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LAPLACE MODEL OF THE FUEL PRESSURE IN THE INJECTION SYSTEM

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Abstract: The study of the unsteady processes in the injection pipe, that connects the injection pump and the injector device, needs a special attention, because it is necessary to find the pressure function depending on the pipe length. This case appears when we are using the light fuels of a low viscosity. The method presented is applied on gasoline injection engines, and the developed model takes into account the influence of his length change when the high pressure is applied on pipe material and liquid fuel. Using the Laplace transform for differential equation system, the mathematical model provides the solution for the pressure field against time length of the injection system.
Keywords: Laplace 2D, modeling, dynamic system, injection system

1. INTRODUCTION

The fuel supplying system to the engine injector is undergoing a non stationary flowing regime. The fuel non stationary flowing in the pipe connecting the injection pump and the injector is affected by the pressure loss in the pipeline as well as by the modifications to the liquid compressibility and the pipe deformation. It is, therefore, necessary to develop a model able to account for the pressure variation in a given point and its variation with time. This model is very useful to more accurately determine the fuel discharge law.

2. THE LAPLACE MODEL

The fuel non-stationary flow through the fuel supplying line to the injector of an injection engine is accompanied by significant variations of both pressure and velocity. Determining the pressure in a given point of the fuel pipeline from the injection pump to the injector implies to write the motion equation for a liquid element of length **l**[m] (Fig 1), regarding the liquid flowing as laminar:

$$m\frac{\partial w}{\partial t}+32\eta\frac{A\,w\,\Delta\ell}{D^2}=A\left(p'-p\right) \tag{1}$$

where:

- m - mass of the liquid considered
- w - mean velocity of the liquid element
- η - dynamic viscosity of the liquid element
- A - area of the fuel pile inner cross section
- $\Delta p=(p' - p)$ the pressure difference between sections 1.1 and 2.2 .
- D - pipe inner diameter

The velocity variation is: $\frac{\partial w}{\partial \ell}\Delta\ell$ and the mean velocity (w') of the liquid element can be written as:

$$w'=w-0.5\frac{\partial w}{\partial \ell}\Delta\ell \tag{2}$$

and the pressure in section 1.1:

$$p' = p + \frac{\partial p}{\partial \ell}\Delta\ell \tag{3}$$

In the previous equations the mass of the liquid was considered:

$$m = A \cdot \rho \cdot \Delta\ell \tag{4}$$

where ρ is the liquid density, regarded as constant.

By substituting relations (2), (3) and (4) in (1), we get:

$$A\rho\frac{\partial\left(w - 0.5\frac{\partial w}{\partial \ell}\Delta\ell\right)}{\partial t}\Delta\ell + 32\eta\frac{A\Delta w}{D^2}\Delta\ell = A\left(p + \frac{\partial p}{\partial \ell}\Delta\ell - p\right) \tag{5}$$

