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COMPARATIVE NUMERICAL STUDY OF FEM AND SPH METHOD FOR BULLET-MULTILAYERED PLATE IMPACT SIMULATION

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Abstract: *This paper presents a numerical simulation of the impact problem of a bullet into a two layered composite plate representing an armor used for the human protection. The approach of this problem is performed using both classical Finite Element Method (FEM) and Smoothed Particle Hydrodynamics (SPH). The aim of the work is to point out that the SPH method is a valuable numerical approach of the impact problems of the composites. The penetration and perforation of a composite that could occur as a result of an impact process are a complex problem which implies many technical and design aspects. This numerical study deals mainly with the impact velocity effect on the delaminating area of the impact zone. The numerical solutions obtained by both methods are very close and the good investigating potential of the SPH method is pointed out.*
Keywords: *SPH, FEM, Impact, Composites, Numerical Simulation*

1. INTRODUCTION

The penetration and perforation of the composite could occur as a result of the impact process. The impact response of materials can range from low (large mass) velocity to high/ballistic (small mass) velocity regimes. A significant development on impact mechanics is presented in [1] and particularly on composite structures in [2].
Large mass impact, known as low velocity impact, results from conditions arising for instance, from tool drops on a product which typically occur at velocities below 10 m/s, while intermediate velocity impact regime occurs for instance for secondary blast debris, hurricane and tornado debris, in the 10m/s to 50 m/s range.
High velocity (ballistic) impact is usually a result of small arms fire, explosive warhead fragments or space debris on a spacecraft and it range from 50 m/s to 1500 m/s. In hyper velocity impact > 2-5 km/s, the projectile is moving at very high velocities and the target material behaves like a fluid.
High velocity impact response is dominated by stress wave propagation through the thickness of the material, in which the structure does not have time to respond. Boundary condition effect can be ignored because the impact event passes before the stress waves reach the boundary.
This paper deals with an application referring to the bullet-armor impact, usually called ballistic impact.
In studying ballistic impacts, it is important to determine the residual velocity of the projectile accurately. From the experimental testing point of view, this is a difficult task because many small particles, fibers, and shear plugs are pushed out by the projectile during penetration. This material can trigger the speed-sensing device being used and yield erroneous values. Moreover, even from a numerical point of view, the particles spread represents an obstacle in obtain the adequate solutions.
Considering the balance of energy reveals important features of ballistic impact, including the effects of laminate thickness, projectile size, shape, and initial velocity. The application of bullet-armor impact presented in this study is performed alternatively by Smoothed Particle Hydrodynamics (SPH) method and Finite Element Method (FEM) in order to estimate and validate SPH method to solve impact problems of bullet targeting a composite plate simulating the armor.
SPH is a numerical simulation meshless method proposed by Lucy in 1977, see [3]. The first applications of this method were connected to astrophysical problems. The method was extended to fluid simulation, especially with free-surface by Monaghan in 1992 and to other fields, see [4] and [6]. The field of applied mechanics is the last one, but it is extensively studied and significant advances have been made, see [5]-[8].
The last preoccupations are focused on coupling SPH with standard numerical procedures, such as the FE method or other meshless techniques because they offer new possibilities to solve complex problems in

engineering. The SPH method is validated as a numerical method in fluid mechanics, but the applied mechanics is concerned it is still in progress.
The SPH advantages seem to be greater then disadvantages, from a lot points of view, especially in some fields, like fluid mechanics and even applied mechanics for those problems that involve large displacements or for materials with a fluid-like behavior or for brittle materials with special properties like ceramics, glass etc.
Some SPH programs exist, but next to these, the SPH method is implemented in the most powerful programs, like ANSYS, beginning with 10^{th} version and the last one is used in this study.

2. SPH FORMULATION

The SPH method is a meshless method, in which the investigated domain is represented by a number of nodes, representing the particles of this domain, having their material and mechanical (mass, position, velocity etc.) characteristics. Each particle represents an interpolation point on which the material properties are known.
The boundary conditions have to be imposed to some of particles, according to the problem analyzed, like in the case of finite element method.
The problem solution is given by the computed results, on all the particles, using an interpolation function. We can say that the fundamentals of SPH theory consist in interpolation theory; all the behavior laws are transformed into integral equations. The kernel function, or smoothing function, often called smoothing kernel function, or simply kernel, gives a weighted approximation of the field variable (function) in a point (particle).
Integral representation of a function $f(x)$, used in the SPH method starts from the following identity:

$$f(x) = \int_{\Omega} f(x')\delta(x - x')dx' \tag{1}$$

where f is a function of a position vector x, which can be an one-, two- or three-dimensional one; $\delta(x - x')$ is a Dirac function, having the properties:

$$\delta(x - x') = \begin{cases} 1 \rightarrow x = x' \\ 0 \rightarrow x \neq x' \end{cases} \tag{2}$$

