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RAILWAYS TRANSITIONS SPLINES WITH G3 CONTACTS

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Abstract: *The paper is a continuation of earlier concerns regarding the roads/railways transitions ends contacts. If in the previous approaches G2 contacts were treated, this paper discuss how to integrate Excel Solver and a low priced CAGD environment (AutoCAD clone) in order to draw G3 contacts railways transition curves, with super-linear monotonic variation of the curvature, using the AutoCAD/IntelliCAD implementation of NURBS (Non-Uniform Rational Basic-Spline). In order to avoid unwanted local alteration of transition shape, the paper uses the particular case of non-uniform non-rational B-splines (NUBS) without interior knots, that is Bézier splines in fact. Besides reaching the highest possible quality level of the ends contacts, these splines are very advantageous when replace the intermediate cubic parabolas of the reversed curves because it can lead to 40% reduction of the transition length.*

Key words: *Bézier, B-spline, NUBS, NURBS, transition curve, curvature, NLP.*

1. Foreword

CAGD environments such as AutoCAD and its clones, extensively used in the design of railways, have spline function which, in the simplest case, unfortunately draws only G1 contacts non-uniform non-rational cubic B-spline (NUBS) segment which, in fact, reduces to a Bézier curve.

As the shape is fair, very easy to draw and afterwards to divide into intermediate points for tracing reasons, there is the temptation, thanks to the linear variation of curvature (Figure 1), to use it instead of the cubic parabola which sadly lack in regular CAGD environments, but is included in very costly AutoDesk Civil 3D, Bentley Microstation and Rail Track, ProVI add-in for AutoCad etc.

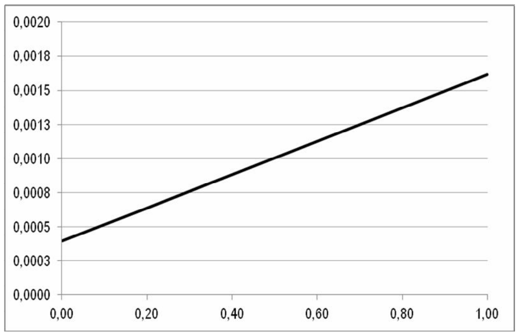

Fig. 1. *100 m long, 4th order (cubic) NUBS curvature (AutoCAD and IntelliCAD implementation) to connect the 0 curvature straight with a 0,002 curvature of the 500 m radius circle (in abscissa Lr = 1 scale is used)*

In Figure 1, while between anchors and corresponding control points there are legs

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equal with a third part of transition chord, the curvature varies linearly, but no G2 contacts.

As long, due to the lack of G2 contacts (fig. 1), the simple application of the original spline is wrong, these three solutions remain:

i) drawing the transition curve like cubic parabola through its coordinates;

ii) approximation of cubic parabola (which almost has G2 contacts) by Bézier splines (following [1], the sufficiently accurate approximation of the clothoid arc begins with the 8^{th}-order Bézier curve - polynomial of minim 7 degree and 8 control points);

iii) since ii) involves the setting-up and running of an optimizing routine, it deserves to spend the effort for a better transition curve with G3 contacts.

The method developed in this paper stands on the point iii) and the following four reasons :

iv) drawing transition curves other than NUBS, in a regular CAGD application (like AutoCAD and its clones), even through LISP or VBA modules, is more complicated than the technique below;

v) CAGD environments dedicated to roads and railways design are so expensive that many design companies not afford;

vi) even if they afford such applications, the training effort for current work with them is great;

vii) access to quality design of the young enthusiasts at the beginning of their career which can afford at most a cheap AutoCAD clone.

