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DYNAMIC MODEL OF THE FREE-THROW IN BASKETBALL GAME

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Abstract: *Today in many sports, sports scientists use movement analysis as a tool to enhance techniques, correct movement errors, assess metabolic costs related to a variety of movements, and aid in rehabilitation.* ***.*** *Biomechanical motion analysis will directly lead to improved performance and injury prevention. In numerous studies (Brancazio 1981; Krausse,1984; Hay 1994) the majority of coaches identify shooting as the most important skill of basketball. In this study we want to create a dynamic model of the free throw based on analysis of video filming.*
Keywords: *dynamic model, modelling, basketball game*

1. INTRODUCTION

To achieve perfection, kinesiological and biomechanical analyses need to be undertaken to understand the affects of the existing techniques on movement, and accordingly new techniques need to be developed.
Analysis of human motion in sport activity involves determining in which way the athletes performances are influence when using optimization methods and specific elements for identify movement.[1]
Biomechanical research of human movement, considered the human body like a mechanical system made from segments (called multibody system – figure 1) on which acting various forces like, muscle force, gravity force, inertia force, reaction force. [3].

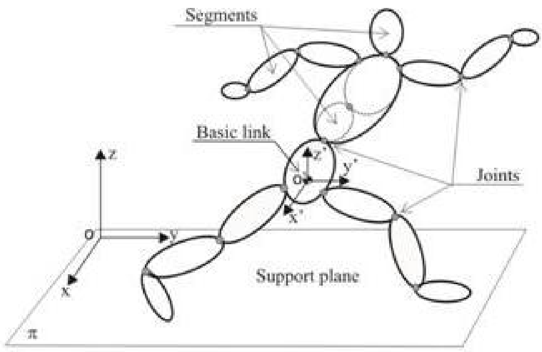


Figure 1 – *Human body in mechanic and decomposition in components part [5]*
The base for biomechanical analysis of sports movement is the antrophometric model.To interpret computerized data for each segment of the body (head, hand, forearm, arm, trunk, pelvis, thigh, calf, foot) depending on the motion carried is the human movement research.

2. RESEARCH METHODS

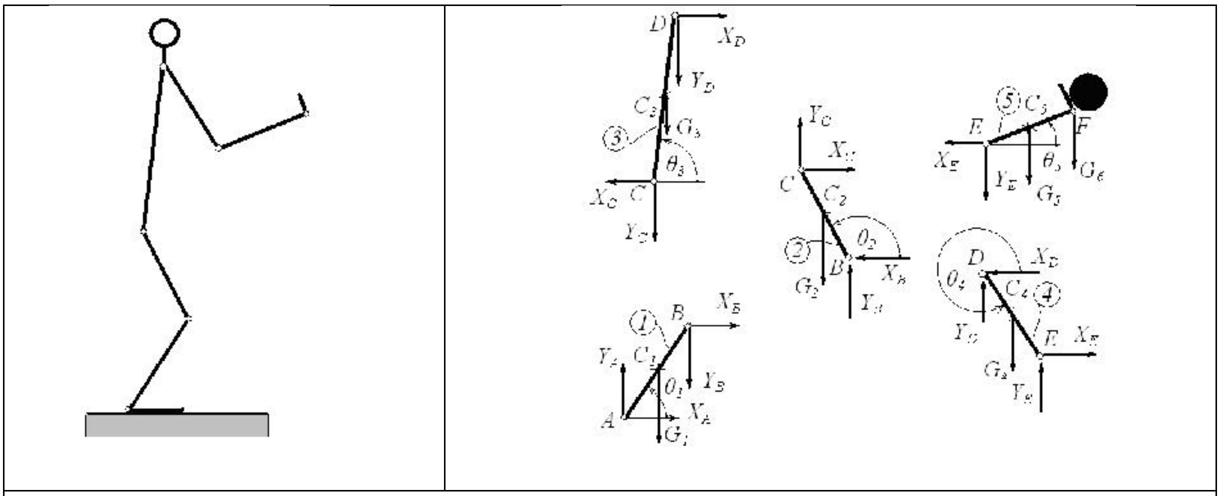
Within the last two decades multi body models have frequently been applied to solve biomechanical problems. Some selected MB applications are for instance simulations for impact analysis (Gruber et al.,1998), investigations of the trampoline jumping (Blajer & Czaplicki, 2001), studies about walking and running (Wojtyra, 2003) or the discussion of forces encountered in bicycle sports (Wangerin et al., 2007). Multi body systems can be used in two different ways: as forward or as inverse models

a) ***Direct dynamics method***: - according to its dynamic parameters which are used in biomechanical system are known, and the objective is to determine the kinematic parameters resulting from movements that arise in the system.
b) ***Inverse dynamics method***: - is to determine dynamic parameters causing movement kinematics system based on biomechanical system that are defined in detail. Using this method is done by the following conditions:

