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STABILITY OF A CIRCULAR PLATE STIFFENED BY A CYLINDRICAL SHELL UNDER THE ASSUMPTION OF NON-AXISYMMETRIC DEFORMATIONS

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Abstract: This paper studies the stability problem of a circular plate stiffened by a cylindrical shell on the outer diameter of the plate. It is assumed that (a) we can use the Kirchhoff theory of plates and shells, and (b) the deformations due to the load are non-axisymmetric. We have the following objectives: (1) to establish the equations that can be used to determine the critical load of a stiffened circular plate (solid or with a hole) (2) computation of the critical load for a stiffened plate with no hole
Keywords: loss of stability, circular plate, stiffened, cylindrical shell, critical load

1. INTRODUCTION

In the engineering practice stability problems of plates loaded in their own plane are especially interesting ones. To the author's knowledge paper [1] by Brian was the first one which dealt with the stability problem of a circular plate. Since then a number of papers have been devoted to this issue. Here we have cited only a few [2], [3] and remark that further references can be found in the papers cited.

A circular plate can be stiffened in various ways. For example we can apply a corrugation to it, or it can be stiffened by a cylindrical shell attached to the plate on its boundary. The present paper investigates the stability of a circular plate provided that the plate is stiffened by a cylindrical shell. This problem was partly solved by Szilassy [4], [5] who set up a differential equation for the rotation field and solved the corresponding eigenvalue problem under the assumption that the shell is subjected to a constant radial load in the middle plane of the plate. The author of the present paper has also investigated this issue by using equations set up for the displacement field [6]. By assuming axisymmetric deformations the critical loads have been determined for various types of support applied on the inner diameter of the plate.

The case of non-axisymmetric deformations has not been investigated yet. Consequently the main objective of the present paper is to determine those equations which provide the critical load if we assume that the load is axisymmetric while the deformations perpendicular due to the load are partly not. As regards the solutions for the cylindrical shell we shall utilize some results of thesis [7] by Jezsó.

2. THE STRUCTURE AND THE LOAD APPLIED

The geometry of the structure consisting of a circular plate and a cylindric shell is shown in Fig. 1. The load is a constant radial load in the middle plane of the plate.

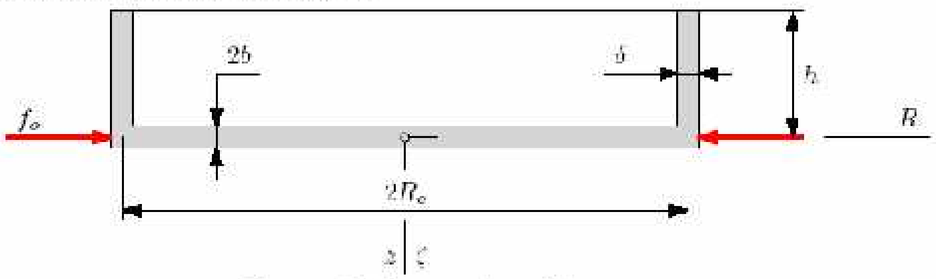

FIGURE 1. Cross section of the structure

We shall assume that the plate and the shell are thin, consequently we can apply the Kirchhoff theory of plates and shells. If the shell and plate are made of the same isotropic material, then E and ν are the Young-modulus and the Poisson ratio, respectively. Separated from each other mentally the two structural elements (the plate and the shell) are shown in Figure 2 where we can also see the in-plane load of the plate.

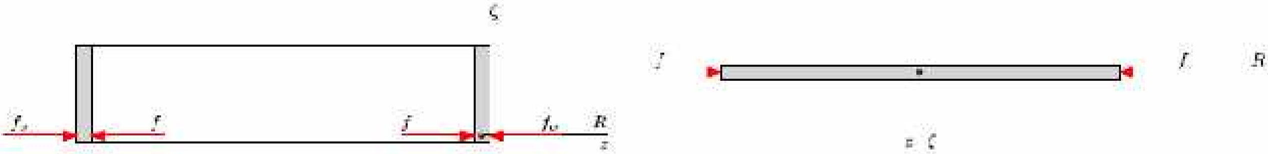

FIGURE 2. The two strucural elements

Fig. 3 shows the coordinate systems and the displacement components on the middle surface of the shell

