RESEARCHES ON THE MATHEMATICAL MODELING OF THE KINEMATICS OF THREE-POINT HITCH COUPLERS USED AT AGRICULTURAL TRACTORS

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ABSTRACT - The paper presents an analytical method applied to elaborate and analyze the kinematics of the three-point hitch couplers used at agricultural tractors. The analysis is grounded on the following basic data: kinematic diagram of the three-point hitch couplers, geometrical dimensions of the elements of the three-point hitch couplers, piston relative speed in the lift cylinder. The analysis is carried out with the aim of determining the speeds of all hitch points as well as the transmission ratio of the three-point hitch couplers.
The mechanism is divided in three kinematic groups. The study and the mathematical modeling of the kinematics is carried out for each distinct group taking into account the piston speed and position in the lift cylinder. The speed of each hitch point and the transmission ratio of the three-point hitch couplers related to the speed of a point positioned on the drawbar or to the speed of the agricultural tractor's center of gravity are further determined.
The paper shows the results of the researches conducted on wheeled tractors U 650 and U 650 DT.

INTRODUCTION

The questions related to the kinematics of three-point hitch couplers are raised when designing these mechanisms with the aim of appropriately choosing their geometrical and kinematic parameters. These parameters must correspond to the most advantageous ratios between the elements of the mechanism that should comply at the same time with the conditions imposed by both national and international standards as well as agri-technical requirements.

For this very aim, starting from the kinematic diagram (fig.1), geometrical dimensions and piston position in the lift cylinder, the following values are further determined: coordinates of fixed (*A*, *C*, *E*, *Q*) and mobile (*B*, *D*, *F K*, *M*,) points of the three-point hitch couplers; coordinates of the center of gravity (point S); coordinates of the instantaneous centre of revolution (CIR, point *I*).

All the coordinates included in this paper are considered to be known and stand for input data for the kinematic calculus. Other input data for this calculus are: *CB*, *CD*, *DK*, *QM*, *MF*, *FE*, *QK*, *FS*, *MS* dimensions, β angle between the *CB* and *CD* lever arms, relative piston speed $v_{BB'}$ to lift cylinder.

In order to deduce the formulas and to determine the parameters mentioned, the following were adopted:

- the construction of the three-point hitch couplers is divided into groups: group of elements *2–3, 4–5* and group of elements *6–7;*
- the system of *xOy* coordinates axis is rigidly connected to the tractor: the *Oy* axis crosses the axis of the tractor's rear axle, and the *Ox* axis is positioned on the supporting surface of the agricultural tractor.

The objectives of the kinematic calculus are:

- calculus of angular speeds of elements 1, 2, ….7 of the three-point hitch couplers;
- calculus of the transmission ration of the three-point hitch couplers.