- The human body is divided into individual anatomical segments (see Figure 1)
- Segments are considered rigid bodies with mass concentrated in their center of gravity
- Parameters of the anthropometric segments are considered constant on movement
- The force of air friction is minimal
- Force of friction with the ground and the joints are considered null
- Travel speed of the whole system is constant

3. DATA PROCESS

To process this images were used Adobe After Effects 7.0 program. After we open the program, the movie or the imagine is imported to be processed. Then will determine what you want to do, in our case, determining the trajectory of a point, the marker placed on the joint.
Each marker is made in this way, obtaining the coordinates of each point of the trajectory (2D - (x, y)), and the data obtained are exported to EXCEL. In MATLAB program, this date are processed, obtained the dynamic model of free throw.



Segmentelor corpului şi mişcările din articulaţie
(AB – gambă, BC – coapsă, CD – trunchhi, DE – braţ, EF – antebraţ)

$x_{C1} = \alpha_1 l_1 c_1$;	$y_{C1} = \alpha_1 l_1 s_1$;
$x_{C2} = l_1 c_1 + \alpha_2 l_2 c_2$;	$y_{C2} = l_1 s_1 + \alpha_2 l_2 s_2$;
$x_{C3} = l_1 c_1 + l_2 c_2 + \alpha_3 l_3 c_3$;	$y_{C3} = l_1 s_1 + l_2 s_2 + \alpha_3 l_3 s_3$;
$x_{C4} = l_1 c_1 + l_2 c_2 + l_3 c_3 + \alpha_4 l_4 c_4$;	$y_{C4} = l_1 s_1 + l_2 s_2 + l_3 s_3 + \alpha_4 l_4 s_4$;
$x_{C5} = l_1 c_1 + l_2 c_2 + l_3 c_3 + l_4 c_4 + \alpha_5 l_5 c_5$;	$y_{C5} = l_1 s_1 + l_2 s_2 + l_3 s_3 + l_4 s_4 + \alpha_5 l_5 s_5$;
$x_6 = l_1 c_1 + l_2 c_2 + l_3 c_3 + l_4 c_4 + l_5 c_5$;	$y_6 = l_1 s_1 + l_2 s_2 + l_3 s_3 + l_4 s_4 + l_5 s_5$;
Coordonatele (x,y) ale punctelor, A,B,C,D,E, F	

$\dot{x}_{C1} = -\alpha_1 l_1 s_1 \omega_1$;	$\dot{y}_{C1} = \alpha_1 l_1 c_1 \omega_1$;
$\dot{x}_{C2} = -l_1 s_1 \omega_1 - \alpha_2 l_2 s_2 \omega_2$;	$\dot{y}_{C2} = l_1 c_1 \omega_1 + \alpha_2 l_2 c_2 \omega_2$;
$\dot{x}_{C3} = -l_1 s_1 \omega_1 - l_2 s_2 \omega_2 - \alpha_3 l_3 s_3 \omega_3$;	$\dot{y}_{C3} = l_1 c_1 \omega_1 + l_2 c_2 \omega_2 + \alpha_3 l_3 c_3 \omega_3$;
$\dot{x}_{C4} = -l_1 s_1 \omega_1 - l_2 s_2 \omega_2 - l_3 s_3 \omega_3 - \alpha_4 l_4 s_4 \omega_4$;	$\dot{y}_{C4} = l_1 c_1 \omega_1 + l_2 c_2 \omega_2 + l_3 c_3 \omega_3 + \alpha_4 l_4 c_4 \omega_4$;
$\dot{x}_{C5} = -l_1 s_1 \omega_1 - l_2 s_2 \omega_2 - l_3 s_3 \omega_3 - l_4 s_4 \omega_4 - \alpha_5 l_5 s_5 \omega_5$;	$\dot{y}_{C5} = l_1 c_1 \omega_1 + l_2 c_2 \omega_2 + l_3 c_3 \omega_3 + l_4 c_4 \omega_4 + \alpha_5 l_5 c_5 \omega_5$

