



The 4th International Conference  
"Computational Mechanics  
and Virtual Engineering"  
COMEC 2011  
20-22 OCTOBER 2011, Brasov, Romania

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## CONDITION MONITORING AND FAULT DIAGNOSIS IN ROTATING MACHINERY USING MODAL TEST & FINITE ELEMENT ANALYSIS

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**Abstract:** Rotor dynamics is the study of vibration behavior in axially symmetric rotating structures. Devices such as engines, motors, disc drives and turbines all develop characteristic inertia effects that can be analyzed to improve the design and decrease the possibility of failure. At higher speeds, the inertia effects of the rotating parts must be represented in order to predict the rotor behavior. Excess vibration can cause noise and wear in structure. It is important to identify all the critical speeds within the range of operation and analyse the damping effects, mass unbalance and other phenomena also their effects in the safe operation.

The experimental technique used thus far is called Modal Testing, a well known and widely used technique in research and industry to obtain the Modal and Dynamic response properties of structures. The technique has recently been applied to rotating structures and some research papers been published, however the full implementation of Modal Testing in active structures and the implications are not fully understood and are therefore in need of much further and more in depth investigations.

The raw data obtained from experiment was used in finite element (FE) model for comparison. Since it has good capability for Eigen analysis and also good graphical facility, and obtained good result. 3-D models result large number of nodes and elements. This paper demonstrates how to extract a plane 2-D model from the 3-D model that can be used with fewer nodes and elements because of ease of use, accuracy and performance.

The aims is to establish a system identification methodology using the analytical/computational techniques and update the model using experimental techniques already established for passive structures but to active rotating structures, which subsequently help to carry out health monitoring as well as further design and development in rotating machinery.

**Keywords:** modelling, vibration, balancing, damping.

### 1. INTRODUCTION

Modal analysis has been used in the last two or three decades in many engineering discipline and technology fields to solve increasingly demanding structural dynamic problems,[1,2].Modal analysis has become a major technology in the quest for determining ,improving and optimization dynamic characteristics of engineering structure and has also discovered profound application for civil and building structure ,biomechanical problems ,space structures ,acoustical instrument, transportation and nuclear plants,[1,3&4].Cotemporary design of complex mechanical ,aeronautical or civil structure requires them to become increasingly lighter ,more flexible and yet strong, [1].Description of the mode shape using just sufficient detail and accuracy to permit there identification and correlation with those the theoretical model. Modes (or resonances) are inherent properties of a structure. Resonances are determined by the material properties (mass, stiffness, and damping properties), and boundary conditions of the structure. Each mode is defined by a natural (modal or resonant) frequency, modal damping, and a mode shape. If either the material properties or the boundary conditions of a structure change, its modes will change. For instance, if mass is added to a vertical pump, it will vibrate differently because its modes have changed. At near the natural frequency of a mode, the overall vibration shape (operating deflection shape) of a machine or structure will tend to be dominated by the mode shape of the resonance. Rotor dynamics is the study of vibration behavior in axially symmetric rotating structures. Devices such as engines, motors, disc drives and turbines all develop characteristic inertia effects that can be analyzed to improve the design and decrease the possibility of failure. At higher rotational speeds, such as in a gas turbine engine, the inertia effects of the rotating parts must be consistently represented in order to accurately predict the rotor behavior [2, 4].

### 1.1. Measurement of (FRF) frequency response function

Regenerated FRF curves, synthesis of (FRF) curves there are two main requirements in the form of response Model, The first being that of regenerating “Theoretical” curve for the frequency response function actually measured and analysis and the second being that of synthesising the other functions which were not measured, (FRF) that isolates the inherent dynamic properties of a mechanical structure.

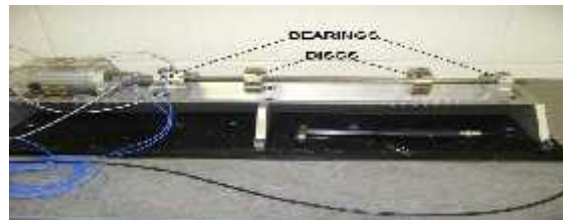
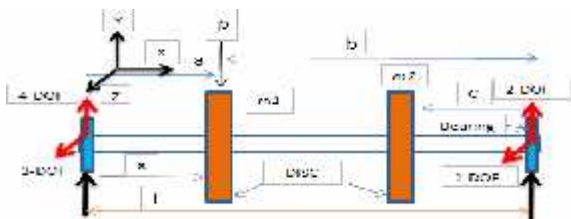
### 1.2. Existing technique used for modelling

Considering the difficulties and limitations in the modelling and/or the modal testing for rotating machines, the most promising avenue is to identify a model of the foundation directly from measured vibration data, and use this with a good FE model for the rotor and a fairly accurate model for the fluid bearings. The research work in this direction was initiated by Lees [5], in 1988, and since then a number of approaches have been proposed by researchers across the world.