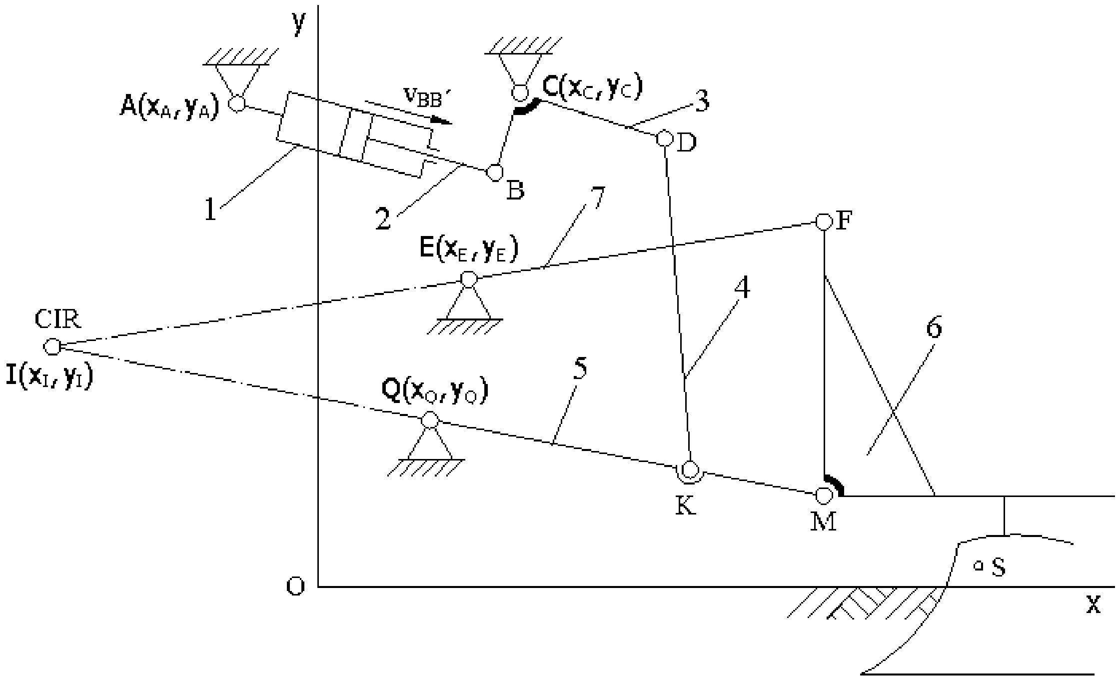

Fig. 1 Diagram for calculating the three-point hitch couplers

KINEMATICS OF THE THREE-POINT HITCH COUPLERS

Group 2-3. Figure 2 is used to determine the angular and linear speeds of elements 2 and 3. The following relations may be successively written starting from the theorem of speeds composition:

$$\bar{v}_B = \bar{v}_{BA} + \bar{v}_{BB'}; \quad \bar{v}_C = \bar{v}_B + \bar{v}_{CB}; \quad \Rightarrow$$

$$v_C = \bar{v}_{BA} + \bar{v}_{BB'} + \bar{v}_{CB}, \tag{1}$$

where $\bar{v}_{BA}$ stands for the vector of relative speed, perpendicular to vector $\overline{AB}$ and equal to the product between the angular speed and vector $\overline{AB}^*$, i.e.

$$\bar{v}_{BA} = \omega_1 \overline{AB}^*.$$

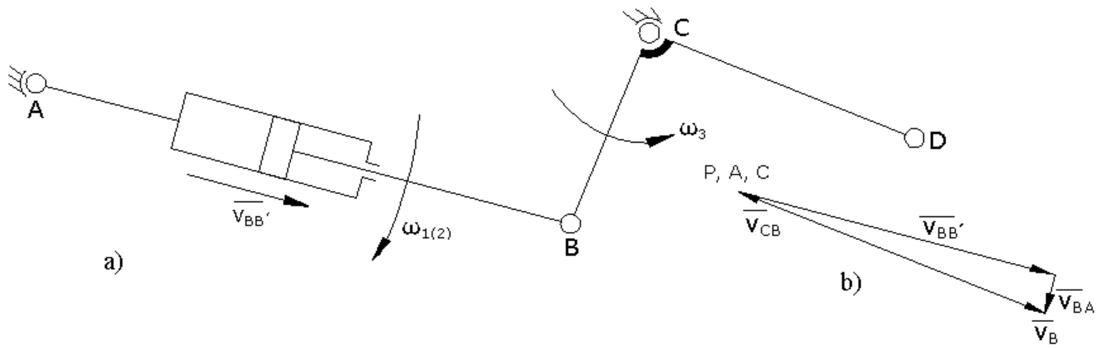

Fig. 2 Diagram for kinematic calculus of group 2—3: a – kinematic diagram; b – speeds plan

Vector $\overline{AB}^*$ is the vector $\overline{AB}$ rotated by 90° counterclockwise and has the following components:

$$\overline{AB_x}^* = -\overline{AB_y}; \quad \overline{AB_y}^* = \overline{AB_x};$$

$$\overline{v_C} = 0; \quad \overline{v_{CB}} = \omega_3 \overline{BC}^*.$$

Vector $\overline{BC}^*$ has the components:

$$\overline{BC}_x^* = -\overline{BC}_y; \quad \overline{BC}_y^* = \overline{BC}_x.$$

Thus, the equation (1) becomes:

$$\omega_1 \overline{AB}^* + \omega_3 \overline{BC}^* + \overline{v_{BB'}} = 0. \tag{2}$$

Representing $\bar{v}_{BB'}$ as product between the scalar value μ and vector $\overline{AB}$, the last relation becomes:

$$\omega_1 \overline{AB}^* + \omega_3 \overline{BC}^* + \mu \overline{AB} = 0. \tag{3}$$

Using the scalar multiplication for the terms of this equation by vector $\overline{BC}$, we obtain:

$$\omega_1 (\overline{AB}^* \cdot \overline{BC}) + \omega_3 (\overline{BC}^* \cdot \overline{BC}) + \mu (\overline{AB} \cdot \overline{BC}) = 0.$$