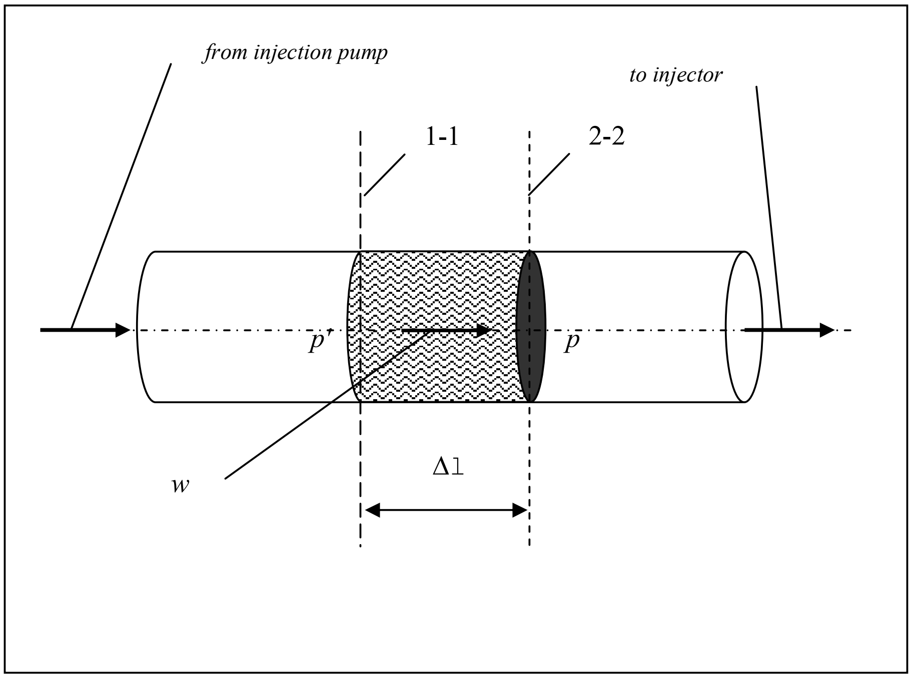

Figure 1: Partial view of fuel charging pipe.

According to Bernoulli equation the pressure difference can also be written as:

$$p' - p = \frac{\rho w^2}{2} \tag{6}$$

which by linearization further yields:

$$\Delta p = -\frac{2\rho w_0 \Delta w}{2}, \text{ and } \Delta w = -\frac{\Delta p}{\rho w_0} \tag{7}$$

where w_0 is the liquid velocity in section 1 1.

From eq.5 it can be obtained:

$$\rho\frac{\partial w}{\partial t} - 0.5\rho\frac{\partial w}{\partial \ell}\frac{\partial \Delta\ell}{\partial t} - 0.5\rho\frac{\partial^2 w}{\partial \ell \partial t} + 32\eta\frac{\Delta p}{\rho w_0 D^2} = \frac{\partial p}{\partial \ell} \tag{8}$$

By neglecting the second-order infinitely small terms, the equation 8 becomes:

$$\rho\frac{\partial w}{\partial t}+32\eta\frac{\Delta p}{\rho w_0 D^2}=\frac{\partial p}{\partial \ell} \tag{9}$$

Due to the high pressure it is necessary to take into account the compressibility effects on the modification of the liquid element length and also on the increase in the pipe volume. If we denote Δ_1 - the variation due to compressibility, Δ_2 - the variation due to pipe deformation we can write the following equations:

$$\Delta_1+\Delta_2=\frac{1}{A}\left(V-\frac{\partial V}{\partial \ell}\Delta\ell\right)dt-\frac{1}{A}Vdt\,,$$

$$\text{or }\ \Delta_1+\Delta_2=-\frac{1}{A}\frac{\partial V}{\partial \ell}\Delta\ell dt \tag{10}$$

According to Hooke's law, the elasticity module can be written as:

$$|E|=V\frac{dp}{dV},$$

where:

$$\Delta_1 A=\frac{1}{E_\ell}\frac{\partial p}{\partial t}\Delta\ell\Delta t \tag{11}$$

in which E_ℓ is the liquid elasticity module.

The equation corresponding to the liquid element deformation due to the pipe deformation is:

$$\Delta V=\pi D^2\Delta_2\,,\ \text{and}\ \Delta_2=\frac{\Delta V}{\pi D^2} \tag{12}$$

The increase in the pipe volume ΔV can be expressed in terms of the increase in the pipe radius by means of the relation:

$$\Delta V=\pi D\Delta R\Delta\ell \tag{13}$$

where ΔR is given by:

$$\Delta R=\frac{R^2}{hE_c}\frac{\partial p}{\partial t}dt \tag{14}$$

where E_c is the pipe elasticity module.