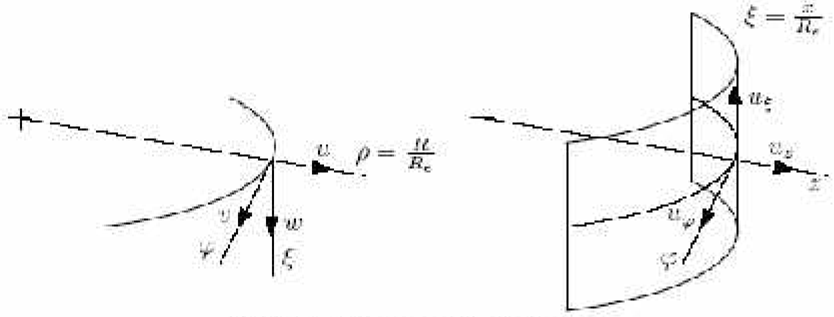

FIGURE 3. The coordinate system

We shall assume that the plane stress problem in the plate due to the load f is axisymmetric i.e., the inner forces N_R, N_φ and $N_{R\varphi}$ satisfy the conditions $N_R = N_\varphi = -f$ and $N_{R\varphi} = 0$. We can calculate the intensity of the distributed forces f from the fact that the radial displacement u of the plate is equal to the radial displacement u_z of the shell. From the axisymmetric part of the shell deformation due to the forces f and f_o we get [4]:

$$u_z|_{\xi=0} = u|_{\rho=1} = -\underbrace{\frac{\nu_c}{2E}\left(\frac{R_e}{\delta}\right)^{\frac{2}{?}} \frac{\cos 2h\beta + \cosh 2h\beta + 2}{\sin 2h\beta + \sinh 2h\beta}}_{\alpha}(f_o - f) = -\alpha\,(f_o - f)\,, \tag{1}$$

where

$$\beta = \nu_2\sqrt{\frac{R_e}{\delta}}\frac{1}{R_e}\,; \quad \nu_2 = \sqrt[4]{3(1-\nu^2)}\,. \tag{2}$$

3. EQUATIONS FOR THE CIRCULAR PLATE

As is well known the vertical displacement w on the middle surface should fulfill the differential equation

$$\bar\Delta\bar\Delta w - \frac{R_e^2 N_R}{I_1 E_1}\bar\Delta w = 0\,, \quad \bar\Delta = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\,, \quad \rho = \frac{R}{R_e}\,, \tag{3}$$

where $I_1 = 2b^3/3$, $E_1 = E/(1-\nu^2)$. Let us expand the solution for w into a Fourier series:

$$w = w_o + \sum_{m=0}^{1}\sum_{n=1}^{\infty} \overset{m}{w}_n(\rho)\cos\left(n\varphi - m\frac{\pi}{2}\right)\,. \tag{4}$$

After substituting the solution into $(3)_1$ we obtain that amplitude functions $w_o(\rho)$ and $\overset{m}{w}_n(\rho)$ have to fulfill the following differential equations

$$L_n(\hat w) + N_R\frac{R_e^2}{I_1 E_1}\left(\frac{d^2}{d\rho^2} + \frac{d}{\rho d\rho} - \frac{n^2}{\rho^2}\right)\hat w = 0\,, \quad m = 0,1 \quad n = 0,1,2,\dots \tag{5a}$$

$$L_n = \frac{d^4}{d\rho^4} + \frac{2}{\rho}\frac{d^3}{d\rho^3} - \frac{1-2n^2}{\rho^2}\frac{d^2}{d\rho^2} + \frac{1-2n^2}{\rho^3}\frac{d}{d\rho} + \frac{n^4-4n^2}{\rho^4}\,. \tag{5b}$$

If $N_R = f = const$ then the above equations have closed form solutions:

$$w_o = c_1 + c_2\ln\rho + c_3 J_o(\sqrt{\mathfrak{F}}\rho) + c_4 Y_o(\sqrt{\mathfrak{F}}\rho)\,, \qquad \mathfrak{F} = R_e^2 f/I_1 E_1 \tag{6a}$$

$$\overset{m}{w}_n = c_1\rho^n + c_2\rho^{-n} + c_3 J_n(\sqrt{\mathfrak{F}}\rho) + c_4 Y_n(\sqrt{\mathfrak{F}}\rho)\,, \qquad m = 0,1;\; n = 1,2,\dots \tag{6b}$$

