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ANALYSIS OF THE DEVELOPED MIXED THERMAL CONVECTION BY USE OF 2ND LAW

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Abstract: *The present study refers to a fully developed thermal mixed convection between two vertical and infinite walls having imposed constant temperatures and for a given flow rate (or pressure gradient). Viscous dissipation and entropy production are successively estimated and then used to find an optimum distance between the plates which minimizes the total entropy production in the flow field. Results indicated that the definition of a entropy criteria to use between different convection regimes is more difficult to apply than a dissipation criteria.*

Key words: *entropy production, optimization, mixed convection.*

1. Introduction

For a long time, the optimization of thermal systems was based on energy considerations. More recently, the approach switched to exergy and entropy analyses that seem more appropriate for what it is called nowadays "sustainable development". Entropy production analysis was particularly used in describing irreversible thermodynamical processes [5]. But this theory, of rich conceptual content, did not offer in applied sciences the expected benefits due to its high complexity and lack of practical impact. It was however relaunched, in particular grace to the Bejan's works of [3], by a simpler and more pragmatic approach that strengthened the optimization based on the limitation of entropy production. However, most of the corresponding works were essentially related to forced convection [6, 7, 8, 14], and much rarely to mixed convection [4].

In addition, several authors emphasized the importance of clear distinguish, based on objective criteria, between forced, mixed and natural convection [12].

The two concerns are considered in the present work that considers a fully developed mixed convection flow between two parallel plates of imposed temperatures. This physical system is not only of theoretical interest, but is part of more engineering applications [1, 2], even if the geometry is more often of annular type [10]. The applications include the efficiency and safety of thermal nuclear plants, refrigeration equipments, and oil wells in profound waters [2]. The mixed thermal convection is also frequently encountered in equipments using renewable energy sources, like solar panels or geothermal heat pumps.

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2. System Description

The physical system considered here consists of a laminar fluid flow between two planar walls placed at $y = 0$ and $y = e$, respectively, whose temperatures T_1 and T_2 are imposed and (fig. 1).

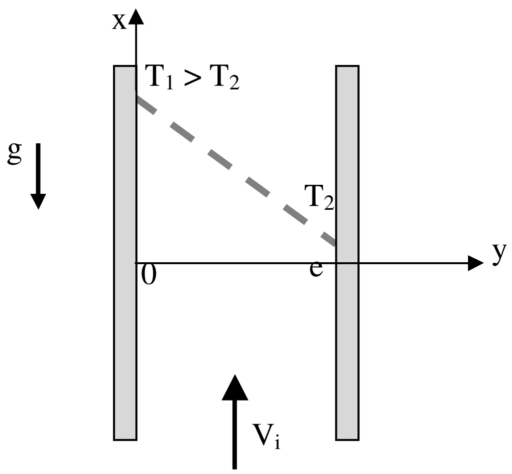

Fig.1. *Fully developed mixed convection flow between two vertical walls*

The fluid is assumed incompressible; the flow is globally upward, of incident velocity V_i. It is assumed that the viscous effects are negligible on the temperature field (in the examples considered here, the Brinkman number – product of Prandtl and Eckert numbers – is of order of magnitude 10^{-7}). Under these conditions, and denoting $\Delta T = T_1 - T_2$ for the temperature difference between the walls, the temperature profile is linear [13]

$$T = T_1 - \frac{\Delta T}{e} y \ \ [\mathrm{K}] \tag{1}$$

The velocity field is one-dimensional and may be expressed as [11]:

$$U = \frac{g\beta}{6\nu e}\Delta T\, y^3 - \frac{g\beta}{4\nu}\Delta T\, y^2 - 6\frac{V_i}{e^2}y^2 + \frac{g\beta}{12\nu}\Delta T\, e\, y + 6\frac{V_i}{e}y \ \ [\mathrm{m/s}] \tag{2}$$

Starting from the above mentioned considerations, the viscous dissipation entropy production is calculated and then used to define a criterion to distinguish between the nature of the convection: natural or mixed.

