# Study on the Possibility to Estimate the Vehicle Side Slip Using Two Independent GPS Receivers

COVACIU Dinu<sup>1, a\*</sup>, PREDA Ion<sup>1,b</sup>, DIMA Dragoş-Sorin<sup>1,c</sup> and CHIRU Anghel<sup>1,d</sup>

<sup>1</sup>Automotive and Transport Department, Transilvania University of Brasov, Blvd. Eroilor, no. 29,

500036 Brasov, Romania

<sup>a</sup>dinu.covaciu@unitbv.ro, <sup>b</sup>pion@unitbv.ro, <sup>c</sup>d.dima@unitbv.ro, <sup>d</sup>achiru@unitbv.ro

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**Abstract.** Slip angle is the difference between the direction a vehicle is travelling and the longitudinal plane of the vehicle body. Knowing vehicle sideslip angle accurately is critical for active safety systems such as Electronic Stability Program (ESP). Vehicle sideslip angle can be measured using optical speed sensors, inertial sensors and/or dual-antenna GPS receivers. These systems are expensive and their use is limited for many users. The goal of this paper is to analyze the possibility to estimate the vehicle sideslip angle, in real-time, by using two low-cost single-antenna GPS receivers.

### Introduction

Slip angle is basically the difference between the travelling direction of a vehicle (known also as course, or course over ground) and the direction of a longitudinal plan of the vehicle body (known also as true heading). The yaw rate or yaw velocity of a vehicle is the angular velocity of this rotation, or rate of change of the heading. The knowledge of the vehicle sideslip and of the yaw rate is critical for some vehicle control systems (like ESP). Many previous studies were conducted in order to find the best solution for measuring the vehicle slip, for both research and practical purposes.

The existing methodologies for sideslip estimation can be categorized into three groups [1]: kinematics-based estimation, kinetic model based estimation, and GPS based estimation. The first method (kinematics based) is robust to vehicle parameter errors, but it is sensitive to sensor bias and disturbances. Kinetic model-based methods require very accurate parameter information, and the small sideslip angle assumption is critical.

In most researches, the solution was to use GPS antennas in combination with inertial sensors: [1], [3], [4], [5]. The principles are presented in detail in [2]. Yaw rate can be measured with a low-cost gyroscope, but for vehicle sideslip measurements the ground speed of different points of the vehicle is required, with a good accuracy. These types of sensors are expensive. During the last years, it was demonstrated that the sideslip can be directly measured with a two-antenna GPS system [1], [6], combined if possible with IMU (Inertial Measurement Unit) sensors. There was also proposed the use of PPP (precise-point-positioning) technology in order to obtain a higher accuracy of GPS measurements [7].

### **Theory of Slip Angle Measurement**

**Kinematic model.** The model used frequently in the study of the lateral dynamics of a vehicle is the "bicycle model" (single-track vehicle model) [8], [9] (Fig. 1). The two left and right front wheels are represented by one single wheel at point A, and the rear wheels are represented by one central rear wheel at point B. The steering angles for the front and rear wheels are represented by  $\delta_f$  and  $\delta_r$  respectively. The center of gravity (CG) of the vehicle is at point CG. The distances of

points A and B from the CG of the vehicle are  $l_f$  and  $l_r$ . The wheelbase of the vehicle is  $L = l_f + l_r$ . Three coordinates are required to describe the planar motion of the vehicle: X, Y and  $\Psi$ . The coordinates X, Y are inertial coordinates describing the location of the CG. The yaw angle  $\Psi$  indicates the orientation angle (heading) of the vehicle. The velocity at the CG of the vehicle is V and makes an angle  $\beta$  with the longitudinal axis of the vehicle. The angle  $\beta$  is called the slip angle of the vehicle.

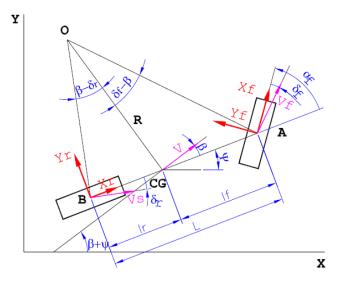


Fig. 1. Simplified model for study of vehicle kinematics

The angles  $\delta_f$  and  $\delta_r$  are between each wheel direction (of the bicycle model) and the direction of the vehicle body. From the triangles OC<sub>G</sub>A and OC<sub>G</sub>B in Fig. 1 result the equations:

$$\frac{\sin(\delta_f - \beta)}{l_f} = \frac{\sin(\frac{\pi}{2} - \delta_f)}{R}$$
(1)

$$\frac{\sin(\beta - \delta_r)}{l_r} = \frac{\sin(\frac{\pi}{2} + \delta r)}{R}$$
(2)

