pmatrix} 0 & 37 & 74 \\ 36 & 73 & 110 \end{pmatrix} \quad r^{\langle i\rangle} := \mathbf{e}\left(i\cdot q_0\right)\cdot R_0 + P^{\langle 0\rangle} \quad r^{\langle k_{1,i}\rangle} := \mathbf{e}\left(\theta + i\cdot q_0\right)\cdot R_0 + P^{\langle 1\rangle} \quad r^{\langle k_{2,i}\rangle} := \mathbf{e}\left(\pi - i\cdot q_1\right)\cdot R_1$

$R^T = (1.618 \quad 1.118) \quad j := 0..KL_{1,2} \quad R_2 := \dfrac{R_0}{2\cdot c(\beta)} \quad R^T = (1.618 \quad 1.118 \quad 0.851) \quad \left(gd(\theta) \quad gd\left(\theta + n\cdot q_0\right)\right) = (108 \quad 180)$

$EXT\left(r^{\langle 0\rangle}, r^{\langle n\rangle}\right) = \begin{pmatrix} 1.118 & 0 \\ 0 & 1.539 \end{pmatrix} \quad EXT\left(r^{\langle k_{1,0}\rangle}, r^{\langle k_{1,n}\rangle}\right) = \begin{pmatrix} 0 & -1.118 \\ 1.539 & 0 \end{pmatrix} \quad r5^{\langle u\rangle} := P^{\langle u\rangle}$

$C := P^{\langle 0\rangle} + \mathbf{e}(3\cdot\beta)\cdot R_2 \quad Rp(q) := \begin{pmatrix} c(q) & -s(q) \\ s(q) & c(q) \end{pmatrix} \quad r5^{\langle p+1\rangle} := C + Rp\left(p\cdot\alpha + 2\cdot\beta - \dfrac{\pi}{2}\right)^{\langle 0\rangle}\cdot R_2 \quad C = \begin{pmatrix} 0 \\ 0.688 \end{pmatrix}$

$R5^{\langle 2\cdot ax+1\rangle} := \mathbf{e}\left(2\cdot\beta + ax\cdot\alpha - \dfrac{\pi}{2}\right)\cdot R_2 \quad is := 0..11 \quad R5^{\langle is\rangle} := R5^{\langle is\rangle} + C \quad V5^{\langle 0\rangle} := V5^{\langle 5\rangle}$

$\left(\left|rs^{\langle 0\rangle}\right| \quad \left|rs^{\langle 1\rangle}\right| - \varphi\right) = (2.044 \quad 0)$

Verificări, închiderii : $P^{\langle 0\rangle} + \mathbf{e}(3\cdot\beta)\cdot R_2 + Rp(0)^{\langle 1\rangle}\cdot R_2 - r^{\langle n\rangle} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad r^{\langle n\rangle} = \begin{pmatrix} 0 \\ 1.539 \end{pmatrix}$

$\mathbf{R5}^{\langle ax\rangle} := V5^{\langle ax\rangle} - C \quad rs^{\langle 0\rangle} + Rp(0)^{\langle 0\rangle} - V5^{\langle 4\rangle} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{e}(\theta) + P^{\langle 0\rangle} - r5^{\langle 4\rangle} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$V5 = \begin{pmatrix} -0.5 & 0.5 & 0.809 & 0 & -0.809 & -0.5 \\ 0 & 0 & 0.951 & 1.539 & 0.951 & 0 \end{pmatrix} \quad V5 - r5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$\mathbf{R5} = \begin{pmatrix} -0.5 & 0.5 & 0.809 & 0 & -0.809 & -0.5 \\ -0.688 & -0.688 & 0.263 & 0.851 & 0.263 & -0.688 \end{pmatrix}$

$\varphi := \left(1+\sqrt{5}\right)\cdot 2^{-1} \quad 1 + \varphi^{-1} - \varphi = 0 \quad \alpha := \mathrm{acos}\left[(2\cdot\varphi)^{-1}\right] \quad \theta := \pi - \alpha \quad gd(\alpha) = 72$

$q := (\alpha \quad \pi)^T\cdot n^{-1} \quad \alpha - 2\cdot\pi\cdot 5^{-1} = 0$

$A_{ax} := -2\cdot\beta + ax\cdot\alpha$

$gd\left(q^T\right) = (2 \quad 5) \quad \beta := \alpha\cdot 4^{-1} \quad gd(3\cdot\beta) = 54$

$gd\left(A^T\right) = (-36 \quad 36 \quad 108 \quad 180 \quad 252 \quad 324)$

$(c(\alpha) \quad c(\beta)) - \left(\dfrac{1}{2\cdot\varphi} \quad \dfrac{\varphi}{2\cdot R_2}\right) = (0 \quad 0)$