The modal model (natural frequencies, mode shapes and modal damping) of a complete machine based on the in-situ experimental modal tests is one possible modelling approach. Irretier [6]. The dynamics of the fluid bearings during machine operation and the rotation of the shaft itself influence the dynamic behaviour of the complete machine, which imposes certain limitations on the modal testing for rotating machines. Bucher and Ewins [7], have discussed these issues in detail. Irretier [8], gave the mathematical background to extract the modal model for rotating structures which are characterised by non-symmetric and time-variant matrices, unlike the assumptions for the experimental modal analysis of stationary structures, namely a linear, time-invariant system where reciprocity holds.

Another alternative is the use of a modal model of the foundation alone together with the FE model of the rotor and the mathematical model for the bearings. Pennacchi et al, [9]. In recent years, with continuing demands for increased performance, many rotating industrial machines are now being designed for operation at high speed, a trend which has resulted in increased mechanical vibration and noise problems [9, 10&11].

## 2. METHODES



**Figure 1:** Schematic of a rotor two discs setup parameter [2,12]. **Picture 1:** Experimental setup for the modal testing.

### 2.1. Equation of motion

#### 2.1.1. Definition of frequency response functions (FRF).

The derivative processes of FRFs are described here and the detail scan are founding. The FRF data of any structure can also be obtained through experimental modal testing or by means of the FE simulation method [12]. For the vibration equation of a system, the mathematical model can be expressed as follows [12]:

$$M \ddot{x} + C \dot{x} + K x = f \tag{1}$$

Where  $M$ ,  $C$ , and  $K$  are the mass, damping, and stiffness matrices of the system, respectively. Moreover,  $x$  and  $f$  are the displacement and external force vectors, respectively. In Eq.(1), the displacement vector of the system can be represented in the modal coordinate with the following mode shape matrix:

$$x_{N \times 1} = \sum_{r=1}^N \eta_r v_r = U_{N \times N} \eta_{N \times 1} \tag{2}$$

Where  $N$  is the total number of components in the modal coordinate ;  $\eta_r$  and  $v_r$  are the components of the modal coordinate and the mode shape vector at the  $r^{\text{th}}$  mode, respectively;  $U = [v_1, v_2, \dots, v_N]$  denotes the mode shape matrix; and  $\eta = [\eta_1, \eta_2, \dots, \eta_N]^T$  refers to the modal coordinate vector. Substituting Eq. (2) into Eq. (1), the equation of spatial motions transformed into the equation of decoupled motion:

$$M U \ddot{\eta} + C U \dot{\eta} + K U \eta = f \tag{3}$$

Multiplying the matrix  $U^T$  at both sides of Eq. (3) produces the following:

$$M \ddot{\eta} + C \dot{\eta} + K \eta = U^T f = \mu \tag{4}$$

Where  $M$ ,  $C$  and  $K$  are the diagonal mass, damping, and stiffness matrices, respectively; and  $\mu$  is the modal force vector. The equation of motion at the  $r^{th}$  mode is represented as follows:

$$M_r \ddot{\eta}_r + C_r \dot{\eta}_r + K_r \eta_r = \mu_r \quad (5)$$

Where  $\mu_r = v_r^T f = \sum_{j=1}^N \phi_{jr} f_j$  and  $\phi_{jr}$  the  $j^{th}$  components of the  $r^{th}$  mode shape vector. By considering the harmonic excitation acting on the system, the component to  $f$  the modal coordinate and the modal force at the  $r^{th}$  mode can be designated as  $\eta_r = \hat{\eta}_r e^{i\omega t}$  and  $\mu_r = \hat{\mu}_r e^{i\omega t}$  respectively. Eq. (2) becomes the summation of a series consisting of the exciting force, components of the mode shape, and the modal parameters:

$$\hat{x}_{n \times 1} = \sum_{r=1}^N \hat{\eta}_r v_r = \sum_{r=1}^N \sum_{j=1}^N \frac{\phi_{jr} \hat{f}_j}{M_r [\omega_r^2 - \omega^2 + 2i\omega \zeta_r \omega_r]} v_r \quad (6)$$