3. Dissipation Function

The local viscous dissipation Φ''' is given for the considered problem by:

$$\Phi'''(y) = \mu\left(\frac{dU}{dy}\right)^2 \ \ [\mathrm{W/m^3}] \tag{3}$$

It results that the viscous dissipation Φ'' over a cross sectional area is easily derived as:

$$\Phi'' = \mu\int_0^e \left(\frac{dU}{dy}\right)^2 dy \ \ [\mathrm{W/m^2}] \tag{4}$$

On the other hand, by use of eq. (1), the viscous dissipation may be expressed as:

$$\Phi'' = 1.39\times 10^{-3}\frac{(\rho g \beta \Delta T)^2 e^3}{\mu} + 12\mu\frac{V_i^2}{e} \ \ [\mathrm{W/m^2}] \tag{5}$$

The last expression is rendered dimensionless by adopting as reference the viscous dissipation associated with an isothermal flow, Φ''° :

$$\Phi''^{\circ} = 12\mu V_i^2 / e \ \ [\mathrm{W/m^2}] \tag{6}$$

The resulting dimensionless expression for viscous dissipation Φ^+ is :

$$\Phi^+ \equiv \frac{\Phi''}{\Phi''^{\circ}} = \frac{1.39\times10^{-3}}{12}\frac{(\rho\, g\, \beta \Delta T)^2 e^4}{\mu^2 V_i^2} + 1 \qquad (7)$$

The equivalent hydraulic diameter for the considered flow is equal to 2*e*; then, the Reynolds and Richardson numbers are, respectively:

$$\mathrm{Re} = \frac{\rho V_i \times 2e}{\mu} \;;\; \mathrm{Ri} = \frac{g\,\beta \Delta T \times 2e}{V_i^2} \qquad (8)$$

It results that Φ^+ depends only on the product Ri·Re, which is the thermal buoyancy coefficient:

$$\Phi^+ = 7.23\times10^{-6}(\mathrm{Ri}\cdot\mathrm{Re})^2 + 1 \qquad (9)$$

Moreover, the eq. (4) or (8) shows that a dissipation caused by the fluid floatability (term including ΔT or Ri·Re) is added to the dissipation that exists in an isothermal flow.

4. Entropy Production

In mixed convection, there are two sources of entropy production: the viscous dissipation Φ'' [W/m^2] that produces a « viscous entropy » σ''_v [W/m^{2}K], and the temperature gradient between the walls, that generates a « thermal entropy » σ''_{th} [W/m^{2}K].

By using eqs. (1), (2), and (3) in the viscous entropy production definition

$$\sigma''_v \equiv \int_0^e \frac{\Phi'''(y)}{T(y)}dy \quad [\mathrm{W/m^2K}], \qquad (10)$$

an analytical expression is obtained that can be integrated numerically. In a first approximation, for small and medium temperature differences ($\Delta T/T$ < 10%), the viscous entropy production may be expressed by dividing the viscous dissipation power to the mean temperature T_m (the temperature field being linear, $T_m = (T_1 + T_2)/2$) [9]:

$$\sigma''_{v,approx.} = \frac{\Phi''}{T_m} = 1.39\times10^{-3}\frac{(\rho\, g\, \beta \Delta T)^2 e^3}{\mu T_m} + 12\frac{\mu V_i^2}{e T_m} \quad [\mathrm{W/m^2K}] \qquad (11)$$

The latter equation has the benefit that it explicitly differentiates between the heat transfer and viscous effects.

On the other hand, the thermal entropy production σ'''_{th} [W/m^{3}K] is defined here as:

$$\sigma'''_{th}(y) \equiv \frac{\lambda}{T^2}\left(\frac{dT}{dy}\right)^2 \quad [\mathrm{W/m^3K}] \qquad (12)$$

For the physical system considered in fig. 1, the temperature gradient is constant [eq.(1)], and the thermal entropy produced over a cross sectional area σ''_{th} [W/m^{2}K] is easily calculated as:

$$\sigma''_{th} \equiv \int_0^e \sigma'''_{th}(y)\,dy = \frac{\lambda}{e}\frac{(\Delta T)^2}{T_1 T_2} \quad [\mathrm{W/m^2K}] \qquad (13)$$

By using a mean temperature in eq. (12), it is approximated for small ΔT's that:

$$\sigma''_{th,approx.} = \frac{\lambda}{e}\left(\frac{\Delta T}{T_m}\right)^2 \quad [\mathrm{W/m^2K}] \qquad (14)$$

From eqs. (10) and (13), a total entropy production can now be calculated:

$$\sigma''_{tot} = \sigma''_v + \sigma''_{th} \quad [\mathrm{W/m^2K}] \qquad (15)$$

For small ΔT's and using the mean temperature, the total entropy production may be approximated as the summation of eqs. (11) and (14):

$$\sigma''_{tot,approx.} = 1.39\times10^{-3}\frac{(\rho g \beta \Delta T)^2 e^3}{\mu T_m} + \frac{12\mu V_i^2}{e T_m} + \frac{\lambda}{e}\left(\frac{\Delta T}{T_m}\right)^2 \qquad (16)$$

Figure 2 represents the total entropy production σ''_{tot} as a function of distance e between the plates for a water laminar flow with an incident velocity in the range 0-0.007 m/s, and a temperature difference between the walls of 10 K.