$\dot{x}_6 = -l_1 s_1 \omega_1 - l_2 s_2 \omega_2 - l_3 s_3 \omega_3 - l_4 s_4 \omega_4 - l_5 s_5 \omega_5$;	$\dot{y}_6 = l_1 c_1 \omega_1 + l_2 c_2 \omega_2 + l_3 c_3 \omega_3 + l_4 c_4 \omega_4 + l_5 c_5 \omega_5$;
$\ddot{x}_{C1} = -\alpha_1 l_1 s_1 \varepsilon_1 - \alpha_1 l_1 c_1 \omega_1^2$;	$\ddot{y}_{C1} = \alpha_1 l_1 c_1 \varepsilon_1 - \alpha_1 l_1 s_1 \omega_1^2$;
$\ddot{x}_{C2} = -l_1 s_1 \varepsilon_1 - l_1 c_1 \omega_1^2 - \alpha_2 l_2 s_2 \varepsilon_2 - \alpha_2 l_2 c_2 \omega_2^2$;	$\ddot{y}_{C2} = l_1 c_1 \varepsilon_1 - l_1 s_1 \omega_1^2 + \alpha_2 l_2 c_2 \varepsilon_2 - \alpha_2 l_2 s_2 \omega_2^2$;
$\ddot{x}_{C3} = -l_1 s_1 \varepsilon_1 - l_1 c_1 \omega_1^2 - l_2 s_2 \varepsilon_2 - l_2 c_2 \omega_2^2 - \alpha_3 l_3 s_3 \varepsilon_3 - \alpha_3 l_3 c_3 \omega_3^2$;	$\ddot{y}_{C3} = l_1 c_1 \varepsilon_1 - l_1 s_1 \omega_1^2 + l_2 c_2 \varepsilon_2 - l_2 s_2 \omega_2^2 + \alpha_3 l_3 c_3 \varepsilon_3 - \alpha_3 l_3 s_3 \omega$
$\ddot{x}_{C4} = -l_1 s_1 \varepsilon_1 - l_1 c_1 \omega_1^2 - l_2 s_2 \varepsilon_2 - l_2 c_2 \omega_2^2 -$ $- l_3 s_3 \varepsilon_3 - l_3 s_3 \omega_3^2 - \alpha_4 l_4 s_4 \varepsilon_4 - \alpha_4 l_4 c_4 \omega_4^2$;	$\ddot{y}_{C4} = l_1 c_1 \varepsilon_1 - l_1 s_1 \omega_1^2 + l_2 c_2 \varepsilon_2 - l_2 s_2 \omega_2^2 +$ $+ l_3 c_3 \varepsilon_3 - l_3 s_3 \omega_3^2 + \alpha_4 l_4 c_4 \varepsilon_4 - \alpha_4 l_4 s_4 \omega_4^2$;
$\ddot{x}_{C5} = -l_1 s_1 \varepsilon_1 - l_1 c_1 \omega_1^2 - l_2 s_2 \varepsilon_2 - l_2 c_2 \omega_2^2 - l_3 s_3 \varepsilon_3 - l_3 c_3 \omega_3^2 -$ $- l_4 s_4 \varepsilon_4 - l_4 c_4 \omega_4^2 - \alpha_5 l_5 s_5 \varepsilon_5 - \alpha_5 l_5 c_5 \omega_5^2$;	$\ddot{y}_{C5} = l_1 c_1 \varepsilon_1 - l_1 s_1 \omega_1^2 + l_2 c_2 \varepsilon_2 - l_2 s_2 \omega_2^2 + l_3 c_3 \varepsilon_3 - l_3 s_3 \omega_3^2 +$ $+ l_4 c_4 \varepsilon_4 - l_4 s_4 \omega_4^2 + \alpha_5 l_5 c_5 \varepsilon_5 - \alpha_5 l_5 s_5 \omega_5^2$;