Where  $x = \hat{x} e^{i\omega t}$  is the displacement vector of the spatial model, and  $f_j = \hat{f}_j e^{i\omega t}$  is the exciting force. In addition,  $\omega_r$  and  $\zeta_r$  stand for the  $r^{th}$  natural frequency and damping ratio (called modal parameters), respectively. Considering just a single component  $f_j$  of the exciting force, the  $i^{th}$  component of the displacement vector can be rewritten as

$$\hat{x}_i = \sum_{r=1}^N \sum_{j=1}^N \frac{\phi_{ir} \phi_{jr} \hat{f}_j}{M_r [\omega_r^2 - \omega^2 + 2i\omega \zeta_r \omega_r]} \quad (7)$$

The (FRF)  $H_{ij}$  is defined as the ratio of the  $i^{th}$  displacement component  $\hat{x}_i$  to the  $j^{th}$  exciting force component  $\hat{f}_j$ . It can also be shown as follows:

$$\frac{\hat{x}_i}{\hat{f}_j} = H_{ij} = \sum_{r=1}^N H_{ij,r} = \sum_{r=1}^N \frac{\phi_{ir} \phi_{jr}}{M_r [\omega_r^2 - \omega^2 + 2i\omega \zeta_r \omega_r]} \quad (8)$$

Where  $H_{ij,r}$  is the peak value of FRF at the  $r^{th}$  mode. Eq.(8) represents the relationship between the single exciting force and displacement. However, the situation of multiple excitations is not considered in the following theory.

### 2.1.2. The Pseudo Mode Shape Method

The simple system of a rotor-bearing-establishment system is shown above (see Fig. 1), that demonstrates the (MBM) operation. The rotor was treated as them other structure, while the foundation and two bearing supports were treated as the sub structure. Considering only the translation (DOF) sin the Y and Z directions at each of the bearing supports, the mode shape vectors of the bearing-foundation structure were represented with a total of four (DOFs). Eq. (8) involves the first four natural Frequencies ( $N=4$ ) that can be expressed as follows [1]:

$$H_{ij} = \frac{\phi_{i1}\phi_{j1}}{M_1[\omega_1^2 - \omega^2 + 2i\zeta_1\omega_1]} + \frac{\phi_{i2}\phi_{j2}}{M_2[\omega_2^2 - \omega^2 + 2i\zeta_2\omega_2]} + \frac{\phi_{i3}\phi_{j3}}{M_3[\omega_3^2 - \omega^2 + 2i\zeta_3\omega_3]} + \frac{\phi_{i4}\phi_{j4}}{M_4[\omega_4^2 - \omega^2 + 2i\zeta_4\omega_4]} \quad (9)$$

Where  $i=1,2,\dots,4$  and  $j=1,2,\dots,4$ . Assume that the first term at the right-hand side of Eq.(9) was dominant in affecting the values of the (FRFs) when  $\omega = \omega_1$  where as the other terms had a weak influence on these FRFs. Thus, the second, third, and fourth terms can be omitted. The simplified form of Eq. (9) can now be presented as follows:

$$H_{ij,1} = \frac{\phi_{i1}\phi_{j1}}{M_1[\omega_1^2 - \omega^2 + 2i\zeta_1\omega_1]} \quad (10) \text{ As}$$

mentioned, with two (DOFs) at each of two bearing supports, there were a total of four (DOFs) for all of the supports of the whole system [10]. After this, a (4\*4) symmetric frequency response function matrix was built as follows:

$$\text{FRF matrix} = \begin{bmatrix} H_{11,1} & & & \\ H_{21,1} & H_{22,1} & & \\ H_{31,1} & H_{32,1} & H_{33,1} & \\ H_{41,1} & H_{42,1} & H_{43,1} & H_{44,1} \end{bmatrix} \quad (11)$$

At the first mode ( $r=1$ ), the available peak values in the (FRF) data were denoted as  $H_{11,1}$ ,  $H_{21,1}$ ,  $H_{31,1}$ ,  $H_{41,1}$ ,  $H_{22,1}$ ,  $H_{32,1}$ ,  $H_{42,1}$ ,  $H_{33,1}$ ,  $H_{43,1}$ , and  $H_{44,1}$ . These values can then be utilised to evaluate the related components.

