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MECHANICAL BEHAVIOUR OF NON-PRE-TENSIONED GLASS FABRIC REINFORCED COMPOSITE TUBES SUBJECTED TO INTERNAL PRESSURE

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Abstract: *The paper presents a theoretical approach regarding the mechanical behaviour of non-pre-tensioned glass fabric reinforced composite tubes, subjected to internal pressure. The computing of stresses and strains is accomplished in the hypothesis of neglecting the temperature- and humidity variations on the composite material structure. The increase of loading capability of pre-tensioned tubes against the non-pre-tensioned ones is situated about 32%.*
Keywords: *glass fabric, plain weave, pre-tensioning, loading capability, pre-tensioning pressure, PMC tubes*

1. INTRODUCTION

The fabrics made from glass fibers are characterized by number of nodes on square centimeter, width, thickness, porosity (eye width), bending strength, surface appearance and applied treatments. The main weave modes are the plain weave (in which the warp- and weft threads pass recurrently one above the other), the twill weave (in which the fabric forms a characteristic model with diagonal lines at the fabric surface) and satin weave (in which the fabric surface is formed from threads belonging either to weft or to warp). The fibers weave mode, their thickness and twisting degree play a significant role on the mechanical features of the composite structure. Regarding the hollow composite structure, the best mechanical features can be obtained in case of structure's reinforcement with thin fabrics, the weave mode being plain weave. The computing of stresses and strains is accomplished neglecting the temperature- and humidity variations on the composite material structure. It is assumed also that the tube plies are orthotropic and the entire tube wall thickness t is much smaller than its curvature radius r.

2. THEORETICAL APPROACH

A composite tube based on polyester resin reinforced with glass fibers (plain weave) is considered, tube made by winding of eight fabric plies and subjected at internal pressure (fig. 1). By t_1 and t_2 the warp respective the weft thickness threads are denoted, the thickness of both threads is considered equal and, by weave mode, they form together a fabric ply with the entire thickness t. A and C represent the axial respective the circumferential tube's direction.
Therefore, the loadings of the tube wall are:

$$\sigma_C = p \cdot \frac{r}{t}, \qquad \textbf{(1)}$$

$$\sigma_A = p \cdot \frac{r}{2t}, \qquad \textbf{(2)}$$

$$\tau_{AC} = 0, \qquad \textbf{(3)}$$

where A and C represent the axial/circumferential tube direction and p is the internal pressure. The composite tube's strains and the one ply stresses are:

$$\varepsilon_A = \frac{1-\upsilon_{\perp II}\cdot\upsilon_{II\perp}}{E_{II}} \cdot \frac{\hat{\sigma}_C}{2K} \cdot \left[\left(t' + t' \cdot \frac{E_{II}}{E_{\perp}}\right) - 2\upsilon_{\perp II}\right], \qquad \textbf{(4)}$$

$$\varepsilon_C = \frac{1-\upsilon_{\perp II}\cdot\upsilon_{II\perp}}{E_{II}}\cdot\frac{\hat{\sigma}_C}{2K}\cdot\left[2\left(t' + t'\cdot\frac{E_{II}}{E_\perp}\right)-\upsilon_{\perp II}\right], \tag{5}$$

$$\gamma_{AC} = 0, \tag{6}$$

$$\sigma_{II\,1} = \frac{\hat{\sigma}_C}{2K}\left[t' + t'\,\frac{E_{II}}{E_\perp}-\upsilon_{\perp II}\upsilon_{II\perp}-2t'\left(\upsilon_{\perp II}-\upsilon_{II\perp}\right)\right], \tag{7}$$

$$\sigma_{II\,2} = \frac{\hat{\sigma}_C}{2K}\left[2(t' + t'\,\frac{E_{II}}{E_\perp}-\upsilon_{\perp II}\upsilon_{II\perp})-t'\left(\upsilon_{\perp II}-\upsilon_{II\perp}\right)\right], \tag{8}$$

$$\sigma_{\perp 1} = \frac{\hat{\sigma}_C}{2K}\left[2(t' + t'\,\frac{E_\perp}{E_{II}}-\upsilon_{\perp II}\upsilon_{II\perp})+t'\left(\upsilon_{\perp II}-\upsilon_{II\perp}\right)\right], \tag{9}$$

$$\sigma_{\perp 2} = \frac{\hat{\sigma}_C}{2K}\left[t' + t'\,\frac{E_\perp}{E_{II}}-\upsilon_{\perp II}\upsilon_{II\perp}+2t'\left(\upsilon_{\perp II}-\upsilon_{II\perp}\right)\right], \tag{10}$$

where:

$$K = t'^2\left(\frac{E_{II}}{E_\perp}+\frac{E_\perp}{E_{II}}-2\right)+1-\upsilon_{\perp II}\cdot\upsilon_{II\perp};\quad t' = \frac{t_1}{t} = \frac{t_2}{t}, \tag{11}$$