$$\begin{Bmatrix} \dot{x}_{c1} \\ \dot{y}_{C1} \\ \omega_1 \\ \dot{x}_{c2} \\ \dot{y}_{c2} \\ \omega_2 \\ \dot{x}_{c3} \\ \dot{y}_{c3} \\ \omega_3 \\ \dot{x}_{c4} \\ \dot{y}_{c4} \\ \omega_4 \\ \dot{x}_{c5} \\ \dot{y}_{c5} \\ \omega_5 \\ \dot{x}_6 \\ \dot{y}_6 \end{Bmatrix} = \begin{bmatrix} -\alpha_1 l_1 s_1 & 0 & 0 & 0 & 0 \\ \alpha_1 l_1 c_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -l_1 s_1 & -\alpha_2 l_2 s_2 & 0 & 0 & 0 \\ l_1 c_1 & \alpha_2 l_2 c_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -\alpha_3 l_3 s_3 & 0 & 0 \\ l_1 c_1 & l_2 c_2 & \alpha_3 l_3 c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -\alpha_4 l_4 s_4 & 0 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & \alpha_4 l_4 c_4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -l_4 s_4 & -\alpha_5 l_5 s_5 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & l_4 c_4 & \alpha_5 l_5 c_5 \\ 0 & 0 & 0 & 0 & 1 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -l_4 s_4 & -l_5 s_5 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & l_4 c_4 & l_5 c_5 \end{bmatrix} \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \end{Bmatrix}$$

$$\begin{Bmatrix} \ddot{x}_{c1} \\ \ddot{y}_{C1} \\ \varepsilon_1 \\ \ddot{x}_{c2} \\ \ddot{y}_{c2} \\ \varepsilon_2 \\ \ddot{x}_{c3} \\ \ddot{y}_{c3} \\ \varepsilon_3 \\ \ddot{x}_{c4} \\ \ddot{y}_{c4} \\ \varepsilon_4 \\ \ddot{x}_{c5} \\ \ddot{y}_{c5} \\ \varepsilon_5 \end{Bmatrix} = \begin{bmatrix} -\alpha_1 l_1 s_1 & 0 & 0 & 0 & 0 \\ \alpha_1 l_1 c_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -l_1 s_1 & -\alpha_2 l_2 s_2 & 0 & 0 & 0 \\ l_1 c_1 & \alpha_2 l_2 c_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -\alpha_3 l_3 s_3 & 0 & 0 \\ l_1 c_1 & l_2 c_2 & \alpha_3 l_3 c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -\alpha_4 l_4 s_4 & 0 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & \alpha_4 l_4 c_4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -l_4 s_4 & -\alpha_5 l_5 s_5 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & l_4 c_4 & \alpha_5 l_5 c_5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} + \begin{bmatrix} -\alpha_1 l_1 c_1 & 0 & 0 & 0 & 0 \\ -\alpha_1 l_1 s_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -l_1 c_1 & -\alpha_2 l_2 c_2 & 0 & 0 & 0 \\ -l_1 s_1 & -\alpha_2 l_2 s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -l_1 c_1 & -l_2 c_2 & -\alpha_3 l_3 c_3 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -\alpha_3 l_3 s_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -l_1 c_1 & -l_2 c_2 & -l_3 c_3 & -\alpha_4 l_4 c_4 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -\alpha_4 l_4 s_4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -l_1 c_1 & -l_2 c_2 & -l_3 c_3 & -l_4 c_4 & -\alpha_5 l_5 c_5 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -l_4 s_4 & -\alpha_5 l_5 s_5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \\ \omega_5^2 \\ \omega_6^2 \end{Bmatrix}$$

$$\{a\} = [A_1]\{\varepsilon\} + [A_2]\{\omega^2\} \quad [m]\{a\} = \{Q\}$$

$$
[m]=\begin{bmatrix}
m_1 &&&&&&&&&&&&&& \\
& m_1 &&&&&&&&&&&&& \\
&& J_1 &&&&&&&&&&&& \\
&&& m_2 &&&&&&&& 0 &&& \\
&&&& m_2 &&&&&&&&&& \\
&&&&& J_2 &&&&&&&&& \\
&&&&&& m_3 &&&&&&&& \\
&&&&&&& m_3 &&&&&&& \\
&&&&&&&& J_3 &&&&&& \\
&&&&&&&&& m_4 &&&&& \\
&&&&&&&&&& m_4 &&&& \\
&&& 0 &&&&&&&& J_4 &&& \\
&&&&&&&&&&&& m_5 && \\
&&&&&&&&&&&&& m_5 & \\
&&&&&&&&&&&&&& J_5
\end{bmatrix}
$$