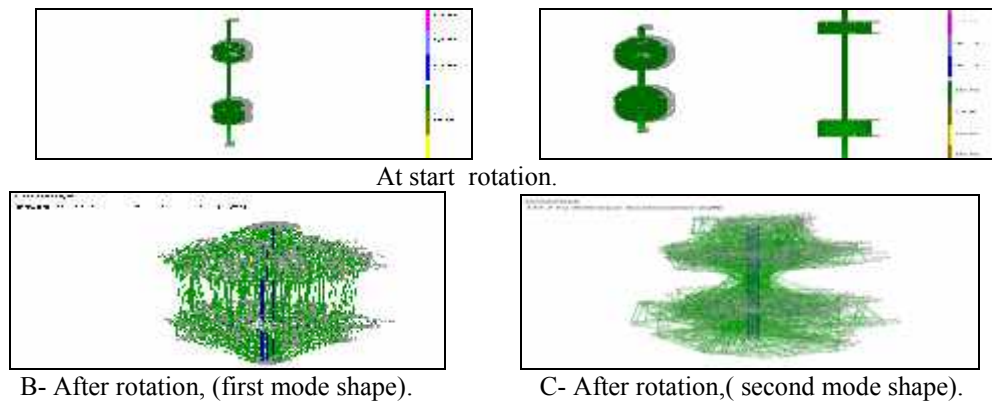
## 2.2. Rotate acquisition and analysis in experimental

The Rotate Acquisition and Analysis software package is part of (m+ p international's) It is designed for trouble shooting and analyzing noise or vibration problems related to the speed characteristics of rotating or reciprocating components of a machine in operation[13,14&15].

### 2.2.1. Test Setup

Experimental modal analysis has grown steadily in popularity since the advent of the digital FFT spectrum analyser in the early 1970's. Today, impact testing (or bump testing) has become widespread as a fast and economical means of finding the modes of vibration of a machine or structure. The test rotor is shown in picture .1. Basically, the rotor

consisted of a shaft with a nominal diameter of 10 mm, with an overall length of 610 mm. Two journal bearings, RK4 Rotor Kit made by Bentley Nevada (the advanced power systems energy services company), could be used to extract the necessary information for diagnostic of rotating machinery, such as turbines and compressor. Been testing the process will be conducted on the rotary machine as the project is based on rotary dynamics reach practical results for the purpose of subsequently applied machinery rotary by using (Smart office program, the smart Office is the software which is used in this project [16]. Then, do the experimental testing using the impact test, installed fixed two accelerometer (model 333B32, sensitivity 97.2 & 98.6 mv/g) in Y&Z direction and roving the hammer (model 4.799.375, S.N24492) on each point for the purpose of generating strength of the movement for the vibration body and the creation of vibration for that with creating a computer when taking readings in file that was dimensions and introducing it with the data within the program (Smart office) [13, 16]. Configurations of testing on the rotary machines, all necessary equipment for test was shown with the design geometry wizard (see Fig. 2).



**Figure 2:** Design of geometry for modal (two discs in effective length), experimental test using smart office.

### 2.3. Simulation of a Model in (ANSYS 12)

A model of rotor system two discs with multi degree of freedom (Y and Z directions) has been used to demonstrate the above capability Fig. 3, 4. A program has been written in (ANSYS 12), Postprocessing commands (/POST26). Applying of gyroscopic effect to rotating structure was carried by using (CORIOLIS) command. This command also applies the rotating damping effect. (CMOMEGA) specifies the rotational velocity of an element component about a user –defined rotational axis. (OMEGA) specifies the rotational velocity of structure about global Cartesian axes. Model the bearings using a spring/damper element (COMBIN 14). Another command which was used in input file (SYNCHRO) that Specifies whether the excitation frequency is synchronous or asynchronous with the rotational velocity of a structure in a harmonic analysis; [14]. The finite element (FE) method used in ANSYS offers an attractive approach to modelling a rotor dynamic system.



A-ANSYS (APDL).

B-(ANSYS workbench).

**Figure 3:** A-Finite element modal rotating machinery (Two discs), three dimensions (3-D),



A- Two discs (2-D).

B- Two discs (2-D).

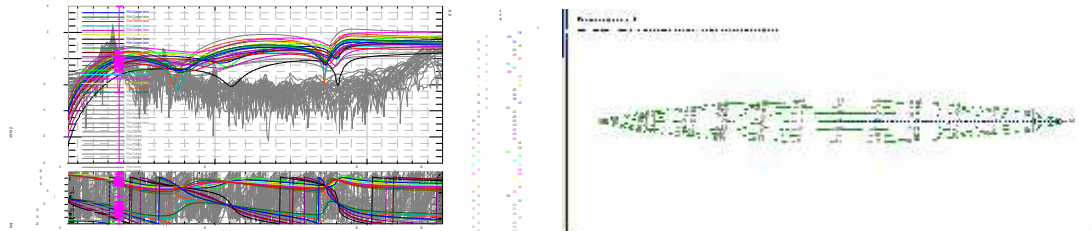
**Figure 4:** A-Finite element modal rotating machinery ANSYS (APDL).Two dimensions (2-D);