Given the last equation, having in view that $(\overline{BC}^* \cdot \overline{BC}) = 0$, we obtain the relation for angular speeds of elements *1* and *2*.

$$\omega_1 = \omega_2 = -\mu \frac{\left(\overline{AB} \cdot \overline{BC}\right)}{\left(\overline{AB}^* \cdot \overline{BC}\right)}, \tag{4}$$

and through coordinates of points

$$\omega_1 = \omega_2 = -\mu \frac{(x_B - x_A)(x_C - x_B) + (y_B - y_A)(y_C - y_B)}{(x_B - x_A)(y_C - y_B) - (y_B - y_A)(x_C - x_B)}. \tag{4′}$$

In order to determine the angular speed of element *3*, the terms of the equation (3) are scalarly multiplied by vector $\overline{AB}$:

$$\omega_1 (\overline{AB}^* \cdot \overline{AB}) + \omega_3 (\overline{BC}^* \cdot \overline{AB}) + \mu \overline{AB} \cdot \overline{AB} = 0.$$

Obtaining:

$$\omega_3 = -\mu \frac{AB^2}{\left(\overline{BC}^* \cdot \overline{AB}\right)} \tag{5}$$

or through coordinates of points:

$$\omega_3 = -\mu \frac{AB^2}{(x_C - x_B)(y_B - y_A) - (y_C - y_B)(x_B - x_A)}. \tag{5′}$$

Group 4-5. Figure 3 is used to analyze the kinematics of this group. The following relations may be successively written for speeds of points *D*, *K* and *Q*:

$$\overline{v_D} = \overline{v_C} + \overline{v_{DC}};$$

$$\overline{v_C} = 0 \left(C \text{ este un punct fix}\right) \Rightarrow \overline{v_D} = \overline{v_{DC}}.$$

$$\overline{v_K} = \overline{v_D} + \overline{v_{KD}} \;\Rightarrow\; \overline{v_K} = \overline{v_{DC}} + \overline{v_{KD}};$$

$$\overline{v_Q} = \overline{v_K} + \overline{v_{QK}} \;\Rightarrow$$

$$\overline{v_Q} = \overline{v_{DC}} + \overline{v_{KD}} + \overline{v_{QK}}. \tag{6}$$

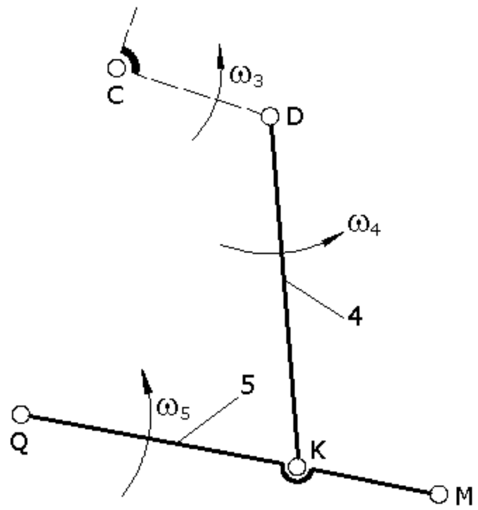

Fig. 3 Diagram for kinematic calculus of group *4 – 5*

In the relation (6) $\overline{v_Q} = 0$ (speed of a fixed point), and the other speeds will be expressed considering the related angular speeds:

$$\omega_3 \overline{CD^*} + \omega_4 \overline{DK^*} + \omega_5 \overline{KQ^*} = 0. \tag{7}$$

Just like for group *2-3*, we obtain the angular speeds ω_4 and ω_5:

$$\omega_4 = -\omega_3 \frac{(x_D - x_C)(y_Q - y_K) - (y_D - y_C)(x_Q - x_K)}{(x_K - x_D)(y_Q - y_K) - (y_K - y_D)(x_Q - x_K)}. \tag{8}$$

$$\omega_5 = -\omega_3 \frac{(x_D - x_C)(y_K - y_D) - (y_D - y_C)(x_K - x_D)}{(x_Q - x_K)(y_K - y_D) - (y_Q - y_K)(x_K - x_D)}. \tag{9}$$