In equation (1), Ω is the function domain, which can be a volume, that contains the x, and where $f(x)$ is defined and continuous. By replacing the Dirac function with a smoothing function $W(x - x', h)$ the integral representation of $f(x)$ becomes:

$$f(x) = \int_{\Omega} f(x')W(x - x', h)dx' \tag{3}$$

where W is the smoothing kernel function, or smoothing function, or kernel function.
The parameter h, of the smoothing function W, is the smoothing length, by which the influence area of the smoothing function W is defined (Figure 1 and Figure 2).

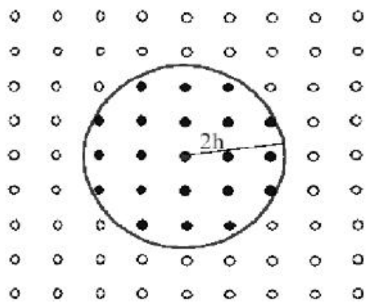

Figure 1: Support domain of W

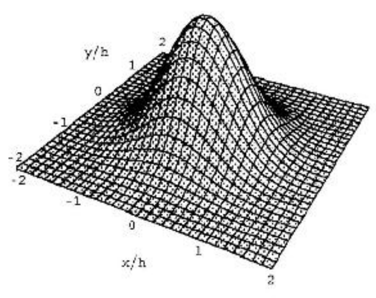

Figure 2: Graphical representation of 2D-Kernel function

As long as Dirac delta function is used, the integral representation, described by equation (1), is an exact (rigorous) one, but using the smoothing function W instead of Dirac function, the integral representation can only be an approximation. This is the reason for the name of kernel approximation. Using the angle bracket $\langle\ \rangle$ this aspect is underlined and the equation (3) can be rewritten as:

$$\langle f(x)\rangle = \int_{\Omega} f(x')W(x - x', h)dx' \tag{4}$$

The smoothing function W is usually chosen to be an even one, which has to satisfy some conditions. The first condition, named normalization condition or unity condition is:

$$\int_{\Omega} W(x-x',h)dx' = 1 \tag{5}$$

The second condition is the Delta function property and it occurs when the smoothing length approaches zero:

$$\lim_{h\to 0} W(x-x',h) = \delta(x-x') \tag{6}$$

The third condition is the compact condition, expressed by:

$$W(x-x',h) = 0 \text{ when } |x-x'| > kh \tag{7}$$

where k is a constant related to the smoothing function for point at x, defining the effective non-zero area.
As the particle approximation is concerned, the continuous integral approximation (4) can be converted to a summation of discrete forms, over all particles belonging to the support domain. Changing the infinitesimal volume dx' with the finite volume of the particle ΔV_j, the mass of the particles m_j can be written,

$$m_j = \Delta V_j \rho_j \tag{8}$$

and finally, relation (3) becomes:

$$\langle f(x) \rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W(x-x_j,h) \tag{9}$$

The particle approximation of a parameter described by a function, for particle i can be expressed by,

$$\langle f(x_i) \rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W_{ij} \tag{10}$$

where,

$$W_{ij} = W(x_i - x_j, h), \tag{11}$$

being the kernel function.

3. NUMERICAL MODELS

The numerical study is accomplished comparatively by FE and SPH models (Figures 3 and 4), for solving the main aspects regarding to the impact problem. Such a problem is a very difficult one, by many reasons.
Among these reasons, some characteristics have to be emphasized: the very short time for developing of some physics phenomena, material and large displacement nonlinearities, behavior of the material beyond the elastic limit, the strain rate and many others. All these aspects are not the aim of this work, but for a part of them, some synthetic presentations will be made in connection of the model construction.