In what follows the Fourier series of a physical quantity – denoted by say Q – will be written in the same form as the series (4):

$$Q = Q_o + \sum_{m=0}^{1}\sum_{n=1}^{\infty}\overset{m}{Q}_n(\rho)\cos\left(n\varphi - m\frac{\pi}{2}\right) . \tag{7}$$

In accordance with this notational convention one can show that the amplitudes of the rotation ψ_φ, the bending moments M_R, M_φ, the torsional moment $M_{R\varphi}$ and the shear force Q_R can all be given in terms of the amplitudes of w as follows:

$$\overset{m}{\psi}_{\varphi n} = -\frac{1}{R_e}\frac{\mathrm{d}\overset{m}{w}_n}{\mathrm{d}\rho} . \tag{8}$$

$$\overset{m}{M}_{Rn} = -\frac{I_1E_1}{R_e^2}\left[\frac{\mathrm{d}^2\overset{m}{w}_n}{\mathrm{d}\rho} + \frac{\nu}{\rho}\left(\frac{\mathrm{d}\overset{m}{w}_n}{\mathrm{d}\rho} - \frac{n^2}{\rho}\overset{m}{w}_n\right)\right] , \tag{9a}$$

$$\overset{m}{M}_{\varphi n} = -\frac{I_1E_1}{R_e^2}\left[\nu\frac{\mathrm{d}^2\overset{m}{w}_n}{\mathrm{d}\rho} + \frac{1}{\rho}\left(\frac{\mathrm{d}\overset{m}{w}_n}{\mathrm{d}\rho} - \frac{n^2}{\rho}\overset{m}{w}_n\right)\right] \tag{9b}$$

$$\overset{m}{M}_{R\varphi n} = \frac{I_1E_1}{R_e^2}(1-\nu)\left[\frac{n}{\rho}\frac{\mathrm{d}\overset{m}{w}_n}{\mathrm{d}\rho} - \frac{n}{\rho^2}\overset{m}{w}_n\right] , \tag{9c}$$

$$\overset{m}{Q}_{Rn} = I_1E_1\frac{1}{R_e^3}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} + \frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho} - \frac{n^2}{\rho^2}\right)\overset{m}{w}_n + \frac{N_R}{R_e}\frac{\mathrm{d}\overset{m}{w}_n}{\mathrm{d}\rho} . \tag{10}$$

4. EQUATIONS FOR THE CYLINDRICAL SHELL

4.1. Field equations of the problem

The governing equations of the cylindrical shell, which stiffens the plate, are presented in a bit more detail. Let u_x, u_φ and u_z be the three displacement coordinates on the middle surface of the shell in the coordinate system shown in Fig. 1. – where $R_e\xi = x$. Deformations on the middle surface are characterized by the axial strains ε_{xx}, $\varepsilon_{\varphi\varphi}$, the shear strain $\varepsilon_{x\varphi}$, the rotation ϑ_x as well as by the elements κ_{xx}, $\kappa_{x\varphi}$ and $\kappa_{\varphi\varphi}$ of the curvature tensor [7]:

$$\varepsilon_{xx} = \frac{1}{R_e}\frac{\partial u_x}{\partial\xi} , \quad \varepsilon_{\varphi\varphi} = \frac{1}{R_e}\left(\frac{\partial u_\varphi}{\partial\varphi} + u_z\right) , \quad \varepsilon_{x\varphi} = \frac{1}{2R_e}\left(\frac{\partial u_x}{\partial\varphi} + \frac{\partial u_\varphi}{\partial\xi}\right) \tag{11}$$

$$\vartheta_x = -\psi_\varphi = -\frac{1}{R_e}\frac{\partial u_z}{\partial\xi} \tag{12a}$$

$$\kappa_{xx} = \frac{1}{R_e^2}\frac{\partial^2 u_z}{\partial\xi^2} , \quad \kappa_{\varphi\varphi} = \frac{1}{R_e^2}\frac{\partial^2 u_z}{\partial\varphi^2} , \quad \kappa_{x\varphi} = \frac{1}{R_e^2}\frac{\partial^2 u_z}{\partial\xi\partial\varphi} \tag{12b}$$