From these, after successive transformations results:

$$\left[\tan(\delta_f) - \tan(\delta_r)\right] \cdot \cos(\beta) = \frac{l_f + l_r}{R}$$
(3)

Knowing that the angular velocity of the vehicle is v/R, representing the rate of change of the vehicle heading (changing in the orientation of vehicle, or the yaw rate of the vehicle body) is

$$\dot{\Psi} = \frac{v}{R} \tag{4}$$

the overall equations of motion will be [8], [9]:

$$X = v \cdot \cos(\Psi + \beta) \tag{5}$$

 $\dot{Y} = v \cdot \sin(\Psi + \beta) \tag{6}$ 

$$\dot{\Psi} = \frac{v \cdot \cos(\beta)}{l_f + l_r} \left( \tan(\delta_f) - \tan(\delta_r) \right)$$
(7)

The input values in this model are  $\delta_f$ ,  $\delta_r$  and v. The velocity v can be assumed to be a time varying function and can be obtained from a longitudinal vehicle model. The slip angle  $\beta$  can be determined when the changes of the coordinates for a small interval of time and the yaw rate are known.

#### Slip angle measurement with two GPS antennas

One of the professional device used for slip angle measurement, based on GPS receivers, is VB20SL (Fig. 2) (part of VBOX family) from Racelogic. The VB20SL is a multi-purpose noncontact speed sensor [11]. Using an advanced dual antenna GPS engine, the VB20SL can calculate not only the speed and direction of travel of the object upon which it is placed, but also an accurate slip angle and pitch or roll angle.



Fig. 2. VB20SL (20 Hz sampling rate)

The installation of the two antennas depends by the variables that should be measured. In order to measure the body slip angle and the pitch angle, the antennas should be placed along the longitudinal axis of the vehicle. In order to measure the body slip angle and the roll angle, the antennas will be placed transversally.

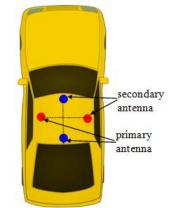


Fig. 3. Using two antennas for slip angle measurement

As shown in Fig. 3, the VB20SL system uses two antennas to measure the slip angle – one designated as the primary antenna and the other one designated as the secondary antenna [12]. The antennas are placed on the vehicle at a set distance apart. The data from both antennas are used to calculate the true heading (which is the direction of a straight line through the antennas, disposed longitudinally, as in Fig. 3, between the blue circles). The greater the separation between the antennas is, the more accurate is the measurement of slip angle. It is also measured the GPS course at the primary antenna (course over ground heading). The slip angle at the primary antenna is the difference between the true heading and the course over ground heading at that point.

Any turning manoeuvre will generate a level of slip angle. The vehicle will still have a true heading and a course over ground, but that "course over ground" is different depending upon where on the vehicle it is measured (depends on the chosen point). The "course over ground" heading of a point at the front of the vehicle will be generally different to that at the back of the vehicle during a turning manoeuvre. So it is very important where is placed the primary antenna. If the primary antenna is placed over the steered wheels themselves, the measured slip angle would be almost identical to the wheels steering angles, less a small amount of tire slip.

In case of oversteering, for example, the slip angle measured at the rear of the car will show a larger slip angle than in a normal steering manoeuvre. The slip angles measured at the middle and front of the car in this case are reduced, but still affected by the angle of the steered wheels. The slip angle of any point on the vehicle body can be obtained by translating the value obtained for the point of measurement, considering the yaw rate and the velocity are known (starting from the relations Eq.5 - Eq.7 given above).

Using precise positioning with two antennas, it is possible to measure the slip angle without the need of a supplementary INS device (inertial navigation system). However, the accuracy of the relative positioning data and also the heading and speed information obtained from the two antennas are critical.

## Slip Angle Measurement with VB20SL

Data recorded with VBOX devices (including VB20SL) are stored in text files, with a structure easy to understand. Data can be processed using VBOXTools software or even a general data processing software, like Excel. A sample test is presented here, with data recorded in a roundabout (see Fig. 4).

