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THE CALCULATION OF THE COEFFICIENT FOR THE REGENERATIVE LOSSES IN STIRLING MACHINES

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Abstract: *The coefficient of regenerative losses, X, is the term that includes all of the losses due to heat transfer in the regenerator. This parameter in turn depends on a large number of variables. Among these are piston speed, cylinder dimensions, regenerator dimensions, materials internal to the regenerator, gas proprieties, and the range of operating conditions. These variables are employed in a new technique for calculating the parameter X.The computed values of X were compared with estimated values of X based on experimental data available in the literature. Agreement between these values was found to be excellent, indicating that the technique for calculating X is accurate. This predictive capability should be a powerful tool in the design of effective Stirling machines..*

Key words: *irreversibility, losses, regeneration, Stirling, efficiency.*

1. Introduction

This paper presents a new technique for calculating the efficiency and power of actual operating Stirling machines. This technique is based on the First Law of Thermodynamics for processes with finite speed [1] and is used in conjunction with a new and novel PV / PX diagram [2,3] and a new method for determining the imperfect regeneration coefficient.

One of the objectives of this paper is to develop the method for determining the imperfect regeneration coefficient X, and to use it for calculating the efficiency and the power output of the Stirling engine.

Initially, the thermal efficiency is written as a function of three basis parameters.

$$\eta_{SE} = \eta_{SE} \cdot \eta_{IIirrev} = \\ = \eta_{CC} \cdot \eta_{II,irrev,\Delta T \cdot \Sigma\Delta Pi}, \quad (1)$$

where

$$\eta_{CC} = 1 - T_0 / T_{H,S} \quad (2)$$

is the Efficiency of a Carnot cycle operating between the same temperature limits as the Stirling engine.

The second law efficiency

$$\eta_{II,irrev,\Delta T} = 1/\left(1 + \sqrt{To/T_{H,S}}\right) \quad (3)$$

takes into account the irreversibility due to the temperature difference between the heat source and the gas in the engine. The second law efficiency

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$$\eta_{IIirrevX} = \left[1 + X\frac{1-\sqrt{T_O/T_{H,S}}}{(\gamma - 1)\ln\varepsilon}\right]^{-1} \quad (4)$$

takes into account the loses in the regenerator due to incomplete regeneration through use of the coefficient of losses, X. The second law efficiency

$$\eta_{IIirrevX} \cdot \Sigma\Delta P_i = \left[1 - \frac{\Sigma\Delta P_i \cdot \Delta V_i}{\eta' P_1 V_1 \sqrt{T_{H,S}/T_0} \cdot \ln\varepsilon}\right]^{-1} \quad (5)$$

with $\eta' = \left(1 - \sqrt{T_0/T_{H,S}}\right) \cdot \eta_{II,irrev,X}$ takes into account the irreversibility losses due to the pressure drop caused by the finite piston speed. The power output of the engine is

$$Pover_{SE} = \eta_{SE} \cdot mRT_{H,g} \cdot \ln\varepsilon \cdot (w/2S), \quad (6)$$

where ε is the compression ratio, w is piston speed, S is the stroke of the piston, and γ is the specific heat ratio.

A major loss in Stirling engines is caused by incomplete regeneration. An analysis for determining this loss is the primary objective of this paper. A second objective is to make a more realistic analysis of the pressure losses through use of a PV / Px diagram as will be described below (for details, see [4]). Finally, the power and efficiency, as determined by this analysis which involves the computation of X, is compared with performance data taken on twelve actual Stirling engines over a range of operating conditions [4].

2. Determination of losses, efficiency and power of the Stirling engine based on an intuitive PV / Px diagram for description of the cycle processes

Computation of pressure losses, work losses, efficiency and power for the processes shown on the new PV / Px diagrams [6] are made using the first law of thermodynamics for processes with finite speed. The first law written to specifically include these conditions is:

$$dU = \delta Q - P_{m,i} \cdot \left[\begin{matrix} 1 \pm aw/c \\ \pm b \cdot \Delta P_{thrott}/(2 \cdot P_{m,i}) \\ \pm f \cdot \Delta P_f / P_{m,i} \end{matrix}\right] dV \quad (7)$$

The irreversible work then is:

$$\delta W_{irrev} = P_{m,i} \cdot \left[\begin{matrix} 1 \pm \frac{aw}{c} \pm b \cdot \frac{\Delta P_{thrott}}{2 \cdot P_{m,i}} \\ \pm \Delta P_f / P_{m,i} \end{matrix}\right] dV \quad (8)$$

when applied to processes with finite speed.