E_{II}, $E_\perp$ and $\upsilon_{\perp II}$ represent the basic elasticity constants and $\hat{\sigma}_C$ is the medium stress that acts in the circumferential direction of the composite tube. At the Poisson ratio, the first index represents the shrinkage direction and the second one is the loading direction that produces this shrinkage.

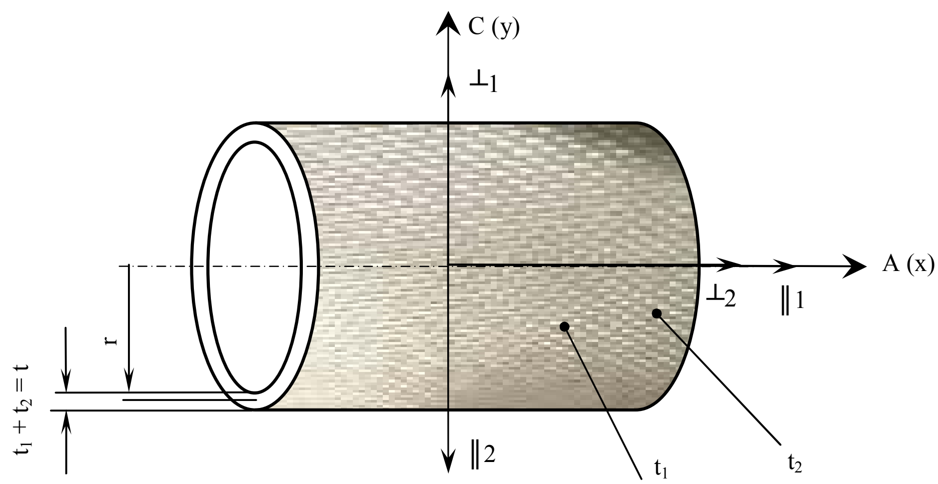


Figure 1: Schematic representation of glass fabric reinforced composite non-pre-tensioned tube

From the experimental results obtained on this kind of tube and from the micromechanical relations:

$$E_{II} = \varphi\cdot E_F + E_M\left(1-\varphi\right), \tag{12}$$

$$E_\perp = \frac{E_M\left(1+0{,}85\cdot\varphi^2\right)}{\left(1-\upsilon_M^2\right)\left[\left(1-\varphi\right)^{1{,}25}+\dfrac{\varphi\cdot E_M\left(1-\upsilon_M^2\right)}{E_F}\right]}, \tag{13}$$

$$G_\# = \frac{G_M\left(1+0{,}6\cdot\varphi^{0{,}5}\right)}{\left(1-\varphi\right)^{1{,}25}+\varphi\dfrac{G_M}{G_F}}, \tag{14}$$

$$\upsilon_{\perp II} = \varphi\cdot\upsilon_F + \upsilon_M\left(1-\varphi\right), \tag{15}$$

$$\upsilon_{II\perp} = \frac{\upsilon_{\perp II}\cdot E_\perp}{E_{II}}, \tag{16}$$

the following input data have been determined and presented in tables 1 – 3. The index F refers to the glass fiber and the index M to the matrix.

Table 1: Basic elasticity constants for UP resin reinforced with glass fabric

	Units	Value
Density	kg/m^3	1750
Young modulus, $E_{\parallel}$	MPa	28100
Young modulus, $E_{\perp}$	MPa	8625
Shear modulus, $G_{\#}$	MPa	3105
Fibers volume fraction, φ	%	35
Poisson ratio, $\upsilon_{\perp\parallel}$	-	0.36
Poisson ratio, $\upsilon_{\parallel\perp}$	-	0.11
Poisson ratio, $\upsilon_{\perp\perp}$	-	0.53

Table 2: Basic elasticity constants for isotropic materials

	E-glass	UP resin
Density [kg/m^3]	2500	1250
Young modulus, E [MPa]	73000	3900
Shear modulus, G [MPa]	29200	1375
Poisson ratio, υ [-]	0.25	0.42

Table 3: Loading and geometric data

	Units	Value
Tube's diameter, D	mm	80
Number of fabric plies, n	-	8
Fabric plies thickness	mm	0.437
Tube wall thickness, t	mm	3.5
Internal pressure, p	MPa	1.5
Fabric winding angle, ω	°	0

The circumferencial stresses in tube's plies loaded at internal pressure are:

$$\sigma_{C\,i} = \frac{p \cdot r_i}{\sum_{i=1}^{8} t_{l\,i}}, \tag{17}$$

where r_i and $t_{l\,I}$ represent the plies radii respective the plies thicknesses given in table 4.