## 3. RESULTS, TABLES & FIGURES

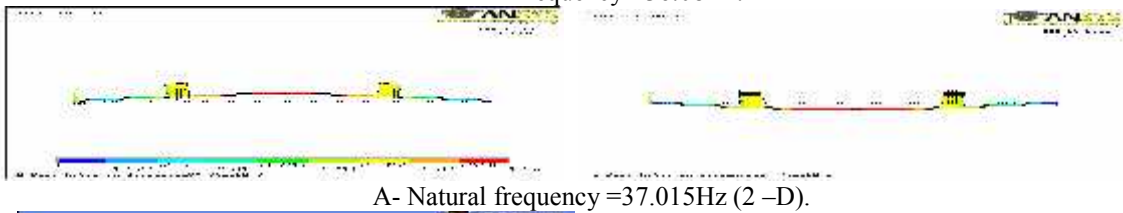
As it can be seen from the experimental the result of natural frequency and mode shape closest match each other, for the two discs Fig. 5, 6 first mode & Fig.7,8 for the second mode & Fig. 9,10 for the third mode shape, we see all the result closest match each other between the experimental and simulation (ANSYS), we can see the result in Table 1 and Fig.11 for comparison. Frequency response function (FRF) of the rotor was found using the impact test mechanism and smart office software. We find the damping ratio ( $\zeta$ ) for different mode shape by curfitting, [10], (multi degree of

freedom system) in experimental part, Table 2 and Fig.12. We draw graph comparison between measured(experimental) & predict natural frequency (ANSYS) ,(Hz) Fig.14 show the slope approximately nearby from 45°. With further simulation we find the relation between the reaction forces with respect time can we see from the Fig. 16-A,B,C,D ,the performance of reaction forces in the right and left bearings with different speeds of rotations.. When comparison with ANSYS Fig.20-C we can prove when add 16 gram mass at the same condition and dimension for two discs, can reduce the amplitude of vibration as well. As it can be seen the results and graphs are matching with each other in both experimental work and ANSYS simulation models for approval when add 16 gram mass can reduce vibration amplitude (FRF).

**3.1. Modelling effect; (two discs).**



**Figure 5:** Stationary load in the middle, in the two discs, (FRF) versus Frequency (Hz), (first mode shape). Natural frequency =36.08Hz.



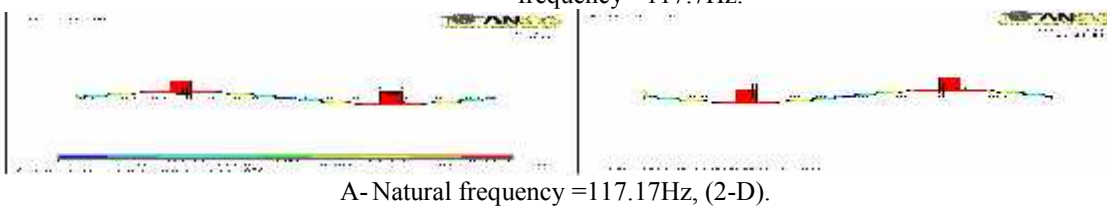
A- Natural frequency =37.015Hz (2 -D).



**Figure 6.** Finite element method simulations, first mode shape two discs, with 2 bearing, B-Natural frequency=37.03Hz (3-D).



**Figure 7:** Stationary load in the middle, in the two discs, (FRF) versus Frequency (Hz), (second mode shape). Natural frequency =117.7Hz.



A- Natural frequency =117.17Hz, (2-D).

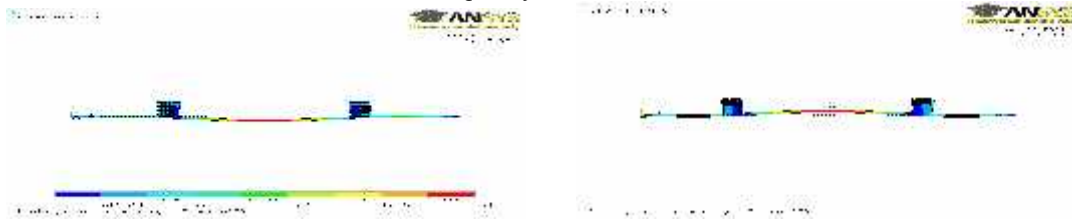


B- Natural frequency =117.21Hz, (3-D).