Group 6 – 7. The kinematics of this group is analyzed in figure 4. The following relations may be successively written for speeds of points *M*, *F* and *E*:

$$\begin{aligned}
&\overline{v_M} = \overline{v_Q} + \overline{v_{MQ}};\\
&\overline{v_Q} = 0 \left(Q \text{ este un punct fix}\right) \Rightarrow \overline{v_M} = \overline{v_{MQ}}.\\
&\overline{v_F} = \overline{v_M} + \overline{v_{FM}} \;\Rightarrow\; \overline{v_F} = \overline{v_{MQ}} + \overline{v_{FM}};\\
&\overline{v_E} = \overline{v_F} + \overline{v_{EF}} \;\Rightarrow\\
&\overline{v_E} = \overline{v_{MQ}} + \overline{v_{FM}} + \overline{v_{EF}}.
\end{aligned} \tag{10}$$

In the relation (10) $\overline{v_E} = 0$ (*E* being a fixed point), and the other speeds will be expressed taking into account the related angular speeds

$$\omega_5 \overline{QM^*} + \omega_6 \overline{MF^*} + \omega_7 \overline{FE^*} = 0. \tag{11}$$

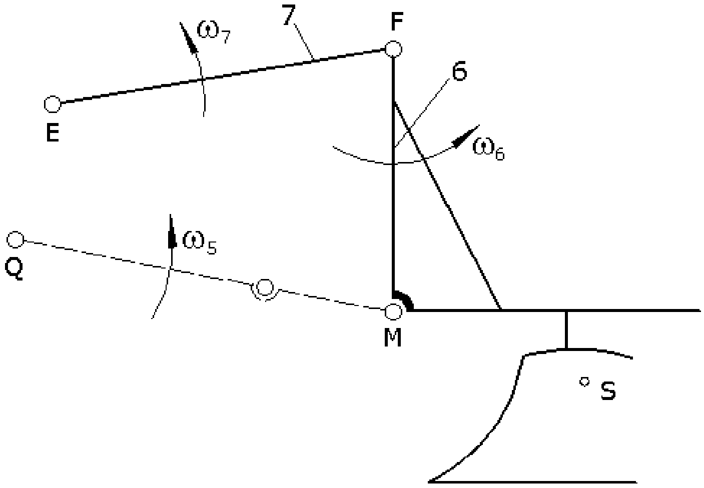

Fig. 4 Diagram for kinematic calculus of group *6 – 7*

Just like for group *2-3*, we obtain the angular speeds ω_6 and ω_7:

$$\omega_6 = -\omega_5 \frac{(x_M - x_Q)(y_E - y_F) - (y_M - y_Q)(x_E - x_F)}{(x_F - x_M)(y_E - y_F) - (y_F - y_M)(x_E - x_F)}. \tag{12}$$

$$\omega_7 = -\omega_5 \frac{(x_M - x_Q)(y_F - y_M) - (y_M - y_Q)(x_F - x_M)}{(x_E - x_F)(y_F - y_M) - (y_E - y_F)(x_F - x_M)}. \tag{13}$$

Linear speeds are calculated by adopting, for exemplification, points *M* and *S* (the latter one being the center of gravity of the system):

$$\overline{v_M} = \omega_6 \overline{IM^*}; \tag{14}$$

$$\overline{v_S} = \omega_6 \overline{IS^*}. \tag{15}$$

The projections of these speeds on coordinate axes are calculated with the relations:

$$\begin{aligned} (v_M)_x &= -\omega_6 IM_y = -\omega_6 (y_M - y_I); \\ (v_M)_y &= \omega_6 IM_x = \omega_6 (x_M - x_I); \\ (v_S)_x &= -\omega_6 IS_y = -\omega_6 (y_S - y_I); \\ (v_S)_y &= \omega_6 IS_x = \omega_6 (x_S - x_I). \end{aligned} \tag{16}$$

Consequently, knowing the angular speeds and the coordinates of the mechanism points for any of its position, determined according to piston position in the lift cylinder, the value of the transmission ratio of the three-point hitch couplers may be calculated for a given position:

$$i_S = \frac{(v_S)_y}{v_{BB'}} = \frac{\omega_6 IS_x}{v_{BB'}} = \omega_6 \frac{(x_S - x_I)}{v_{BB'}}. \tag{17}$$

Provided the center of gravity is placed on the drawbar which, at its turn, is placed on the ends of lower links, the relation used to determine the transmission ratio becomes:

$$i_M = \frac{(v_M)_y}{v_{BB'}} = \frac{\omega_6 IM_x}{v_{BB'}} = \omega_6 \frac{(x_M - x_I)}{v_{BB'}}. \tag{18}$$

The last two relations emphasize that the value of the transmission ratio of the three-point hitch couplers depends on the position of the instantaneous centre of revolution of the carried implement and on the position of its center of gravity.