Thus we get the expression of the liquid element deformation due to the pipe deformation as:

$$\Delta_2=\frac{D}{h}\frac{1}{E_c}\frac{\partial p}{\partial t}\Delta\ell\, dt \tag{15}$$

Equations (11) and (15) are replaced in equation (10) to obtain:

$$\frac{1}{A}\frac{\partial V}{\partial \ell}=\frac{\partial w}{\partial \ell}=-\left(\frac{1}{E_\ell}+\frac{D}{h}\frac{1}{E_c}\right)\frac{\partial p}{\partial t} \tag{16}$$

To simplify the calculations the notations **a** and **c** are introduced like:

$$a^2 = \left[\rho \left(\frac{1}{E_\ell} + \frac{D}{h} \frac{1}{E_c} \right) \right]^{-1}$$

$$c = \frac{16\mu}{\rho w_0 D^2}$$

where μ is kinematic viscosity of the liquid element.

By means of the above notations the equations (9) and (16) become:

$$\rho \frac{\partial w}{\partial t} = \frac{\partial p(t,x)}{\partial x} + 2 \cdot c \cdot p(t,x) \tag{17a}$$

$$\frac{\partial w}{\partial x} = -\frac{1}{\rho a^2} \frac{\partial p(t,x)}{\partial t} \tag{17b}$$

By applying the Laplace transform according to the time coordinate we get the following system of equations:

$$\rho s W(s) = \frac{\partial P(s,x)}{\partial x} + 2c \cdot P(s,x) \tag{18a}$$

$$\frac{\partial W(s)}{\partial x} = -\frac{1}{\rho \cdot a^2} \cdot s \cdot P(s,x) \tag{18b}$$

where from, by replacing the second order differential equation for pressure as:

Error!+2c **Error!**+**Error!**$P(s,x)=0$ (19)

can be obtained.

In equation (19) if we denote $\mathbf{q}^2$=$\mathbf{c}^2$ - **Error!** , we obtain the solution to the differential equation:

$$P(s,x) = C_1 \cdot e^{(q-c)x} + C_2 \cdot e^{(q+c)x} \tag{20}$$

The integration constants C_1 and C_2 are determined from the boundary conditions of the fuel supplying system:

$P(t,x)|_{x=0} = P(t,0);$

and:

$P(t,x)|_{x=L} = P_c$,

where P_c is pressure at the end of the compression in the engine cylinder.

After substituting the expressions for the integration constants, we obtain the solution to the differential equation for the pressure in the fuel supplying pipeline as:

$$P(s,x) = \left[P(s,0) - \frac{\left(P_c(s) - P(s,0) \cdot e^{(q+c)L}\right)}{\left(e^{(q-c)L} + e^{-(q+c)L}\right)} \right] e^{(q-c)x} + \left[\frac{\left(-P_c(s) - P(s,0) \cdot e^{(q-c)L}\right)}{\left(e^{(q-c)L} + e^{-(q+c)L}\right)} \right] e^{(q+c)x} \tag{21}$$

If we consider that the value of the pressure $P_c \ll P(s,0)$, for example the value of the pressure at the end of the compression in the engine cylinder is much lower than the pressure at the end of the compression in the injection pump, then the ratio of the pressure in a pipe point and the pressure at the inner end, namely the pump discharge, becomes:

$$\frac{P(s,x)}{P(s,0)} = \left[\frac{e^{q(L-x)} - e^{-q(L-x)}}{e^{qL} - e^{-qL}} \right] \cdot e^{-cx} \tag{22}$$

Since the product (***q*** *L*) takes high values, this product value in equation (22) can be so simplified as to yield:

$$\frac{P(s,x)}{P(s,0)} = \exp(-cx) \cdot \exp\left(-x\sqrt{c^2 - \frac{a^2}{s^2}}\right) \tag{23}$$

Going back to the original variables of the system, ***Ω****(t,x)*, the complete solution of the pressure field in the fuel supplying pipeline to an injection engine is obtained. This calls for a complex development of the Laplace reverse transform applied to (23).

3. CONCLUSIONS

In order to investigate the response of the system injection pump, supplying pipeline, injector, it is recommended to use equation (22).

The solution of the pressure field for the fuel supplying pipeline makes it possible to further determine the variation of the fuel pressure against time and in any point of the pipe connecting the injection pump and the injector.

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