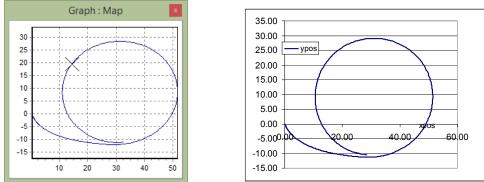


Fig. 4. The track represented in VBOXTools (left) and Excel (right)

Sample data are presented in Table 1. There are three main categories of data: measured (examples: satellites, latitude, longitude, speed, heading, height), calculated (examples: travelled distance, x and y coordinates in meters, absolute heading, slip angle, centre line deviation, radius of turn) and taken from CAN interface (examples: heading 2, true heading). In this case data taken from CAN interface are provided by the VB20SL engine, and are obtained thanks to the secondary GPS antenna.

Table 1 - Sample data from VB20SL

Time	Vel.	Head.	Slip Angle	turn- radius	Centre- Line-Dev.	Х	Y	True Head
0	1.066	157.45	0.00	1.01	0.31	0.00	0.00	312.10
0.05	1.079	152.79	0.00	5.85	0.31	0.01	-0.02	312.27
0.1	1.06	154.17	106.37	1.19	0.32	0.01	-0.03	311.96
0.15	1.411	155.32	132.76	0.99	0.33	0.02	-0.05	311.93
0.2	1.271	156.2	144.94	50.57	0.33	0.03	-0.07	311.77
0.25	1.47	154.23	149.31	1.38	0.34	0.04	-0.09	311.65
0.3	1.615	157.87	152.59	1.72	0.35	0.05	-0.11	311.54
0.35	1.612	153.96	154.42	4.32	0.36	0.06	-0.13	311.50
0.4	1.8	155.12	155.63	1.37	0.37	0.07	-0.15	311.44
0.45	1.939	154.73	156.08	1.16	0.38	0.08	-0.18	311.43
0.5	1.975	157.95	156.02	7.60	0.39	0.09	-0.20	311.20
0.55	2.073	154.5	156.71	4.50	0.40	0.10	-0.23	311.06

In Fig. 5 below is given the graphical representation of calculated slip angle (orange), measured speed (blue), heading (gray) and true heading (red). The same variables, with slip angle calculated in Excel, are shown in Fig. 6. The value of "true heading" is taken from VB20SL - is calculated internally, based on the heading and velocity values measured with the two GPS sensors. The "heading" value is given by the primary GPS antenna and slip angle is calculated.

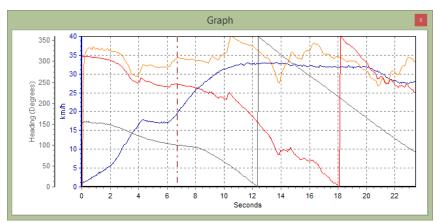


Fig. 5. Diagrams of speed, heading and slip angle, as function of time (VBOXTools)

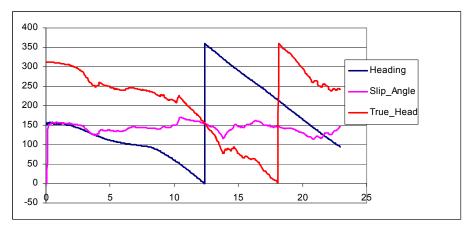


Fig. 6. Diagrams of heading, true heading and slip angle, calculated in Excel

Other calculated data are centre line deviation and radius of turn. Radius of turn and speed will give the yaw rate (Eq.4). Centre line deviation indicates the vehicle deviation from a specified centre line of the track. The centre line can be configured by the user. The VBOXTools software includes the capability to perform centre line deviation tests in both live and post-processed modes. By knowing the heading when driving straight ahead, the software can calculate any deviation from the reference line [13]. This is done using the speed and heading parameters, and as such is very accurate (within a few cm) for tests of short durations (less than one minute). It should be noted that the heading value recorded by a VBOX is more accurate at faster speeds, so this test is best done over 30 km/h, according to the VBOX documentation. The centre line deviation test is usually made for straight roads and is meant to indicate when the car is not driving straight ahead or when he make an overtaking maneuver.

## Slip Angle Measurement with Two Independent Receivers

The VB20SL system has an update rate of 20 Hz. Higher update rates are available only for the systems with single antenna. On the market can be found also some single antenna devices with a 20 Hz update rate, and many devices with 10 Hz or 5 Hz sampling rate. A problem at high sampling rates is to synchronize the outputs of the two receivers. For low sampling rates, the synchronization

is not so important. This is why we chosen for tests two GPS 18x-5Hz receivers from Garmin, with an update rate of 5 Hz [14].