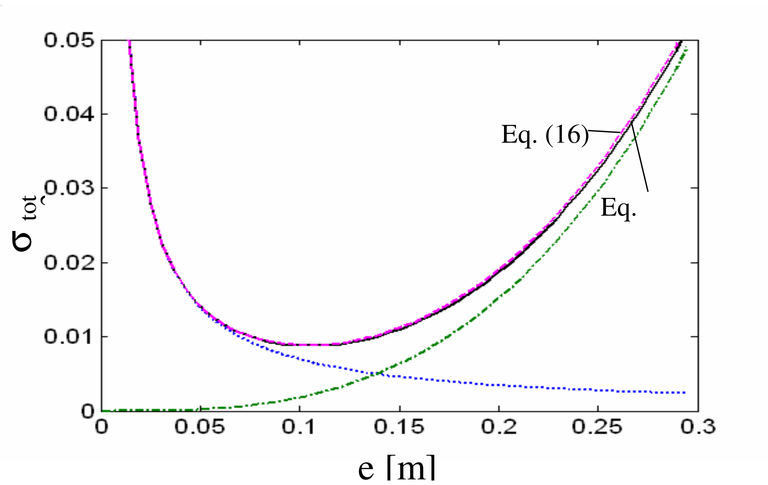

Fig. 2. *Entropy production as a function of distance e, for a water laminar flow.* $V_d = 0 - 0.007$ *m/s* ; $T_m = 293$ *K* ; $\Delta T = 10$ *K.*

It is shown that the velocity influence is not even visible on the plotted curves. Results from eq. (16) are also shown, indicating no significant difference from eq. (15) for ΔT as high as 50K. Due to its explicit content, this equation will be prefered in the subsequent analysis.

Figure 3 presents the three terms of eq. (16), aiming to show the relative importance of each term on the total sum. It results that the second term is really negligible and the competition is between the first and the third term. The sum σ''_{tot} is dominated by the thermal component at small separation distances e, but by the buoyancy entropy at large e-values.

Most interestingly is that the total entropy production $\sigma''_{tot}(e)$ presents a minimum value for the separation distance corresponding to $\partial\sigma''_{tot}/\partial e = 0$, that will be called « optimum » and denoted by e_{opt}. When using eq, (16), it results an explicit expression for it:

$$e_{opt} = 3.9352\left[\mu\,\frac{12\mu V_i^2 + \lambda\dfrac{\Delta T^2}{T_m}}{(\rho g \beta \Delta T)^2}\right]^{1/4} \qquad (17)$$

In other words, there is an optimal distance between the walls for which the total entropy production is minimal. This result is due to the inverse effects of thermal entropy and buoyancy entropy indicated in fig. 3. Figure 2 indicated no influence of incident velocity on the total entropy production, for all velocity values considered and laminar flow conditions. In this case, if $V_i = 0$ in eq. (17), the temperature difference disappear from the mathematical expression and the optimal distance e_{opt} associated with pure natural convection is only a function of the mean temperature and the corresponding thermophysical properties:

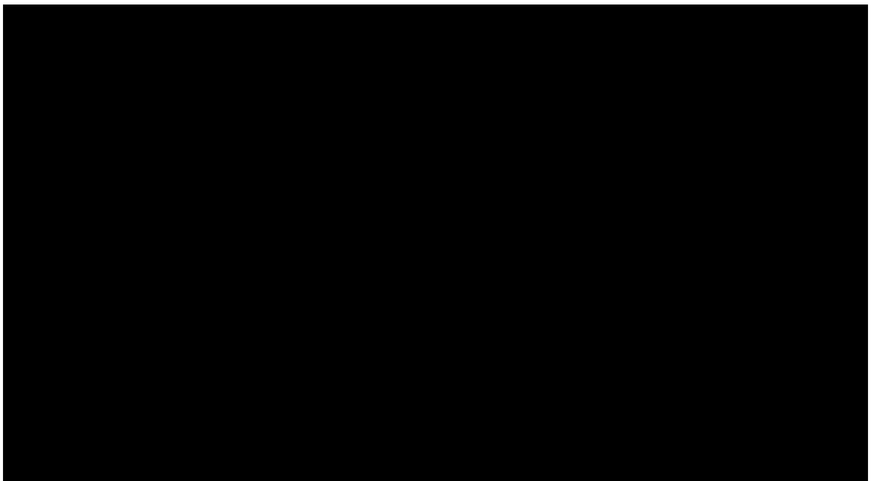

Fig. 3. *Buoyancy entropy (1st term), isothermal (2nd term), thermal (3rd term) and total* T_m = 293 *K*; ΔT =10 *K* ; V_d = 0,001 *m/s* [eq.(15)]

$$e_{opt}^{nc} = 3.9352\left[\frac{\mu\lambda}{(\rho g \beta)^2 T_m}\right]^{1/4} \qquad (18)$$

Figure 4 presents the variation of optimal distance e_{opt} with the mean temperature, for laminar flows and different fluids. For example, $e_{opt} = 8{,}8 \cdot 10^{-2}\, m$ for a water flow at T_m = 300 K.

5. Selection Criteria

The concepts of forced, natural or mixed convection are without doubt clear as definitions but do not exactly reflect the physics. The forced and natural convections are just limiting cases of the mixed convection, and in practical applications the convection mechanism is frequently of mixed type even if it is approximated otherwise. In addition, the literature offers little or incomplete information on how exactly to distinguish between the three convective regimes.

For the physical system presented in fig.1, previous works propose a number of selection criteria [11–13] that are based on ratios between the present forces. The present study proposes other criteria, based on dissipation and entropy production.

- **Dissipation criteria**

Analysis of eq. (4) or (8) suggest that two approaches for the selection criteria: comparison of the buoyancy dissipation (term on ΔT or on Ri·Re) with the dissipation in an isothermal flow, or with the total dissipation power.

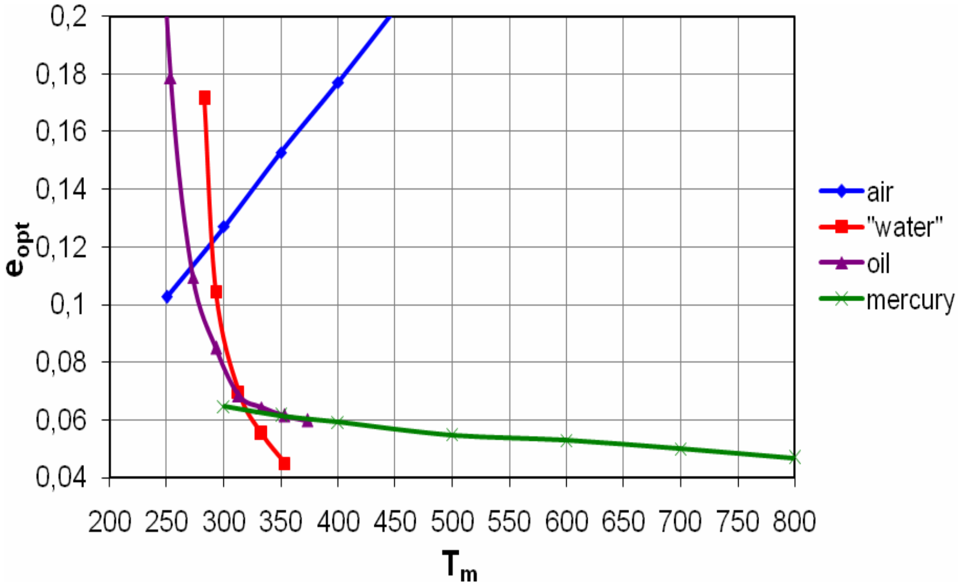

Fig. 4. *Optimal distance as a function of mean temperature, for laminar flows and different fluids*

In the first case, the selection criterion becomes:

$$\Gamma_d = \frac{7.23\times10^{-6}(\mathrm{Ri}\cdot\mathrm{Re})^2}{1} \qquad (19)$$
$$= 7.23\times10^{-6}(\mathrm{Ri}\cdot\mathrm{Re})^2$$

The interval of possible values for Γ_d is [0, +∞]. Then, forced convection may be associated with cases when the buoyancy dissipation is inferior to 5% of the isothermal dissipation (i.e., $\Gamma_d < 0{,}05$), and vice-versa, natural convection may be associated with cases when isothermal dissipation is less than 5% of the buoyancy dissipation (i.e., $\Gamma_d > 20$). The selection value of 5% is avidently arbitrary, but it is a reasonable value to be adopted by convention.