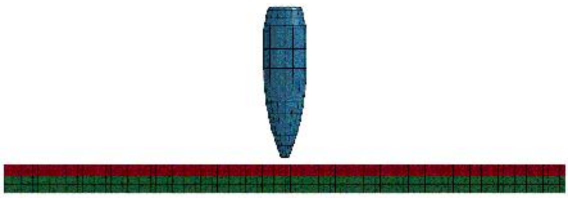

Figure 3: FE model

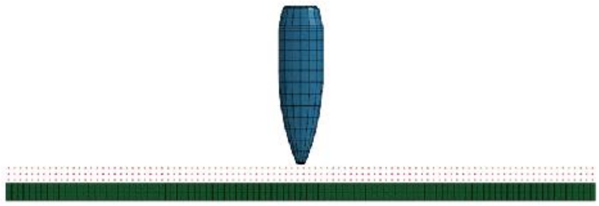

Figure 4: SPH model

The geometric model consists of a 7.62 caliber bullet impacting a 10cm×10cm×0.6cm composite (multilayered) plate, simulating the armor, made up of two isotropic layers: an exterior one of steel and interior one of aluminum. In the case of SPH model, the exterior steel layer consists of particles. Such a problem (normal impact of a bullet with a plane plate) can be considered a structure with two symmetric planes or an axisymmetric structure. A simple model can be adopted, represented only by a half or by a quarter of a 3D structure (using 3D finite elements), or represented by a plane structure (using 2D axisymmetric elements).
Because the considered example is not a large one, we preferred a 3D completed models, presented in the Figures 3 and 4. The 3D finite element model consists of 1131 nodes and 1008 elements (SOLID3D) for bullet, and of 30603 nodes and 20000 elements (SOLID3D) for each layer. The 3D SPH model is a model, which consists in finite elements for bullet and aluminum plate and SPH particles for steel plate. A model using only SPH particles is possible, but not every time is the best solution; from different reasons, not discussed here, the most used models consists in finite elements and SPH elements, like the models adopted by us.

The impact velocity of 420 m/s is considered, being the case of a medium distance of firing. As the material models are concerned, three material models were used: plastic-kinematic (Cowper-Symonds) material model for the bullet, elastic-plastic-hydro material model for the aluminum plate and Johnson-Cook material model for the steel plate. For each material models specific constants were used according to the experimental results and the values presented in the technical literature.

4. MATERIAL MODELS

One of the most used material model, adopted for the target (often for the projectile or hammer too), is the Elastic Plastic with Kinematic Hardening Model, being strain rate dependent plasticity for isotropic materials. The strain rate is taken into account by Cowper-Symonds model using the coefficients C and P, having the same name. The yield function σ_y is given by [9]:

$$\sigma_y = \left[1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{\frac{1}{P}}\right]\left(\sigma_0 + \beta E_p \varepsilon_p^{ef}\right) \tag{12}$$

where σ_0 is the initial yield stress, ε_p^{ef} is the effective plastic strain, E_p is the plastic hardening modulus which is given by:

$$E_p = \frac{E_T E}{E - E_T}, \tag{13}$$

β being the hardening parameter that can vary between 0 and 1 depending on plasticity type (0 for kinematic and 1 for isotropic respectively) and E_T is the tangent modulus. For this model, the user has to specify the failure strain for which elements will be eliminated.

According to the flow rule, the direction of plastic straining can be evaluated. The hardening rule describes the changing of the yield surface with progressive yielding, so that the conditions for subsequent yielding can be established.

Another material model equally used for specimen (target) and hummer (projectile) modeling is Johnson and Cook Plasticity Model, [9], which express the flow stresses:

$$\sigma_y = \left(A + B\bar{\varepsilon}^{p^n}\right)\left(1 + c\ln\dot{\varepsilon}^*\right)\left(1 - T^{*m}\right) \tag{14}$$

where A, B, C, n, and m are user defined input constants, $\bar{\varepsilon}^p$ is the effective plastic strain, and:

$$\dot{\varepsilon}^* = \frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_0} \tag{15}$$

is the effective plastic strain rate for $\dot{\varepsilon}_0$=1 s^{-1}, and:

$$T^* = \frac{T - T_{room}}{T_{melt} - T_{room}} \tag{16}$$

T being the temperature (by empirical assumption *T* represents 90% of the plastic work, T_{room} is the room environment temperature and T_{melt} is the melting temperature of the material). The strain at fracture is given by:

$$\varepsilon^f = \left[D_1 + D_2 \exp D_3\sigma^*\right]\left[1 + D_4 \ln\varepsilon^*\right]\left[1 + D_5 T^*\right] \tag{17}$$

where $D_1 \ldots D_5$ are input constants and σ^* is the ratio of pressure divided by effective stress:

$$\sigma^* = \frac{p}{\sigma_{eff}} \tag{18}$$

The fracture occurs when the damage parameter D, relation (19), reaches the value 1.