The corresponding inner forces and bending moments are obtained from the Hooke law:

$$N_{xx} = E_1\delta\left(\varepsilon_{xx} + \nu\varepsilon_{\varphi\varphi}\right) , \quad N_{\varphi\varphi} = E_1\delta\left(\varepsilon_{\varphi\varphi} + \nu\varepsilon_{xx}\right) , \quad N_{x\varphi} = E_1\delta\left(1-\nu\right)\varepsilon_{x\varphi} , \tag{13}$$

$$M_{xx} = E_1\frac{\delta^3}{12}\left(\kappa_{xx} + \nu\kappa_{\varphi\varphi}\right) , \quad M_{\varphi\varphi} = E_1\frac{\delta^3}{12}\left(\kappa_{\varphi\varphi} + \nu\kappa_{xx}\right) , \quad M_{x\varphi} = E_1\frac{\delta^3}{12}\left(1-\nu\right)\kappa_{x\varphi} . \tag{14}$$

The above equations are associated with the equilibrium equations

$$\frac{\partial N_{xx}}{\partial\xi} + \frac{\partial N_{x\varphi}}{\partial\varphi} + R_e p_x = 0 , \tag{15a}$$

$$\frac{\partial N_{x\varphi}}{\partial\xi} + \frac{\partial N_{\varphi\varphi}}{\partial\varphi} + \frac{1}{R_e}\left(\frac{\partial M_{x\varphi}}{\partial\xi} + \frac{\partial M_{\varphi\varphi}}{\partial\varphi}\right) + R_e p_\varphi = 0 , \tag{15b}$$

$$\frac{\partial^2 M_{xx}}{\partial\xi^2} + 2\frac{\partial^2 M_{x\varphi}}{\partial\xi\partial\varphi} + \frac{\partial^2 M_{\varphi\varphi}}{\partial\varphi^2} - R_e N_{\varphi\varphi} + R_e p_z = 0 . \tag{15c}$$

Observe that we have as many equations as there are unknowns (fifteen equations in fifteen unknowns).

4.2. Solution in terms of the Galerkin function

For $p_x = p_\varphi = 0$ the fundamental equations (obtained after we have eliminated the intermediate variables) set up for the displacement coordinates u_x, u_φ and u_z will be fulfilled identically if we calculate the displacement coordinates from the Galerkin function ϕ using the relations

$$u_x = \frac{\partial^3\phi}{\partial\xi\partial\varphi^2} - \nu\frac{\partial^3\phi}{\partial\xi^3}; \quad u_\varphi = -\frac{\partial^3\phi}{\partial\varphi^3} - (2+\nu)\frac{\partial^3\phi}{\partial\xi^2\partial\varphi}, \quad u_z = \nabla^2\nabla^2\varphi\,, \tag{16}$$

in which ϕ should satisfy the differential equation

$$\nabla^2\nabla^2\nabla^2\nabla^2\varphi - 4\beta^2\frac{\partial^4\phi}{\partial\xi^4} - \frac{4\beta^4R^2}{E\delta}p_z = 0\,,\ \nabla^2 = \frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\varphi^2}\,,\ \beta^4 = 3\left(1-\nu^2\right)\frac{R_k^2}{\delta^2}\,, \tag{17}$$

since the distributed load p_z exerted on the shell is zero in the present problem [7]. With regard to equation (4) we assume that ϕ is expanded into a Fourier series:

$$\mathcal{F}(\xi,\varphi) = \mathcal{F}_o(\xi) + \sum_{m=0}^{1}\sum_{n=1}^{\infty}\overset{m}{\mathcal{F}}_n(\xi)\cos\left(n\varphi - m\frac{\pi}{2}\right)\,. \tag{18}$$