The work expression for the finite speed isothermal irreversible compression process 12 (Fig.1) can be integrated using the Direct Method [5] to obtain:

$$W_{12,irrev} = \int_1^2 P_{m,i} dV + \int_1^2 \left[\begin{matrix} \frac{aw}{c} + \frac{b\Delta P_{thrott}}{2P_{m,cpr,i}} + \\ + \Delta P_f / P_{m,cpr,i} \end{matrix}\right] \cdot P_{m,cpr,i} dV \quad (9)$$

The work losses may be calculated for the compression process 12 by using eq. (9):

$$W_{12,losses} = W_{12,irrev} - W_{12,rev} = \left[\begin{matrix} \frac{aw}{c} P_{m,cpr,i} + \\ + \frac{b\Delta P_{thrott}}{2} + \Delta P_f \end{matrix}\right] \cdot (V_2 - V_1). \quad (10)$$

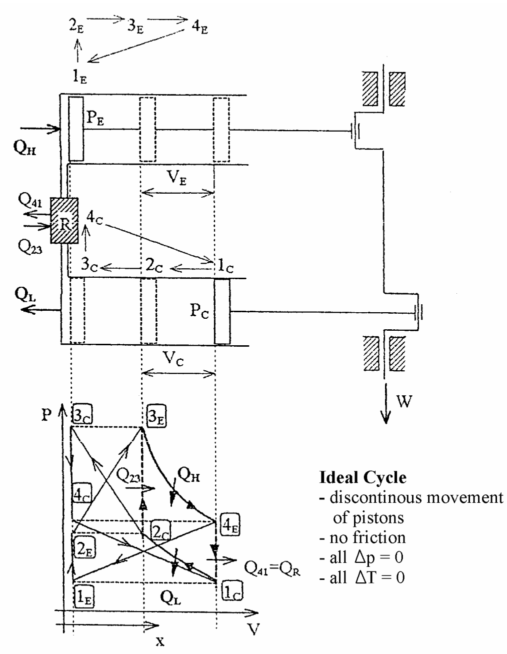

Fig. 1. *The new PV/Px diagram of the ideal Stirling cycle*

Computing and summing the losses due to finite speed of the pistons, throttling of the gas through the regenerator, and mechanical friction for the whole Stirling engine cycle, and introducing them in eq. (5), it becomes:

$$\eta_{II,irrev,\Sigma\Delta Pi} = 1 - \left[\begin{array}{l}\left(\frac{w}{w_{S,L}}\right)\gamma\left(1+\sqrt{\tau}\right)\ln\varepsilon + \\ 5N\left(\frac{w}{w_{S,L}}\right)^2 + \\ + \frac{3(0{,}94+0{,}045w)}{4p_4}\end{array}\right] / \left(\tau\eta' \ln\varepsilon\right) \qquad (11)$$

The heat input during the expansion process is also irreversible due to finite speed. In order to take account of this influence, an adjusting parameter z is introduced:

$$Q_{34} = z \cdot mRT_{H,g} \ln \varepsilon. \qquad (12)$$

Finally, the real power output of the engine, eq.(6) becomes:

$$Power_{SE,irrev} = \eta_{SE} \cdot zmRT_{H,g} \cdot \ln\varepsilon(w/2S). \qquad (13)$$

where the value of z was evaluated at 0,8 by comparison with available experimental data for twelve Stirling engines.

3. A method for calculating the coefficient of regenerative losses, X, in Stirling engine

The analysis resulted in differential equations that were then integrated. This integration is based on either a lump analysis, which gives pessimistic results, X_1, or on a linear distribution of the temperature in the regenerator matrix and gas (see fig. 2), which gives optimistic results, X_2.

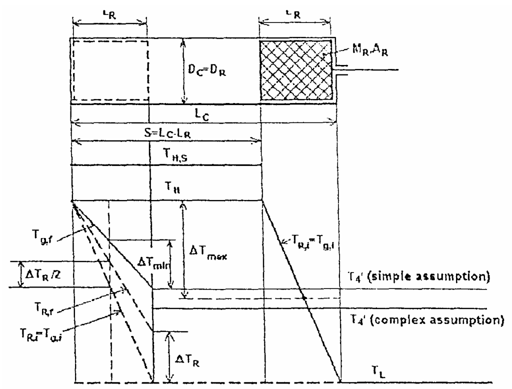

Fig. 2. *Gas and matrix temperature distribution in the regenerator*

The resulting expressions for are:

$$\begin{cases} X_1 = \dfrac{1+2M+e^{-B}}{2(1+M)}; \\ X_2 = \dfrac{M+e^{-B}}{(1+M)}. \end{cases} \qquad (14)$$

where:

$$\begin{cases} M = \dfrac{m_g c_{v,g}}{m_R c_R}; \\ B = (1+M)\dfrac{hA_R}{m_g c_{v,g}} \cdot \dfrac{S}{w}. \end{cases} \quad (15)$$

$$h = \frac{0{,}395\left(\dfrac{4P_m}{RT_L}\right) w_g^{0{,}424} c_P(T_m) v(T_m)^{0{,}576}}{(1-\tau)\left[1-\dfrac{\pi}{4[(b/d)-1]}\right] D_R^{0{,}576}\cdot \mathrm{Pr}^{2/3}} \quad (16)$$

with m_g is the mass of the passing through the regenerator, m_R is the mass of the screens of the regenerator, A_R is the surface area of the wires in the regenerator, v is the viscosity of the working gas, and h is the convective heat transfer coefficient in the regenerator (based on correlation given in).

The sensitivity of X_1 and X_2 to changes in operating variables such as the piston speed was determined. The computed values of X_1 and X_2 were compared with values of X determined from experimental data available in the literature [3-6]. The results based on the theory were found to predict the values from experimental data by using the following equation:

$$X = yX_1 + (1-y)X_2, \quad (17)$$

where y is an adjusting parameter with the value of 0,72.

The loss due to incomplete regeneration as determined through use of eq. (17) is the final loss to be considered in the analysis. The second law efficiency due to irreversibilities from incomplete regeneration is:

$$\eta_{II,irrev,X} = \left[1+\begin{pmatrix}0{,}72X_1+\\ +0{,}28X_2\end{pmatrix} \left(1-\sqrt{T_0/T_{HS}}\right)/ \left(R/c_v(T)\cdot \ln\varepsilon\right) \right]^{-1} \quad (18)$$

In Fig. 3-5 the variation of the coefficient of regenerative losses with the piston speed is represented for several values of the analysis parameters (d, S, porosity), and Fig. 6 illustrates the convective heat transfer coefficient dependence upon the piston speed.

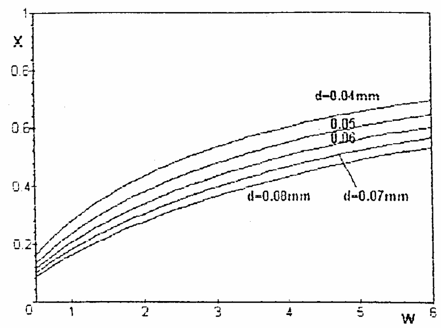

Fig. 3. *Coefficient of regenerative losses versus the piston speed for several values of the wire diameter (D_C = 60mm, D_R = 60mm, P_m = 50 bar, S = 30mm, N = 700, τ = 2)*

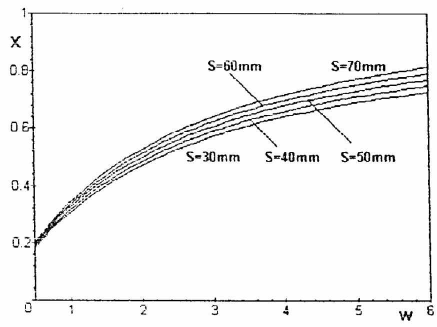

Fig. 4. *Coefficient of regenerative losses versus the piston speed for several values of the piston stoke (DC = 60mm, DR = 50mm, Pm = 50bar, d = 0.05mm, N = 700, τ = 2)*

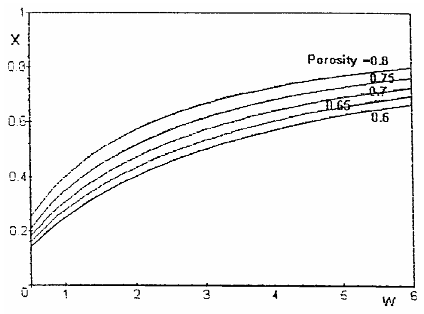

Fig. 5. *Coefficient of regenerative losses versus the piston speed for several values of matrix porosity (DC* = 60mm, *DR* = 50mm, *Pm* = 50 bar, *S* = 30mm, *d* = 0.05mm, *N* = 700, τ = 2*)*