$$D = \sum \frac{\Delta\bar{\varepsilon}^p}{\varepsilon^f} \tag{19}$$

The elastic-plastic-hydro material model often is used when the material has a hydrodynamic behavior, specially under strong shock like an impact. This behavior is up to material type and velocity level. The yield strength calculus depends on the values of effective plastic strain (EPS) and effective stress (ES) are defined or not. If ES and EPS are undefined, the yield strength is calculated as [9]:

$$\sigma_y = \sigma_0 + E_h\bar{\varepsilon}^p + \left(a_1 + a_2 p\right)\max[p,0] \tag{20}$$

The quantity E_h is the plastic hardening modulus defined in terms of Young's modulus, E, and the tangent modulus, E_t [9]:

$$E_h = \frac{E_t E}{E - E_t} \qquad (21)$$

If ES and EPS are specified, the yield stress is given by a relation (22), which is obtained by mathematical and graphic manipulation.

$$\sigma_y = f\left(\bar{\varepsilon}^p\right) \qquad (22)$$

Johnson-Cook Plasticity Model, like many others, is accompanied by an equation of state (EOS). In our examples a Gruneisen EOS was used. For compressed materials, the Gruneisen equation of state, with cubic shock velocity-particle velocity defines a pressure [9]:

$$p = \frac{\rho_0 C^2 \mu \left[1 + \left(1 - \frac{\gamma_0}{2}\right)\mu - \frac{a}{2}\mu^2\right]}{\left[1 - (S_1 - 1)\mu - S_2 \frac{\mu^2}{1+\mu} - S_3 \frac{\mu^3}{(1+\mu)^2}\right]^2} + (a\mu + \gamma_0)E \qquad (23)$$

Many others EOS (linear-polynomial, JWL, ratio of polynomials, tabulated etc.) are implemented and the using of each one of them depends on some aspects, which are not the subject of this paper.
For projectile modeling, the Rigid Material Model [9] is often used. Such approximation of a deformable body is a preferred modeling technique in many real work applications, because the calculus time can be significant smaller. In many cases we are interested in what happens with the target and fewer with the projectile.
The elements which are considered rigid are bypassed in the element processing and no storage is allocated for storing history variables, so the rigid material model is a very cost efficient one.
Some material properties have to be given by the user. Young's modulus E and Poisson's ratio υ being used for determining sliding interface parameters if the rigid body interacts in a contact definition. Density ρ is necessary for calculus of the inertial properties.
In all cases, unrealistic values of the material constants may cause some solving difficulties.

5. NUMERICAL RESULTS

A comparison between the FE and SPH model for the case of 420 m/s bullet velocity can be performed studying the figures 5 and 6 representing the field of von Mises stresses. It is observed a similar distribution of effective stress in both cases and a certain zone of detachment of the two layers with a bigger displacement of the aluminium one as it was expected.

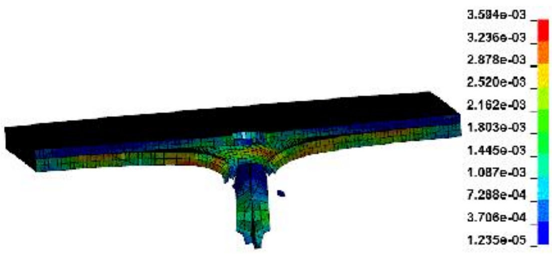

Figure 5: FE model - von Mises stress field

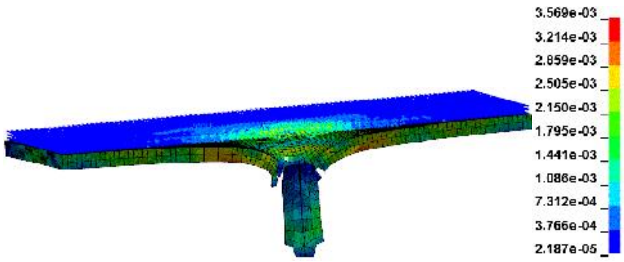

Figure 6: SPH model - von Mises stress field

As we can see in Figure 5 and 6, the maximum stress values (3.594e-3 and 3.569e-3 $gcm/\mu s^2$, or 359.40 and 356.90 MPa) are very closed, the error for SPH model being only -0.69%. The fundamental measure units were [g] for mass, [cm] for length and [s]e-6 for time.
Concerning the bullet velocity, a similar profile is obtained by those two methods as it is noticed in figures 7 and 8, respectively. At the same time (100e-6 seconds), the residual bullet velocities are also very closed (355.25 and 353.70 m/s^2). The error regarding residual bullet velocities calculated using FE and SPH models is -1.69%.