$(\alpha \quad \beta) - (4 \quad 1)\cdot\dfrac{\pi}{10} = (0 \quad 0)$

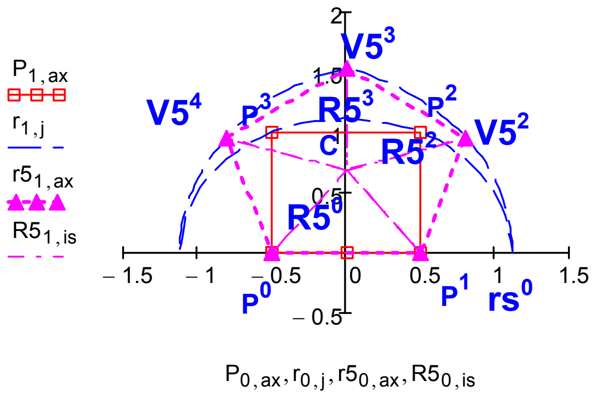


Fig. 16 Pentagonul

$$\varphi := \left(1+\sqrt{5}\right)\cdot 2^{-1} \qquad \left(1+\sqrt{5}\right)\cdot 2^{-1} = 1.618034$$

$$\left(\varphi \quad \varphi^2 \quad \varphi^3\right) - \left(\varphi^{-1} \quad 1 \quad \varphi\right)\cdot(\varphi+1) = (0 \quad 0 \quad 0)$$

$$s(\beta) - \frac{1}{2\cdot\varphi} = 0 \qquad s(3\cdot\beta) - \frac{\varphi}{2} = 0 \qquad c(4\cdot\beta) - \frac{1}{2\cdot\varphi} = 0$$

$$Rp(q) := \begin{pmatrix} c(q) & -s(q) \\ s(q) & c(q) \end{pmatrix}$$

$$\frac{Rp(\beta)^T}{\left(\sqrt{5}+1\right)}\cdot\begin{bmatrix} \sqrt{\left(\sqrt{5}+2\right)\cdot\sqrt{5}} & -1 \\ 1 & \sqrt{\left(\sqrt{5}+2\right)\cdot\sqrt{5}} \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\alpha := \operatorname{acos}\left[(2\cdot\varphi)^{-1}\right] \qquad \beta := \alpha\cdot 4^{-1} \qquad 10\cdot\beta - \pi = 0$$

$$\alpha - 2\cdot\pi\cdot 5^{-1} = 0 \qquad (4 \quad 6 \quad 10)\cdot\beta - (\alpha \quad \pi-\alpha \quad \pi) = (0 \quad 0 \quad 0)$$

$$\left(s(\beta) \quad c(\beta)\right) - \left(\frac{1}{2\cdot\varphi} \quad \frac{\sqrt{4\cdot\varphi+3}}{2\cdot\varphi}\right) = (0 \quad 0)$$

$$\left(s(2\cdot\beta) \quad c(2\cdot\beta)\right) - \left[\frac{\sqrt{4\cdot\varphi+3}}{2\cdot(\varphi+1)} \quad \frac{5\cdot\varphi+3}{6\cdot\varphi+4}\right] = (0 \quad 0)$$

$$\mathbf{e}(\beta) - \frac{1}{2\cdot\varphi}\cdot\begin{pmatrix} \sqrt{4\cdot\varphi+3} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{e}(\beta) - \begin{bmatrix} \sqrt{\left(\sqrt{5}+2\right)\cdot\sqrt{5}} \\ 1 \end{bmatrix}\cdot\left(\sqrt{5}+1\right)^{-1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{e}(2\cdot\beta) - \begin{bmatrix} 2+\sqrt{5} \\ \sqrt{\left(\sqrt{5}+2\right)\cdot\sqrt{5}} \end{bmatrix}\cdot\left(3+\sqrt{5}\right)^{-1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{e}(3\cdot\beta) - \frac{1}{4\cdot\left(2+\sqrt{5}\right)}\cdot\begin{bmatrix} \sqrt{\left(\sqrt{5}+2\right)\cdot\sqrt{5}}\cdot\left(1+\sqrt{5}\right) \\ 7+3\cdot\sqrt{5} \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{e}(3\cdot\beta) - \frac{1}{4}\cdot\begin{bmatrix} \sqrt{\left(\sqrt{5}+2\right)\cdot\sqrt{5}}\cdot\left[\dfrac{1+\sqrt{5}}{\left(2+\sqrt{5}\right)}\right] \\ \dfrac{7+3\cdot\sqrt{5}}{\left(2+\sqrt{5}\right)} \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{e}(4\cdot\beta) - \begin{pmatrix} 1 \\ \sqrt{5+2\cdot\sqrt{5}} \end{pmatrix}\cdot\frac{1}{\left(1+\sqrt{5}\right)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{e}(2\cdot\beta) - \frac{1}{2}\cdot\begin{bmatrix} \dfrac{5\cdot\varphi+3}{(3\cdot\varphi+2)} \\ \dfrac{\sqrt{4\cdot\varphi+3}}{(\varphi+1)} \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{e}(3\cdot\beta) - \frac{1}{4\cdot\varphi}\cdot\begin{bmatrix} \dfrac{\sqrt{4\cdot\varphi+3}\cdot(10\cdot\varphi+6)}{8\cdot\varphi+5} \\ \dfrac{42\varphi+26}{8\cdot\varphi+5} \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{e}(4\cdot\beta) - \frac{1}{2\cdot\varphi}\cdot\begin{pmatrix} 1 \\ \sqrt{4\cdot\varphi+3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{e}(5\cdot\beta) - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$gd\left(q^{T}\right) = (2\ \ 5)$ $KL = \begin{pmatrix} 0 & 37 & 74 \\ 36 & 73 & 110 \end{pmatrix}$