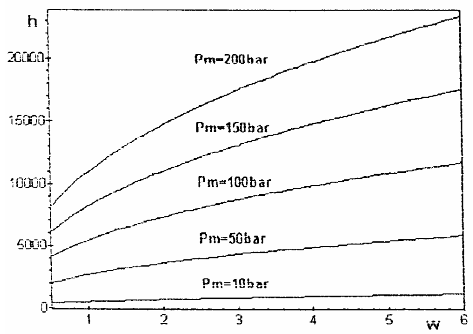

Fig. 6. *Convective heat transfer coefficient in the regenerator versus the piston speed for several values of the average pressure of the working gas (DR*=50, *b/d* =1.5, τ = 2*)*

4. Comparison of analytic results with the operating performance of actual Stirling engines

The results of computations of efficiency and power output based on this analysis are compared to performance data taken from twelve operating Stirling engines in Figs. 7-8 and in Table I.

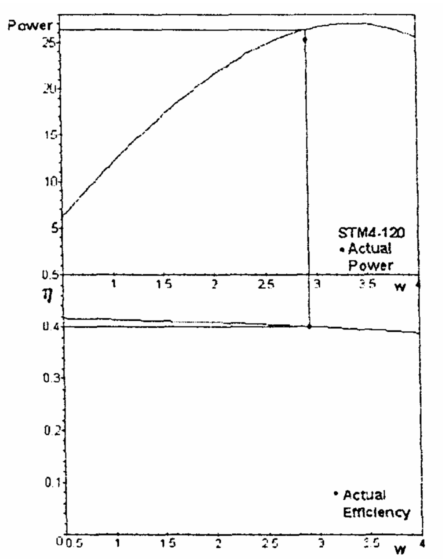

Fig. 7. *Comparison of the analysis results with actual performance data for the STM*4-120 *Stirling engine* [6]

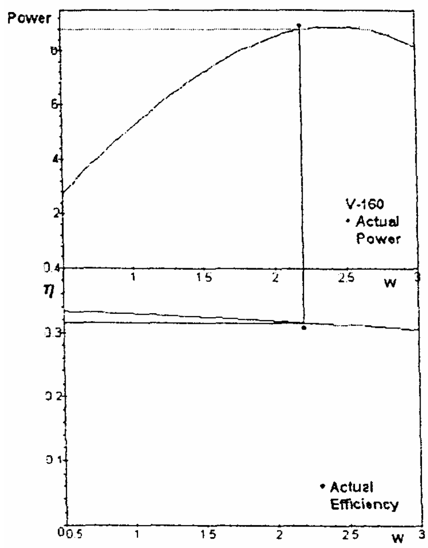

Fig. 8. *Comparison of the analysis results with actual performance data for theV-160 Stirling engine* [6]

Comparison between the analytical results and actual engine performance data Table 1

Stirling Engine	Actual Power [kW]	Calculated Power [kW]	Actual Efficiency	Calculated Efficiency
NS-03M, regime 1 (economy)	2.03	2.182	0.359	0.3392
NS-03M, regime 1 (max. power)	3.81	4.196	0.31	0.3297
NS-03T, regime 1 (economy)	3.08	3.145	0.326	0.3189
NS-03T, regime 1 (max. power)	4.14	4.45	0.303	0.3096
NS-30A, regime 1 (economy)	23.2	29.45	0.375	0.357
NS-30A, regime 1 (max. power)	30.4	33.82	0.33	0.3366
NS-30S, regime 1 (economy)	30.9	33.78	0.372	0.366
NS-30S, regime 1 (max. power)	45.6	45.62	0.352	0.3526
STM4-120	25	26.36	0.4	0.4014
V-160	9	8.825	0.3	0.308
4-95 MKII	25	28.4	0.294	0.289
4 – 275	50	48.61	0.42	0.4119
GPU-3	3.96	4.16	0.127	0.1263
MP1002 CA	200W	193.9W	0.156	0.1536
Free Piston Stirling Engine	9	9.165	0.33	0.331
RE-1000	0.939	1.005	0.258	0.2285

This figures show that there is high degree of correlation between this analysis and the operational data. This indicates that this analysis can be used to accurately calculate X and of other losses. Therefore, this analysis can be used to accurately predicting Stirling engine performance under a wide range of conditions. This capability should be of considerable value in Stirling engine design and in the prediction the performance of a particular Stirling engine over a range of operating speed.

The strong correlation between the analytical results and actual engine performance data also indicates that the Direct Method of using the first law for processes with finite speed is a valid method of analysis for irreversible cycles.

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