APPLICATIONS OF THEORETICAL RESEARCHES

In this part of the paper the authors present part of the results arisen from the theoretical researches carried out on the agricultural tractor U 650 DT, which is representative for the Romanian tractor fleet. The following input data, corresponding to the second category (SR ISO 730-1) were used:

- coordinates of fixed points (in mm): *A*(-1, 1156), *C*(230, 1143), *Q*(230, 495), $E \in \{(400, 857); (400, 891); (400, 923); (400, 964)\}$;
- dimensions of elements (in mm): *AB* = 160, *CB* = 94,8, *CD* = 260, *DK* = 760, *QM* = 900, *MF* = 850, *FE* = 637, $QK \in \{299, 459, 559\}$, *MS* = 820;
- values of angles: α = 25° (inclination of the lift cylinder to the horizontal) β = 124° (angle between *CB* and *CD* lever arms)

The research results are presented in a diagram: variation of the transmission ratio i_M (fig.5 and 7); variation of the transmission ratio i_S (fig. 6 and 8); variation of speeds at point *M* (fig. 9).

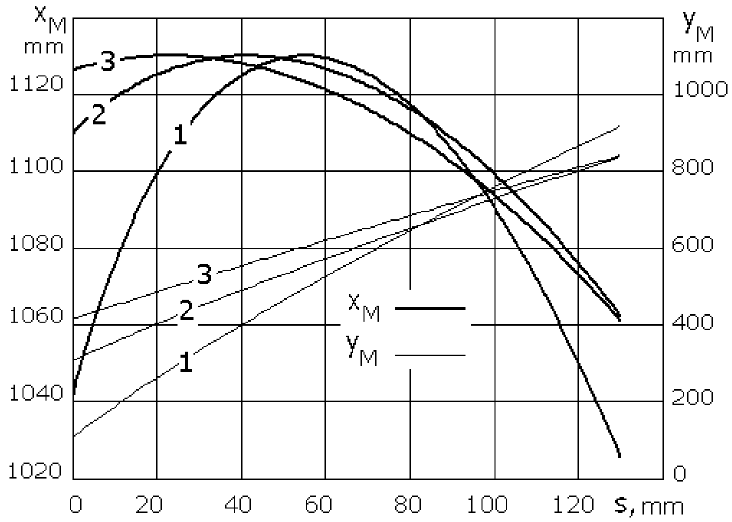

Fig. 5 Variation of the transmission ratio i_M depending on piston position for 4 positions of point *E*: 1 – *E*(400, 857); 2 – *E*(400, 891); 3 – *E*(400, 923); 4 – *E*(400, 964)

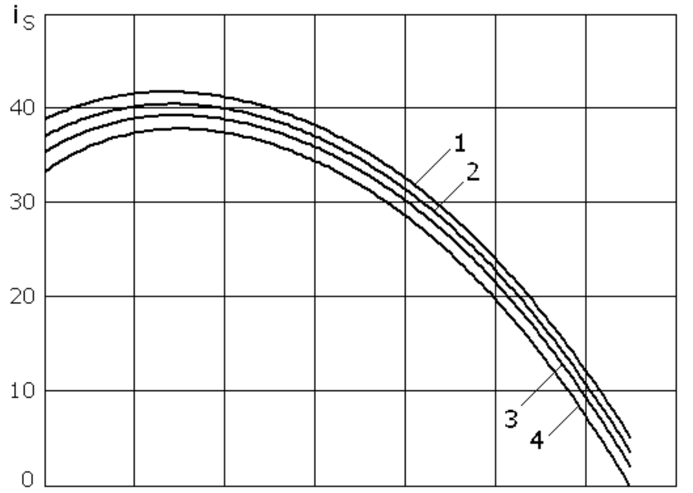

Fig. 6 Variation of the transmission ratio i_S depending on piston position for 4 positions of point *E*: 1 – *E*(400, 857); 2 – *E*(400, 891); 3 – *E*(400, 923); 4 – *E*(400, 964)