One can show that

$$\frac{d^8\overset{m}{\mathcal{F}}_n}{\partial\xi^8} - 4n^2\frac{d^6\overset{m}{\mathcal{F}}_n}{\partial\xi^6} + 6n^4\frac{d^4\overset{m}{\mathcal{F}}_n}{\partial\xi^4} - 4n^6\frac{d^2\overset{m}{\mathcal{F}}_n}{\partial\xi^2} + n^8\overset{m}{\mathcal{F}}_n + 4\beta^4\frac{d^4\overset{m}{\mathcal{F}}_n}{\partial\xi^4} = 0\,. \tag{19}$$

The characteristic polynomial of this equation takes the form

$$(\lambda_n^2 - n^2)^4 = -4\beta^4\lambda^4\,. \tag{20a}$$

The roots of this polynomial are as follows:

$$\lambda_{n1} = -\beta_{n1} + i\alpha_{n1};\ \lambda_{n2} = -\beta_{n2} + i\alpha_{n2};\ \lambda_{n3} = \bar\lambda_{n1};\ \lambda_{n4} = \bar\lambda_{n2};\ \lambda_{n,4+l} = -\lambda_{nl};\ l = 1,\dots,4\,, \tag{20b}$$

where

$$\beta_{n1} = b_n + \frac{\beta}{2}\,,\ \beta_{n2} = b_n - \frac{\beta}{2}\,,\ \alpha_{n1} = \frac{\beta}{2} + a_n\,,\ \alpha_{n2} = \frac{\beta}{2} - a_n\,, \tag{20c}$$

$$b_n = \frac{\beta}{2}\sqrt{\sqrt{1+4\left(\frac{n}{\beta}\right)^4} + 2\left(\frac{n}{\beta}\right)^2}\,,\quad a_n = \frac{\beta}{2}\sqrt{\sqrt{1+4\left(\frac{n}{\beta}\right)^4} - 2\left(\frac{n}{\beta}\right)^2}\,. \tag{20d}$$

It can be shown that the real solution of equation (19) takes the form

$$\overset{m}{\mathcal{F}}_n = \sum_{k=1}^{2}\Big[\overset{m}{K}_{nk}\sinh(\beta_{nk}\xi)\sin(\alpha_{nk}\xi) + \overset{m}{M}_{nk}\sinh(\beta_{nk})\cos(\alpha_{nk}\xi) + \overset{m}{P}_{nk}\cosh(\beta_{nk}\xi)\sin(\alpha_{nk}\xi) + \overset{m}{S}_{nk}\cosh(\beta_{nk}\xi)\cos(\alpha_{nk}\xi)\Big]\,. \tag{21}$$

where the quantities $\overset{m}{M}_{nk},\dots,\overset{m}{J}_{nk}$ are altogether eight integration constants. Every physical quantity can be written in a form similar to that of equation (7) – we should write ξ instead of ρ there. Omitting the long hand made calculations we shall present only those physical quantities here which are involved in the boundary and continuity conditions prescribed on the circle in which the two middle surfaces intersect each other.

$$\overset{m}{u}_{x\,n} = -n^2\overset{m}{\mathcal{F}}_n^{(1)} - \nu\overset{m}{\mathcal{F}}_n^{(3)};\quad \overset{m}{u}_{\varphi\,n} = -n^3\overset{m}{\mathcal{F}}_n - (2+\nu)\overset{m}{\mathcal{F}}_n^{(2)}\,, \tag{22a}$$

$$\overset{m}{u}_{z\,n} = n^4\overset{m}{\mathcal{F}}_n - 2n^2\overset{m}{\mathcal{F}}_n^{(2)} + \overset{m}{\mathcal{F}}_n^{(4)}\,, \tag{22b}$$

$$\overset{m}{\vartheta}_{x\,n} = -\overset{m}{\psi}_{\varphi\,n} = \frac{1}{R_c}\left(n^4\overset{m}{\mathcal{F}}_n^{(1)} - 2n^2\overset{m}{\mathcal{F}}_n^{(3)} + \overset{m}{\mathcal{F}}_n^{(5)}\right)\,, \tag{23}$$

$$\overset{m}{N}_{xx\,n} = \frac{2\delta E}{R_c}\overset{m}{\mathcal{F}}_n^{(2)},\quad \overset{m}{N}_{x\varphi\,n} = \frac{2\delta E}{R_c}\overset{m}{\mathcal{F}}_n^{(3)}\,, \tag{24}$$