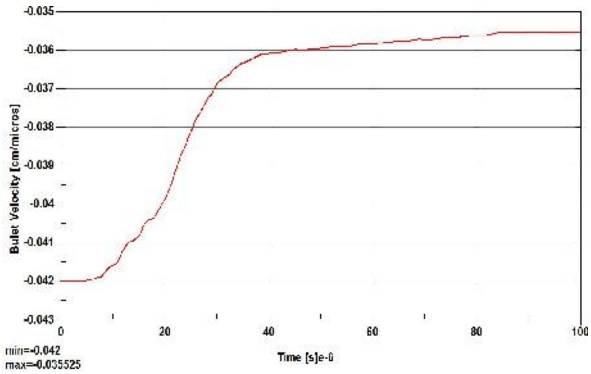

Figure 7: FE model – time bullet velocity

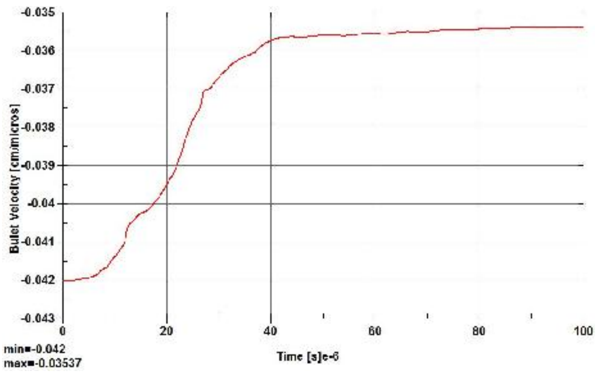

Figure 8: SPH model – time bullet velocity

Many other parameters like displacements, accelerations, different forms of energies (internal, kinematic or total) in connection with each plate or with the bullet can also be calculated, represented as a function of time and compared. All these parameters obtained by FEM or SPH method are in a good concordance.

6. CONCLUSION

This paper dealing with the SPH method represents a validation of this method for using in the applied mechanics, in special dynamic problems, like an impact problem of a bullet into a multilayered composite plate.
Many other aspects can be also studied, numerical simulated but for each aspect a more space requires.
The SPH method is also fitted for a right numerical simulation of some materials, like glass, ceramics etc., adopting also appropriated material models.
The case of plate layers which can be deformed only together, (layers without friction or moving between them) can be also treated, in a similar way, but the problem of the contact and the friction between layers does not exist and the computer time is shorter.
This paper did not present all the advantages of SPH method, but we consider it to be an incentive to be used in a larger area, specially in applied mechanics. In our country, this method is still little used by different reasons.

REFERENCES

[1] Stronge W.G., Impact Mechanics, Cambridge Univ. Press, 2000.
[2] Abrate S., Impact on Composite Structures, Cambridge Univ. Press, 1998.
[3] Lucy, L., B., A numerical approach to the testing of the ssion hypothesis, Astrophysical Journal, (82), pp.1013-1024, 1977.
[4] Monaghan, J., J., Smoothed Particle Hydrodynamics, In Ann. Rev. Astron. Astrophys., 1992, pp. 543-574
[5] Liu, G. R., Liu, M. B., Smoothed Particle Hydrodynamics, World Scientific Publishing Co.Pte.Ltd., 2009
[5] Buruchenko, S., K., Smoothed Particle Hydrodynamics: Some Results, Вычислительнцые, техцнологии, Том 7, No. 1, 2002.
[6] Nastasescu, V., Barsan, Gh., Metoda SPH, Editura Academiei Forţelor Terestre, Sibiu, 2012
[7] Ma, W., L., Pang, B., J., Zhang, W., Gai, B., Z., Numerical Simulation of Major Debris-Cloud Features Produced by Projectile Hypervelocity Impact on Bumper, Hypervelocity Impact Research Center, Harbin Institute of Technology, P.O. Box 3020, No.2, Yikuang Street, Harbin 150080, P. R. China, 2008.
[8] Tanaka, K., Numerical Study on the High Velocity Impact Phenomena by Smoothed Particle, Hydrodynamics (SPH), 1-1-1 Umezono, Centtral 2 Tsukuba, Ibaraki 305-8568 JAPAN, 2005.
[9] LS-DYNA Keyword User's Manual, Version 971, May 2007.