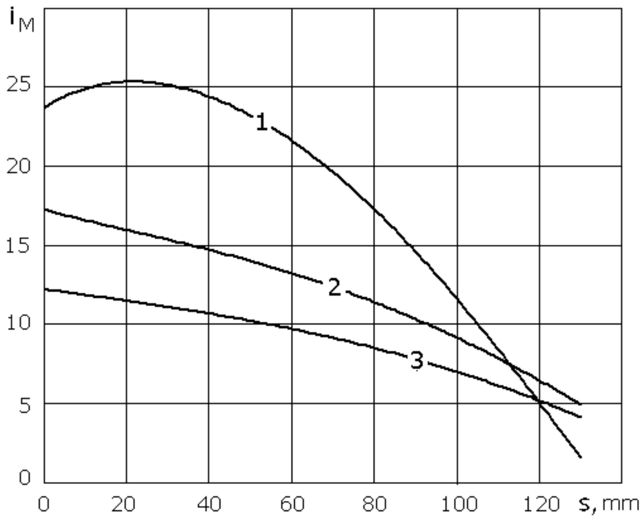

Fig. 7 Variation of the transmission ratio i_M depending on piston position for 3 values *QK*: 1 – *QK* = 299 mm; 2 – *QK* = 459 mm; 3 – *QK* = 559 mm

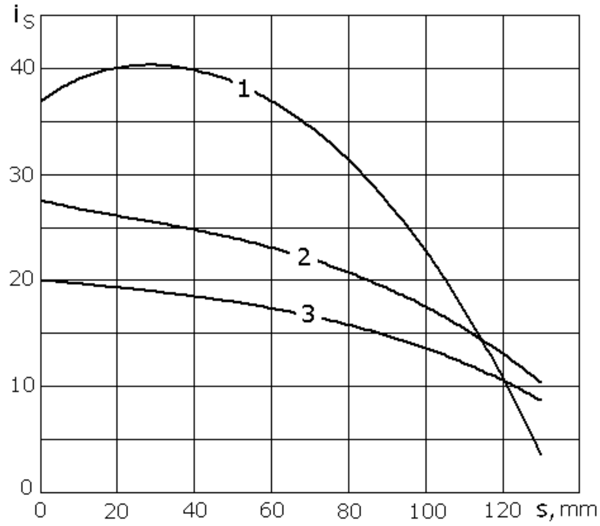

Fig. 8 Variation of the transmission ratio i_S depending on piston position for 3 values *QK*: 1 – *QK* = 299 mm; 2 – *QK* = 459 mm; 3 – *QK* = 559 mm

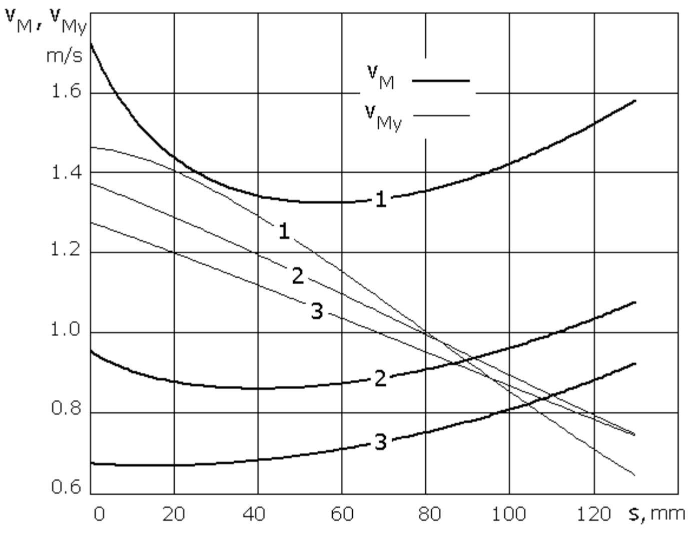

Fig. 9 Variation of speeds v_M and vertical components v_{My} depending on piston position for 3 values *QK*: 1 – *QK* = 299 mm; 2 – *QK* = 459 mm; 3 – *QK* = 559 mm

CONCLUSIONS

- The calculus algorithm allows us to determine all kinematic parameters of the mechanism (linear speeds, angular speeds, transmission ratios) for any three-point hitch couplers.
- The transmission ratios are aggregative parameters used to assess the performance of the three-point hitch couplers on account of the fact that they are defined considering both the kinematic parameters specific to the mechanism and the parameters of the lift cylinder.

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