2. Assumptions

The transition curves problem is treated only in terms of the types of contacts;

The AutoCAD (and its clones) NURBS implementation is used. For example, for a single segment of curve, NURBS AutoCAD requires only entering the coordinates of anchors and extreme tangents, the coordinates of control points and knots being calculated based on the input data;

The aim is to obtain a non-uniform and non-rational B-spline curve or a non-rational Bézier curve with G3 contacts (to avoid any confusion, we note that although the two types of curves are calculated differently for roads/railways transition curves the non-uniform and non-rational B-spline curves reduce to non-rational Bézier curves);

A monotonic S shaped curvature is to be achieved.

3. Solution

First, it choose a fair form for the theoretical curvature between 0 and 1/R. In fig. 3 the theoretical curvature is a spline of 6th order (polynomial of 5 degree with 6 control points Pi) given by the following relationships:

- NUBS form

$$C_5(t) = \sum_{i=0}^{5} N_{i,5}(t) P_i \qquad 0 \le t \le 1 \tag{1}$$

where $N_{i,5}(t)$ are B-spline basis functions

$$N_{i,0}(t) = \begin{cases} 1 & if\ u_i \le t \le u_{i+1} \\ 0 & otherwise \end{cases} \tag{2}$$

$$N_{i,5}(t) = \frac{t - u_i}{u_{i+5} - u_i} N_{i,4}(t) + \frac{u_{i+6} - t}{u_{i+6} - u_{i+1}} N_{i+1,4}(t)$$

where $0 \le u_i \le u_{i+1} \le co$, u_i are *knots* and *co* is NUBS chord.

- Bézier form

$$C_5(t) = \sum_{i=0}^{5} B_{i,5}(t) P_i \qquad 0 \le t \le 1 \tag{3}$$

where $B_{i,5}(t)$ are Bernstein basis polynomials

$$B_{5,i}(t) = \frac{5!}{i! \cdot (5-i)!} \cdot t^i \cdot (1-t)^{5-i} \quad (4)$$

$$C_5(t) = (1-t)^5 P_0 + 5t(1-t)^4 P_1 + 10t^2(1-t)^3 P_2 + 10t^3(1-t)^2 P_3 + 5t^4(1-t)P_4 + t^5 P_5 \qquad 0 \le t \le 1 \quad (5)$$

transition curve is calculated as a 8th order spline (polynomial of 7 degree with 8 control points Pi) whose Bézier form is that is:

$$C_7(t) = \sum_{i=0}^{7} B_{7,i}(t) \cdot P_i \quad cu \quad 0 \le t \le 1 \quad (6)$$

or

$$C_{x7}(t) = (1-t)^7 x_0 + 7t(1-t)^6 x_1 + 21t^2(1-t)^5 x_2 + 35t^3(1-t)^4 x_3 + 35t^4(1-t)^3 x_4 + 21t^5(1-t)^2 x_5 + 7t^6(1-t)x_6 + t^7 x_7 \quad (7)$$

$$C_{y7}(t) = (1-t)^7 y_0 + 7t(1-t)^6 y_1 + 21t^2(1-t)^5 y_2 + 35t^3(1-t)^4 y_3 + 35t^4(1-t)^3 y_4 + 21t^5(1-t)^2 y_5 + 7t^6(1-t)y_6 + t^7 y_7 \quad (8)$$

A NLP shall be defined with $x_1 \div x_6$ and $y_1 \div y_6$ variables and objective function

$$F_{ob} = \sum_{t=0}^{1} \left(k_{t,Real} - k_{t,Theoretic}\right)^4 \quad (9)$$

where the real curvature is calculated as

$$k_{t,Real} = \frac{C_{1,x}(t) \cdot C_{2,y}(t) - C_{2,x}(t) \cdot C_{1,y}(t)}{\sqrt{\left(C_{1,x}^2(t) + C_{1,y}^2(t)\right)^3}} \quad (10)$$

and several constraints regarding ends tangents.

$C_{1,x}(t)$ and $C_{1,y}(t)$ are first derivatives of $C_{x7}(t)$ and $C_{y7}(t)$, while $C_{2,x}(t)$ and $C_{2,y}(t)$ are second derivatives. NLP is solved by Excel Solver.