$$\overset{m}{M}_{xx\,n} = \frac{\delta E}{2\beta^4}\left[n^4\overset{m}{\mathcal{F}}_n^{(2)} - 2n^2\overset{m}{\mathcal{F}}_n^{(4)} + \overset{m}{\mathcal{F}}_n^{(6)} - \nu n^2\left(n^4\overset{m}{\mathcal{F}}_n - 2n^2\overset{m}{\mathcal{F}}_n^{(2)} - \overset{m}{\mathcal{F}}_n^{(4)}\right)\right]\,, \tag{25}$$

$$\overset{m}{Q}_{xz\,n} = \frac{\delta E}{2\beta^4R_c}\left[-n^6\overset{m}{\mathcal{F}}_n^{(1)} + 3n^4\overset{m}{\mathcal{F}}_n^{(3)} - 3n^2\overset{m}{\mathcal{F}}_n^{(5)} + \overset{m}{\mathcal{F}}_n^{(7)}\right]\,. \tag{26}$$

5. BOUNDARY- AND CONTINUITY CONDTIONS

A solution for the amplitude of the displacement field on the middle surface of the plate contains 4, while a solution for $\overset{m}{\mathcal{F}}_n$ involves 8 integration constants. In what follows we shall present those boundary- and continuity conditions, which provide the integration constants. We shall start from the inner boundary of the plate.

If the plate has no hole in it then the displacement $\overset{m}{w}_n(\rho = 0)$ and the rotation $\overset{m}{\vartheta}(\rho = 0)$ has to be finite. If there is a hole in the plate, the boundary conditions depend on the supports applied. However, two boundary conditions can be prescribed on the inner boundary only.

The shell and plate deform together on the intersection line of the middle surfaces of the shell and the plate so it is clear that the following kinematic continuity conditions should be fulfilled:

$$\overset{m}{v}_{\xi n}(\xi = 0) = -\overset{m}{w}_n(\rho = 1)\,, \qquad \overset{m}{u}_{\varphi n}(\xi = 0) = \overset{m}{v}_n(\rho = 1) = 0\,, \tag{27a}$$

$$\overset{m}{v}_{zn}(\xi = 0) - \overset{m}{u}_n(\rho = 1) = \begin{cases} 0 & n \neq 0 \\ \text{const.} & n = 0 \end{cases}\,, \qquad \overset{m}{\vartheta}_{rn}(\xi = 0) = \overset{m}{\vartheta}_{\varphi n}(\rho = 1)\,. \tag{27b}$$

Observe, that the second and third conditions reflect the fact that the plane stress problem is axisymmetric.

For the shear force $\overset{m}{Q}_{xzn}$ we can not prescribe any condition, since $\overset{m}{u}_{zn}(\xi = 0) = 0$.

As regards the axisymmetric part, equation

$$f_o + Q_{xzo} - f = 0 \tag{28}$$

should also be fulfilled.

Since $\overset{m}{v}_n(\rho = 1) = 0$ we can not prescribe continuity conditions for the inner forces $\overset{m}{N}_{R\varphi n}$ and $\overset{m}{N}_{x\varphi n}$. However the axisymmetric parts of these quantities are equal to zero.

It follows from the global equilibrium of the structure that the axisymmetric part of the shear force should meet the condition $Q_{Ro} = 0$. Otherwise the continuity condition

$$\overset{m}{Q}_{Rn}(\rho = 1) - \overset{m}{N}_{xxn}(\xi = 0) = 0 \tag{29a}$$

should be fulfilled.

As regards the bending moments equation

$$\overset{m}{M}_{Rn}(\rho = 1) - \overset{m}{M}_{xxn}(\xi = 0) = 0\,. \tag{29b}$$

is the continuity condition.