**Figure 8:** Finite element method simulations, two discs with 2 bearing second mode shape,



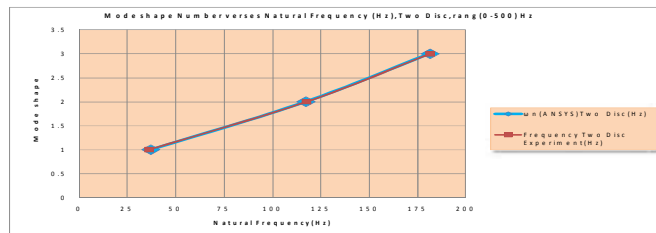
**Figure 9:** Stationary load in the middle, in the two discs, (FRF) versus frequency (Hz), (third mode shape). Natural frequency = 181.6Hz.



**Figure 10:** Finite element method, two discs with 2 bearing (2-D), third mode shape, Natural frequency = 181.34Hz, (2-D).

**Table 1:** Comparison between natural frequency outcome from experiment & ANSYS, (two discs).

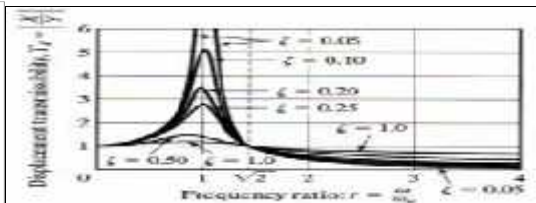
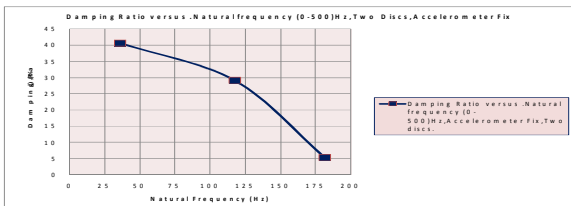
Mode shape	$\omega_n$ (ANSYS)Two Disc(Hz)	Frequency Two Disc Experiment(Hz)	Error %
1	37.015	36.08037	2.525003377
2	117.17	117.7	-0.452334215
3	181.34	181.6	-0.143377082



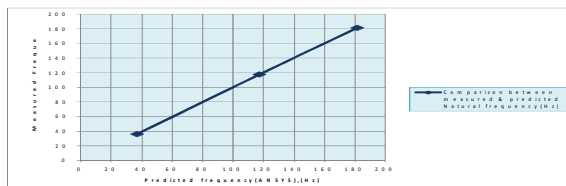
**Figure 11:** Natural frequency, experiment versus ANSYS, (two discs).

**Table 2:** Natural frequency and damping ratio ( $\zeta$ ) for two discs rang (0-500) Hz. (Experimental part).

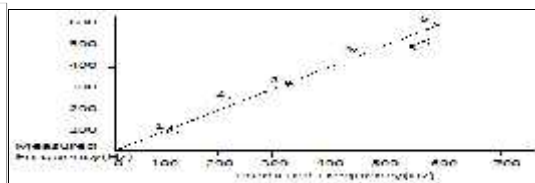
Name	Frequency (Hz)	Damping ratio ( $\zeta$ )%	Modal A[kg/s]
Model 1	36.08037	40.715	1.276808E-03 + I 2.003253E-03
Mode 2	117.74	29.095	5.904288E-04 + I 1.617448E-03
Mode 3	181.6	5.337	2.515677E-04 + I 3.83315 E-05