$$[m]\left([A_1]\{\varepsilon\}+[A_2]\{\omega^2\}\right)=\{Q\}$$

$$
[m][A_1]=\begin{bmatrix}
-\alpha_1 m_1 l_1 s_1 & 0 & 0 & 0 & 0 \\
\alpha_1 l_1 m_1 c_1 & 0 & 0 & 0 & 0 \\
J_1 & 0 & 0 & 0 & 0 \\
-m_2 l_1 s_1 & -\alpha_2 m_2 l_2 s_2 & 0 & 0 & 0 \\
m_2 l_1 c_1 & \alpha_2 m_2 l_2 c_2 & 0 & 0 & 0 \\
0 & J_2 & 0 & 0 & 0 \\
-m_3 l_1 s_1 & -m_3 l_2 s_2 & -\alpha_3 m_3 l_3 s_3 & 0 & 0 \\
m_3 l_1 c_1 & m_3 l_2 c_2 & \alpha_3 m_3 l_3 c_3 & 0 & 0 \\
0 & 0 & J_3 & 0 & 0 \\
-m_4 l_1 s_1 & -m_4 l_2 s_2 & -m_4 l_3 s_3 & -\alpha_4 m_4 l_4 s_4 & 0 \\
m_4 l_1 c_1 & m_4 l_2 c_2 & m_4 l_3 c_3 & \alpha_4 m_4 l_4 c_4 & 0 \\
0 & 0 & 0 & J_4 & 0 \\
-m_5 l_1 s_1 & -m_5 l_2 s_2 & -m l_3 s_3 & -m_5 l_4 s_4 & -\alpha_5 m_5 l_5 s_5 \\
m_5 l_1 c_1 & m_5 l_2 c_2 & m_5 l_3 c_3 & m_5 l_4 c_4 & \alpha_5 m_5 l_5 c_5 \\
0 & 0 & 0 & 0 & J_5
\end{bmatrix}
$$

$$[M]=[A_1]^T[m][A_1]$$

$M_{11}=(m_1\alpha_1^2+m_2+m_3+m_4+m_5)l_1^2+J_1$ $M_{12}=l_1l_2c_{2-1}(\alpha_2m_2+m_3+m_4+m_5)$; $M_{13}=l_1l_3c_{3-1}(\alpha_3m_3+m_4+m_5)$; $M_{14}=l_1l_4c_{4-1}(\alpha_4m_4+m_5)$; $M_{15}=l_1l_5c_{5-1}(\alpha_5m_5)$;	$M_{22}=(\alpha_2^2m_2+m_3+m_4+m_5)l_2^2+J_2$; $M_{23}=l_2l_3c_{3-2}(\alpha_3m_3+m_4+m_5)$; $M_{24}=l_2l_4c_{4-2}(\alpha_4m_4+m_5); M_{25}=l_2l_5c_{5-2}(\alpha_5m_5)$;
$M_{33}=(\alpha_3^2m_3+m_4+m_5)l_3^2+J_3$; $M_{34}=l_3l_4c_{4-3}(\alpha_4m_4+m_5)$; $M_{35}=l_3l_5c_{5-3}(\alpha_5m_5)$; $M_{44}=(\alpha_4^2m_4+m_5)l_4^2+J_4$; $M_{45}=l_4l_5c_{5-4}(\alpha_5m_5)$; $M_{55}=(\alpha_5^2m_5)l_5^2+J_5$;	