Table 4

Plies	Plies radii	Plies thicknesses
1	0.218	0.437
1-2	0.437	0.875
1-3	0.656	1.312
1-4	0.875	1.75
1-5	1.093	2.187
1-6	1.312	2.625
1-7	1.531	3.062
1-8	1.75	3.5

According to relations (17) and with help of data presented in tables 1 – 3, the stresses and strains in tube's plies can be computed. The results are presented in tables 5 and 6.

Table 5: Composite non-pre-tensioned tube's stresses and strains reinforced with glass fabric, loaded at 1.5 MPa internal pressure

	Ply 1	Plies 1-2	Plies 1-3	Plies 1-4
ε_A [-]	0.0024	0.0012	0.0008	0.0006
ε_C [-]	0.0067	0.0033	0.0022	0.0017
γ_{AC} [-]	0			
σ_C [MPa]	137.89	69.32	46.46	35.03
$\sigma_{II\,1}$ [MPa]	93.86	47.18	31.62	23.84
$\sigma_{II\,2}$ [MPa]	206.87	103.99	69.7	52.56

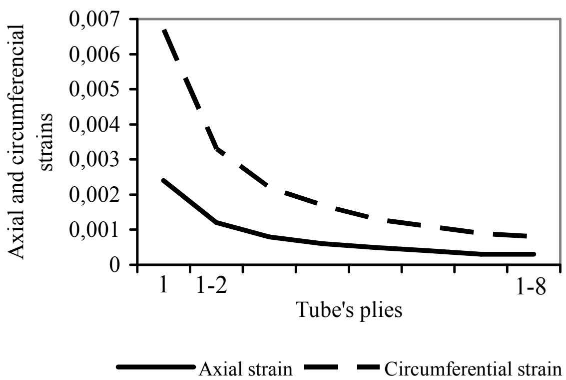
	Ply 1	Plies 1-2	Plies 1-3	Plies 1-4
$\sigma_{\perp 1}$ [MPa]	69.03	34.7	23.26	17.53
$\sigma_{\perp 2}$ [MPa]	44.08	22.15	14.85	11.2
$\tau_{\# 1,2}$ [MPa]	0			

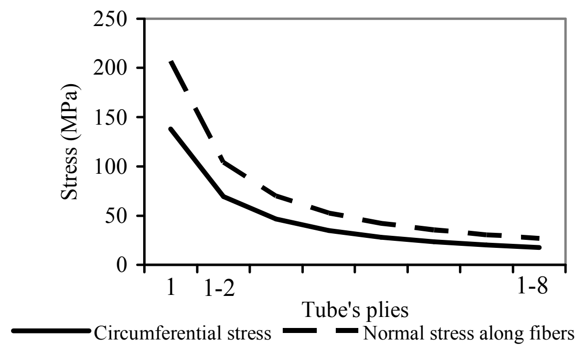
Table 6: Composite non-pre-tensioned tube's stresses and strains reinforced with glass fabric, loaded at 1.5 MPa internal pressure

	Plies 1-5	Plies 1-6	Plies 1-7	Plies 1-8
ε_A [-]	0.0005	0.0004	0.0003	0.0003
ε_C [-]	0.0013	0.0011	0.0009	0.0008
γ_{AC} [-]	0			
σ_C [MPa]	28.17	23.6	20.34	17.89
σ_{II1} [MPa]	19.18	16.06	13.84	12.17
σ_{II2} [MPa]	42.27	35.41	30.51	26.84
$\sigma_{\perp 1}$ [MPa]	14.1	11.81	10.18	8.95
$\sigma_{\perp 2}$ [MPa]	9	7.54	6.5	5.71
$\tau_{\# 1,2}$ [MPa]	0			

3. CONCLUSIONS

The distributions of axial-, circumferential strains and stresses in tube's plies are presented below.





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