**Figure 12:** Damping ratio( $\zeta$ ) versus natural frequency (0-500)Hz, **Figure 13:** Variation of Td with r,[3].

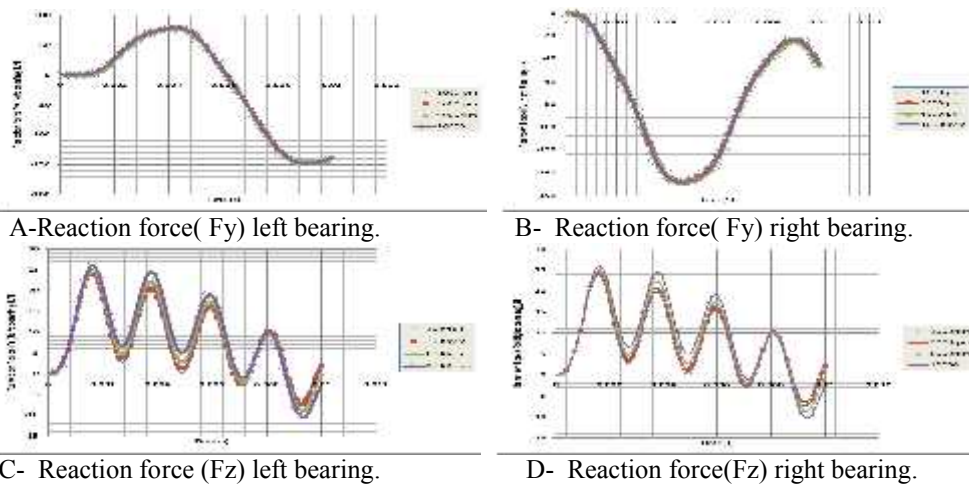


**Figure 14:** Comparison between measured & predicted frequency (Hz).



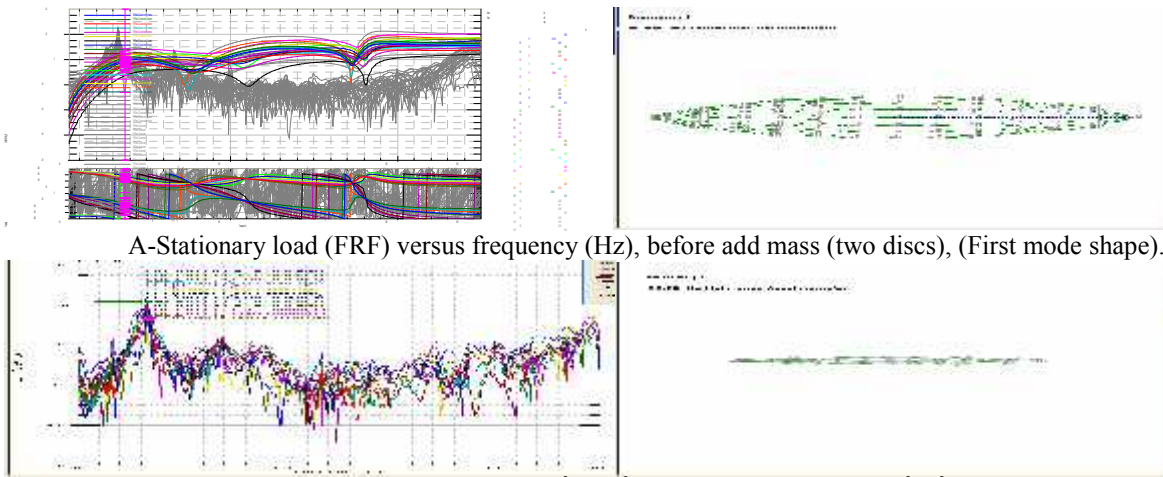
**Figure 15:** measured & predicted frequency (Hz),[12].

### 3.2. Reaction forces in the left and right bearings(two discs)

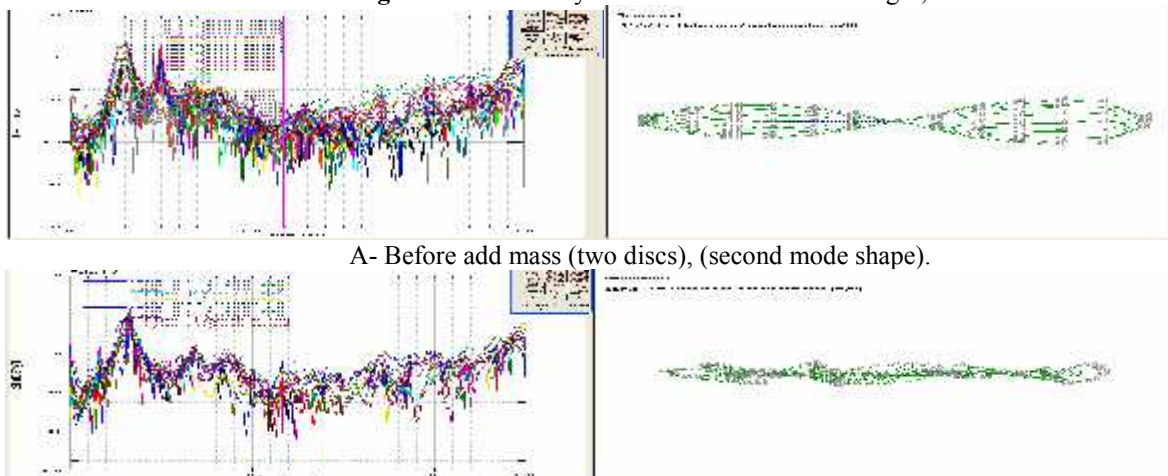


**Figure 16.** Relation between reaction bearing forces with time at different speed of rotation(two discs)

**3.3. Unbalance effect;** Add the mass at speed of rotation (two discs in the effective length).



**Figure 17:** Stationary load 2 discs in effective length;



**Figure 18:** Stationary load (FRF) versus frequency (Hz);



A- Before add mass (two discs), (third mode shape).