$$\{Q\}=\begin{Bmatrix} X_A+X_B \\ Y_A-Y_B-G_1 \\ M_1+X_A\alpha_1 l_1 s_1-X_B(1-\alpha_1)l_1 s_1-Y_A\alpha_1 l_1 c_1-Y_B(1-\alpha_1)l_1 c_1 \\ -X_B+X_C \\ Y_B+Y_C-G_2 \\ M_2-X_B\alpha_2 l_2 s_2-X_C(1-\alpha_2)l_2 s_2+Y_B\alpha_2 l_2 c_2-Y_C(1-\alpha_2)l_2 c_2 \\ -X_C+X_D \\ -Y_C-Y_D-G_3 \\ M_3-X_C\alpha_3 l_3 s_3-X_D(1-\alpha_3)l_3 s_3+Y_C\alpha_3 l_3 c_3-Y_D(1-\alpha_3)l_3 c_3 \\ X_E-X_D \\ Y_E+Y_D-G_4 \\ M_4+X_D\alpha_4 l_4 s_4+X_E(1-\alpha_4)l_4 s_4-Y_D\alpha_4 l_4 c_4-Y_E(1-\alpha_4)l_4 c_4 \\ -X_E \\ -Y_E-G_5-G_6 \\ M_5-X_E\alpha_5 l_5 s_5+Y_E\alpha_5 l_5 c_5 \end{Bmatrix}
\begin{bmatrix} -\alpha_1 l_1 s_1 & 0 & 0 & 0 & 0 \\ \alpha_1 l_1 c_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -l_1 s_1 & -\alpha_2 l_2 s_2 & 0 & 0 & 0 \\ l_1 c_1 & \alpha_2 l_2 c_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -\alpha_3 l_3 s_3 & 0 & 0 \\ l_1 c_1 & l_2 c_2 & \alpha_3 l_3 c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -\alpha_4 l_4 s_4 & 0 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & \alpha_4 l_4 c_4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -l_4 s_4 & -\alpha_5 l_5 s_5 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & l_4 c_4 & \alpha_5 l_5 c_5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[m][A_2]=\begin{bmatrix} -\alpha_1 m_1 l_1 c_1 & 0 & 0 & 0 & 0 \\ -\alpha_1 m_1 l_1 s_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -m_2 l_1 c_1 & -\alpha_2 m_2 l_2 c_2 & 0 & 0 & 0 \\ -m_2 l_1 s_1 & -\alpha_2 m_2 l_2 s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -m_3 l_1 c_1 & -m_3 l_2 c_2 & -\alpha_3 m_3 l_3 c_3 & 0 & 0 \\ -m_3 l_1 s_1 & -m_3 l_2 s_2 & -\alpha_3 m_3 l_3 s_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -m_4 l_1 c_1 & -m_4 l_2 c_2 & -m_4 l_3 c_3 & -\alpha_4 m_4 l_4 c_4 & 0 \\ -m_4 l_1 s_1 & -m_4 l_2 s_2 & -m_4 l_3 s_3 & -\alpha_4 m_4 l_4 s_4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -m_5 l_1 c_1 & -m_5 l_2 c_2 & -m_5 l_3 c_3 & -m_5 l_4 c_4 & -\alpha_5 m_5 l_5 c_5 \\ -m_5 l_1 s_1 & -m_5 l_2 s_2 & -m_5 l_3 s_3 & -m_5 l_4 s_4 & -\alpha_5 m_5 l_5 s_5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[M']=[A_1]^T[m][A_2]$$

$M'_{11}=0$; $M'_{12}=(\alpha_2 m_2+m_3+m_4+m_5)l_1 l_2 s_{1-2}$; $M'_{13}=(\alpha_3 m_3+m_4+m_5)l_1 l_3 s_{1-2}$; $M'_{14}=(\alpha_4 m_4+m_5)l_1 l_4 s_{1-4}$; $M'_{15}=\alpha_5 m_5 l_1 l_5 s_{1-5}$	$M'_{21}=(\alpha_2 m_1+3)m_1 l_1 l_2 s_{2-1}$; $M'_{22}=0$; $M'_{23}=(\alpha_3+2)m_3 l_2 l_3 s_{2-3}$; $M'_{24}=(\alpha_4+1)m_4 l_2 l_4 s_{2-4}$; $M'_{25}=\alpha_5 m_5 l_2 l_5 s_{2-5}$;
$M'_{31}=l_1 l_3(\alpha_3 m_3+m_4+m_5)s_{3-1}$; $M'_{32}=l_2 l_3(\alpha_3 m_3+m_4+m_5)s_{3-2}$; $M'_{33}=0$	$M'_{34}=l_3 l_4(\alpha_4 m_4+m_5)s_{3-4}$; $M'_{35}=\alpha_5 m_5 l_1 l_5 s_{3-5}$
$M'_{41}=l_1 l_4(\alpha_4 m_4+m_5)s_{4-1}$; $M'_{42}=l_2 l_4(\alpha_4 m_4+m_5)s_{4-2}$; $M'_{43}=l_3 l_4(\alpha_4 m_4+m_5)s_{4-3}$; $M'_{44}=0$; $M'_{45}=\alpha_5 m_5 l_4 l_5 s_{4-5}$	$M'_{51}=l_1 l_5(\alpha_5 m_5)s_{5-1}$; $M'_{52}=l_2 l_5(\alpha_5 m_5)s_{5-2}$; $M'_{53}=l_3 l_5(\alpha_5 m_5)s_{5-3}$; $M'_{55}=0$; $M'_{54}=\alpha_5 m_5 l_4 l_5 s_{5-4}$