$A := \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{\varphi} \end{pmatrix}$ $r^{\langle k_{a,i} \rangle} := A^{a} \cdot \mathbf{e}(2 \cdot i \cdot q_1)$

$Ep := Rp(0)$ $re^{\langle 1+3\cdot u \rangle} := Ep^{\langle 0 \rangle}$ $re^{\langle 6 \rangle} := re^{\langle 0 \rangle}$

$re^{\langle 2+3\cdot u \rangle} := Ep^{\langle 1 \rangle} \cdot \left(\sqrt{\varphi}\right)^{u+1}$

$re^{\langle 2 \rangle} = \begin{pmatrix} 0 \\ 1.272 \end{pmatrix}$ $re^{\langle 5 \rangle} = \begin{pmatrix} 0 \\ 1.618 \end{pmatrix}$

$re = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1.272 & 0 & 0 & 1.618 & 0 \end{pmatrix}$

$rd^{\langle jd \rangle} := \mathbf{e}(2 \cdot jd \cdot \beta)$ $rs^{\langle jd \rangle} := \mathbf{e}(6 \cdot jd \cdot \beta)$

$gd(\beta) = 18$ $rd^{\langle 10 \rangle} := rd^{\langle 0 \rangle}$ $rs^{\langle 10 \rangle} := rs^{\langle 0 \rangle}$

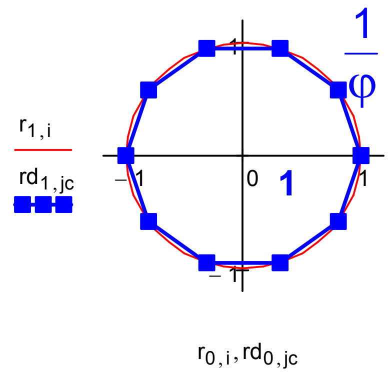


Fig. 17

Decagonul convex cu latura $\frac{1}{\varphi}$

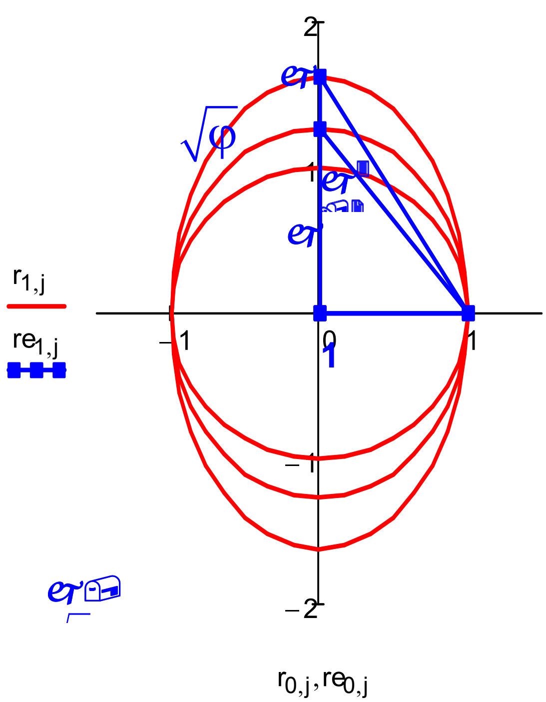


Fig. 18

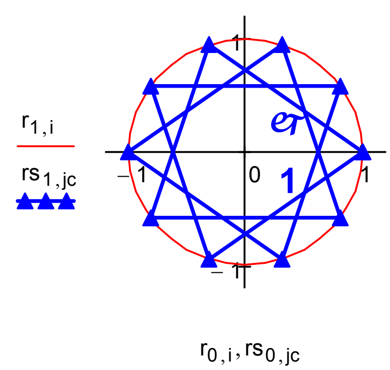


Fig. 19

Decagonul stelat cu latra □ înscrise în cercul trigonometric

I / Grig ! **→ 16 : 49 !** *□ mic = □ de aur cu laturile*
1, $\sqrt{\varphi}$ *,□, Elipsa mare = de aur, cu semiaxe* ***1,*** *□* **!**