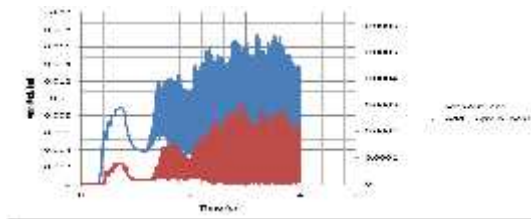
B-After add the mass (8 gram at disc 1, (45°, 225°) & 8 gram at disc 2, (180°, 0°), at speed of rotation, Two Disc (third mode shape).

**Figure 19:** Stationary load (FRF) versus frequency (Hz);

### 3.4. ANSYS Results



A- Displacement versus time before adds mass. B-Displacement versus time after add mass 8gram to each disc.



C- Merge comparison.

**Figure 20:** Amplitude verses time,( A-With out load ,B-After add 16 gram mass,C-Merge);

## 4. DISCUSSION AND CONCLUSIONS

In this paper investigate the behaviour of a rotor system with stationary load in the middle (Two discs) .Modern experimental modal analysis techniques have been reviewed. The three main topics pertaining to modal testing; (FRF) measurement techniques, excitation techniques and modal parameter estimation (curve fitting) methods were covered. The simulation values obtained from the (ANSYS) Fig.6, 8 &10 in two discs to find natural frequency and draw mode shape are perfectly nearby to approval values obtained from the experimental that shown Fig. 5,7& 9 for two discs, Comparison value with ANSYS ,are shown in Table 1 above and is clear in the Fig. 11.From the Fig.12 notes the decreased the damping ratio ( $\zeta$ ) caused increased Natural frequency until reach maximum amplitude when the system

reach resonance  $\omega = \omega_n$ , when damping ration ( $\zeta$  approximately = 0), (free vibration) is clear in Fig.13, [3].To

comparison the experimental value against the predicted on for each of the modes included at shown in Fig.14. In this way it is possible to see not only the degree of correlation between the two sets of results, but also the nature (and possible case) of any discrepancies which do exist. The points plotted should lie on or close to straight line of slope Fig. 15, [12],

Fig. 17, 18&19 shows the (FRF) of stationary rotor setup (A), versus achieved unbalanced stationary frequency response function of the setup (B), as it can be seen from the figures above, the (FRF) amplitude of the system has reduced by 19 db for first mode, 6db for second mode, 5db for third mode, due to applying an excessive 16 grams of loading to the system at the same speed. That mean the amplitude is reducing in the shaft after add mass in the disc 1 with angle (45°, 225°) & in disc 2 with angle (180°, 0°), As it can be seen from three figures different mode shape, the experiment shows that by applying a negligible excessive loading to the system the value of the natural frequency mode



shape does not change significantly for added masses. We expect the noise to be reduced as well due to reduced excess vibration.

For further studies, there is no need to make more experiments about this study while (ANSYS) gives accurate results. We used (ANSYS) to find the relation between the reaction bearing forces ( $N$ ) with respect to time. As we can see from Fig. 16, A, B, C & D, we see the reaction force in the Y direction is approximately the same value when increasing the speed for left and right bearings while in the Z direction we note the reaction force increasing a little bit at the beginning when the motor runs up, and we see the maximum reaction force value in the Y direction for the right bearing that means the right bearing carries the maximum load while the force in the Z direction starts high and then decreases slowly. During this study, the performance of reaction bearing forces in both bearings can aid in the design of low-noise rotor-bearing systems and reduce the reaction force in the bearing to make the bearing last longer by lubrication. In order to investigate the effects of design parameters on the noise of rotor-bearing systems, the effects of radial clearance and width of bearing, lubricant viscosity. For various rotational speeds, it is found that, as a general rule, the noise of the bearing decreases as the lubricant viscosity increases, the width of the bearing increases, and the radial clearance of the bearing decreases. A simple mathematical model has been used, however more elaborate models based on a much larger degree of freedom may be used based on suppleness or stiffness influence coefficients. The mathematical models may also be used to refine the measured data and help in removal of contaminated data. It is therefore feasible to create a mathematical model as a database for various systems for condition monitoring during their life time of the machines.

## ACKNOWLEDGMENTS

The authors are deeply appreciative to the Kingston University for supporting this research.

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