4. Example

In the next example are presented the steps of algorithm:

1. It is considered a transition curve whose anchors are P_0 (x_0= 0.0000, y_0= 0.0000) and P_7 (x_7= 100.0000, y_7= 2.4000). The transition links the alignment of k_{AR}=0.0000 curvature with the circle of k_{RC}=0,0020 curvature (R=500 m) having the extreme tangents T_{AR}=0.0000 and T_RC=0,0720.
2. It draws a standard 4th order B-spline (2 anchors and 2 intermediate control points) with the above data using *spline* command.
3. Using *splinedit* command edit the curve and then increase the order from 4 to 8 with *refine* and *elevate order* commands. CAGD environment will select the resulted 8 control points.
4. Copy the control points coordinates into Excel sheet where the optimizing problem is already defined ($P_1 \div P_6$ control points coordinates are the variables of the optimizing problem) and solve the NLP (fig. 2, fig. 3 and fig. 4).
5. Go back into the CAGD, select the curve, *splinedit* and *Move vertex* commands are executed and introduce new coordinate values of control points $P_1 \div P_6$. It is noted that there is a change of curve shape.

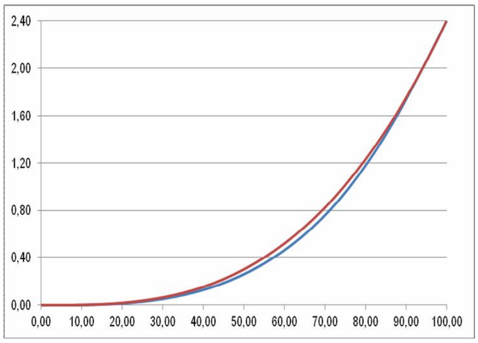

Fig. 2. *Initial (red) and final (blue) C7(t)*

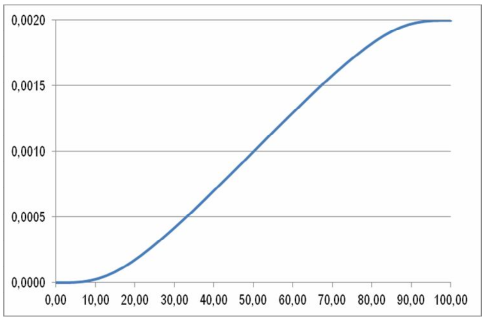

Fig. 3. *Overlapped theoretical and real $C_7(t)$ curvatures.*

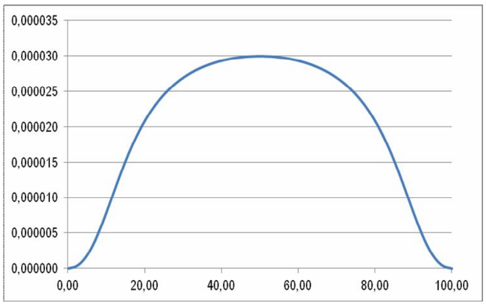

Fig. 4. *Real curvature variation.*

Steps 3÷5 can be easily implemented in a VBA module working inside CAGD as the above commands are available in VBA and Excel can be automatically integrated with AutoCAD and its clones. It follows that the final solution of the problem reduces to manual execution of step 2 and launch the VBA module for the remaining steps.

5. Conclusions

Low cost CAGD environments can be used both for approximating clothoid or cubic parabola arcs with almost G2 contacts, as well as for drawing transition curves with G3 contacts ;

The best use of splines with G3 contacts is the case of reversed curves for high speed tracks when the transition length can reduce with 40%. For all other curves the transition spline is longer than cubic parabola or clothoid.

Rational non-uniform B-splines or rational Bézier splines are not favorable for transition curves.

6. Future Work

In the future is interesting to take into account:

- a NLP improvement for a faster solution;
- establishing the theoretical curvature and the transition length based on vehicle-track interaction criteria.

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