For a fixed value of Γ_d, eq. (18) indicates that the corresponding thermal buoyancy coefficient is:

$$\mathrm{Ri}\cdot\mathrm{Re} = 372\sqrt{\Gamma_d} \qquad (20)$$

The previous set values of Γ_d lead to the following:

$\Gamma_d < 0{,}05 \Rightarrow \mathrm{Ri}\cdot\mathrm{Re} < 83$

$\Gamma_d > 20 \Rightarrow \mathrm{Ri}\cdot\mathrm{Re} > 1663$

The second approach defines another selection criteria, K_d, that can vary between 0 and 1:

$$K_d = \frac{7.23\times10^{-6}(\mathrm{Ri}\cdot\mathrm{Re})^2}{7.23\times10^{-6}(\mathrm{Ri}\cdot\mathrm{Re})^2 + 1} \qquad (21)$$

Under these conditions, it is reasonable to assume forced convection if the buoyancy dissipation is inferior to 5% of the total dissipation (i.e., $K_d < 0{,}05$), and assume natural convection if the same term represents more than 95% of the total dissipation term ($K_d > 0{,}95$).

Equation (20) can be rewritten as:

$$\mathrm{Ri}\cdot\mathrm{Re} = \left[\frac{1}{7.23\times10^{-6}}\frac{K_d}{1-K_d}\right] \qquad (22)$$

It results that the above selection criteria become:

$K_d < 0{,}05 \Rightarrow \mathrm{Ri}\cdot\mathrm{Re} < 85$

$K_d > 0{,}95 \Rightarrow \mathrm{Ri}\cdot\mathrm{Re} > 1621$

The comparison of the selection values obtained for the product Ri·Re indicates magnitude of the selection values are that the two approaches are basically equivalent. Moreover, the orders of comparable with those derived in previous studies based on the friction force at the surface (resp. 15.2 and 5470 in [13]), or using other criteria (resp. 8.3 and 506, or 8.3 and 3325, or 38 and 2318 [12]).

• Entropy criteria

Another ways to set selection criteria is based on entropy production sources. The first approach is again a comparison in eq. (15) of the buoyancy entropy (1st term) and the isothermal entropy (2nd term). It results that the ratio of the two terms is exactly the criteria Γ_d previously defined and the use of entropy does not bring anything new.

A second approach is to divide the buoyancy entropy at the total entropy production, defining thus the criteria K_s:

$$K_S = \frac{1.39\times10^{-3}\dfrac{(\rho g \beta \Delta T)^2 e^3}{\mu T_m}}{1.39\times10^{-3}\dfrac{(\rho g \beta \Delta T)^2 e^3}{\mu T_m} + \dfrac{12\mu V_i^2}{e T_m} + \dfrac{\lambda}{e}\left(\dfrac{\Delta T}{T_m}\right)^2} \tag{23}$$

or its inverse:

$$\frac{1}{K_S} = 1 + 8.63\times10^3 \frac{\mu^2 V_i^2}{(\rho g \beta \Delta T)^2 e^4} + \frac{1}{T_m}\frac{\lambda\mu}{1.39\times10^{-3}(\rho g \beta)^2 e^4} \tag{24}$$

The last expression can be rewritten as:

$$\frac{1}{K_S} = 1 + \frac{1}{(\mathrm{Ri}\cdot\mathrm{Re})^2}\left[1.38\times10^5 + 0.115\times10^5 \frac{\Delta T}{T_m}\frac{1}{Br}\right] \tag{25}$$

where Br is the Brinkman number ($Br = \mu V_i^2 / \lambda \Delta T$).

Equation (24) can now be used to set selection values like K_S = 0.05 and K_S = 0.95, as it was done for K_d, but this would lead to expressions difficult to use. It seems then reasonable to not use entropy criteria as selection criteria between the three types of heat convection.

6. Conclusion

The study leads to analytical expressions for the viscous dissipation and total entropy production in laminar mixed convection associated with flow between parallel walls, expressions that put in evidence the influence of the thermal buoyancy. Results indicate an optimal separation distance between the walls for which the total entropy production is minimal. In addition, it was found a selection criterion useful to assess the convection mode (forced, mixed or natural) and it is reasonably based on dissipation term contributions. The limiting values are of the same order of magnitude as the values previously obtained based on other selection criteria.

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