$$[M']=$$

$$\begin{bmatrix} 0 & (\alpha_2 m_2+m_3+m_4+m_5)l_1 l_2 s_{1-2} & (\alpha_3 m_3+m_4+m_5)l_1 l_3 s_{1-2} & (\alpha_4 m_4+m_5)l_1 l_4 s_{1-4} & \alpha_5 m_5 l_1 l_5 s_{1-5} \\ (\alpha_2 m_2+m_3+m_4+m_5)l_1 l_2 s_{2-1} & 0 & (\alpha_3 m_3+m_4+m_5)l_2 l_3 s_{2-3} & l_2 l_4(\alpha_4 m_4+m_5)s_{2-4} & \alpha_5 m_5 l_2 l_5 s_{2-5} \\ l_1 l_3(\alpha_3 m_3+m_4+m_5)s_{3-1} & l_2 l_3(\alpha_3 m_3+m_4+m_5)s_{3-2} & 0 & (\alpha_4 m_4+m_5)l_3 l_4 s_{3-4} & \alpha_5 m_5 l_3 l_5 s_{3-5} \\ l_1 l_4(\alpha_4 m_4+m_5)s_{4-1} & l_2 l_4(\alpha_4 m_4+m_5)s_{4-2} & l_3 l_4(\alpha_4 m_4+m_5)s_{4-3} & 0 & \alpha_5 m_5 l_4 l_5 s_{4-5} \\ l_1 l_5(\alpha_5 m_5)s_{5-1} & l_2 l_5(\alpha_5 m_5)s_{5-2} & l_3 l_5(\alpha_5 m_5)s_{5-3} & \alpha_5 m_5 l_4 l_5 s_{5-4} & 0 \end{bmatrix}$$

$$[M]\{\varepsilon\}+[M']\{\omega^2\}=[A_1]^T\{Q\}$$

$$Q_1 = M_1 - l_1c_1(\alpha_1G_1 + G_2 + G_3 + G_4 + G_5 + G_6) \quad ;$$
$$Q_2 = M_2 - l_2c_2(\alpha_2G_2 + G_3 + G_4 + G_5 + G_6) \quad ;$$
$$Q_3 = M_3 - l_3c_3(\alpha_3G_3 + G_4 + G_5 + G_6) \quad ;$$
$$Q_4 = M_4 - l_4c_4(\alpha_4G_4 + G_5 + G_6) \quad ;$$
$$Q_5 = M_5 - l_5c_5\alpha_5(G_5 + G_6) \quad ;$$

$$\{Q'\} = \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{Bmatrix} - \begin{Bmatrix} l_1c_1(\alpha_1G_1 + G_2 + G_3 + G_4 + G_5 + G_6) \\ l_2c_2(\alpha_2G_2 + G_3 + G_4 + G_5 + G_6) \\ l_3c_3(\alpha_3G_3 + G_4 + G_5 + G_6) \\ l_4c_4(\alpha_4G_4 + G_5 + G_6) \\ l_5c_5\alpha_5(G_5 + G_6) \end{Bmatrix} = \{M^{ext}\} - \{M_G\}$$

$$[M]\{\varepsilon\} + [M']\{\omega^2\} = \{M^{ext}\} - \{M_G\} \quad \{M^{ext}\} = [M]\{\varepsilon\} + [M']\{\omega^2\} + \{M_G\}$$

4. CONCLUSION:

After achievement the filming and their analysis were elaborated kinograms throw which corroborated with the dynamic model can be a strength in modeling free throws in basketball game. Making specialized programs in kinematic analysis for sport performance can be very helpful for coaches and athletes during training stages. This model should allow the specialist to be able to compare and extract the necessary information in a short time.

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