Since the boundary of the shell with coordinates $\xi = h/R_e$ is free, the following boundary conditions should be satisfied:

$$\overset{m}{N}_{xxn} = 0\,, \qquad \overset{m}{N}_{x\varphi n} + \frac{1}{R}\overset{m}{M}_{x\varphi n} = 0 \tag{30a}$$

$$\overset{m}{M}_{xxn} = 0\,, \qquad \overset{m}{Q}_{xzn} \quad \frac{n}{R_e}\overset{m}{M}_{x\varphi n} = 0\,. \tag{30b}$$

The boundary- and continuity conditions (27), (29) and (30) provide twelve homogenous equations for the twelve integration constants. These equations involve f as a parameter. The critical value of f can be determined from the condition that the system determinant should vanish.

6. NUMERICAL EXAMPLE

We have made numerical computations for a solid plate. The graphs in Fig. 4 provide the critical load of the plate in terms of the height h of the stiffening shell. It is clear from Fig. 4 that the stiffening significantly increases the critical load as the height h is increased till it reaches a certain limit. The curves show the critical load for the axisymmetric deformation and the first 3 members of the Fourier-series. We see that the lowest value of the critical load belongs to axisymmetric deformation. These computations have been made with the values $E = 2 \cdot 10^5$ MPa, $\nu = 0.3$, $R_e = 40$ mm, $\delta = 2b = 4$ mm.

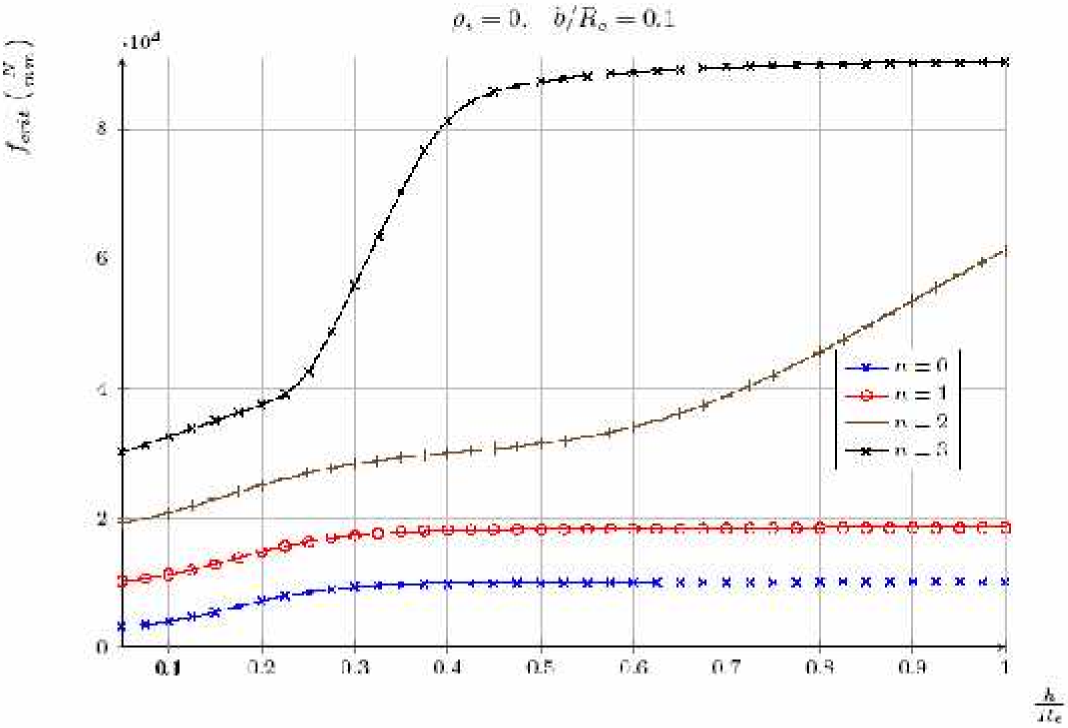

FIGURE 4. Critical load for the plate

7. CONCLUDING REMARKS

The present paper has established the equations that can be used to determine the critical load of a circular plate (solid or with a hole) stiffened by a cylindrical shell under the assumption of non-axisymmetric deformations. We have had a difficulty in clarifying what the continuity conditions are between the two separate elements of the structure. We have also presented a solution for a plate with no hole assuming axisymmetric and non-axisymmetric deformations. Further numerical examinations are in progress for a plate with a hole.

8. ACKNOWLEDGEMENT

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