The output of the GPS 18x-5Hz receiver consists in successive sequences of text information, according to NMEA 0183 [15] specification. These data are primarily collected and processed using a custom software application integrated in an in-house made data acquisition system, DS-5 [16]. Data stored by this system in the output text files were not enough for this study, so that it was necessary to update the software, to store also the heading values. An upgraded version of the software is capable also to take data from two receivers simultaneously, display some data on the screen and record data in two separate files, one for each receiver. The user interface of the data acquisition software is shown in Fig. 7.

Eventually the data recorded on a test looks like this:

25.6352237,45.6397665,605.6,14:58:08.6, 11.15,358.2,1,07 25.6352237,45.6397727,605.5,14:58:08.8, 11.28,356.8,1,07 25.6352242,45.6397785,605.5,14:58:09.0, 11.30,002.1,1,07 25.6352240,45.6397842,605.5,14:58:09.2, 11.30,002.1,1,07 25.6352230,45.6397898,605.4,14:58:09.4, 12.17,357.6,1,07 25.6352213,45.6397957,605.4,14:58:09.6, 12.24,358.5,1,07 25.6352198,45.6398015,605.3,14:58:09.8, 12.24,358.5,1,07 25.6352193,45.6398078,605.1,14:58:10.0, 12.98,001.0,1,07

where the fields, comma separated, are: longitude, latitude, altitude, time, speed, heading, a quality indicator (1 means standard 3D fix, 2 means differential signal was available) and the number of satellites in view.

It can be noticed the resolution of the available data: 7 decimals for the geographic coordinates, 0.1 meters for altitude, 0.01 km/h for speed, 0.1 degrees for heading.

COM ports:	and a second	Received Data			Calculated Data		
Bisconnec	t Date (DMY):	311214		Distance:	<u></u>		
/ehicle	Time (UTC):	103517.4 103516.4		Total time:	( <u></u> );		
Bec 2	a			Max speed:	-		
Rec. 2 <u>S</u> tart	Latitude:	45.6396657 N 25.6352248 E		Mean speed:			
	Longitude:						
ality	Altitude:	594.6	594.6				
10	1 to a diversi	100.1	102.2	Accel:			
Satellites	Heading:	182.1	183.3	Slip angle:	-		
0.9 HDOP	Speed:	0.00	0.00	Yaw rate:	1 <u>111</u> 1		
10 0.9 HDOP 0.9 D D	Records:	798	798				
NIA	IEA Strings (\$GPRMC)						

Fig. 7. The user interface of the data acquisition software

The heading value represents the course over ground heading for that receiver (antenna). Zero degrees means the traveling direction is North.

From this point, the data processing was done using a custom AutoLisp application (under AutoCAD) [16] and MsExcel.

### **Data Collecting and Processing**

The two antennas were mounted in two versions: on the longitudinal axis of the vehicle, and transversally, above the approximate section that includes the CG (center of gravity) of the vehicle (as represented in Fig. 3).

With the two antennas aligned with the longitudinal axis, the registered tracks are represented in Fig. 8. The secondary antenna (front) was mounted approximately above the CG of the vehicle body.

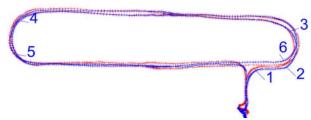


Fig. 8. Track recorded with two antennas aligned with the longitudinal axis of the car body

The speed versus distance diagram is presented in Fig. 9. There are small differences between curves because the two antennas are travelling slightly different paths.

In Fig. 10 is shown the speed versus time diagram. The numbers above the time line correspond to the numbers marked in Fig. 8. So it can be seen the speed measured by the two antennas at the same moment, depending by the car position, when turning. A detail is given also in Fig. 10, showing that the speed measured by the front receiver is a little higher than the speed measured by the rear receiver, in a left turn of the vehicle (between the marked points 2 and 3 as in Fig. 8).

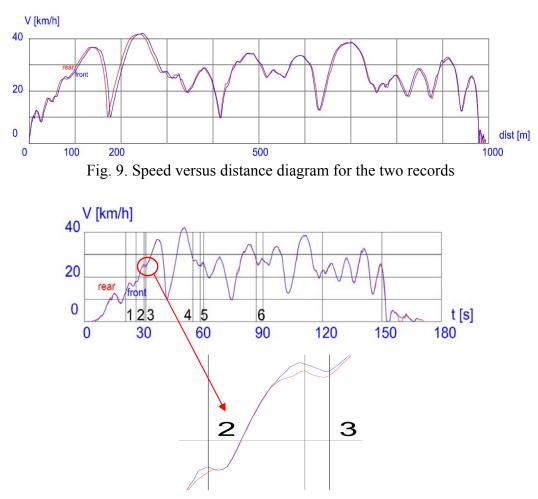
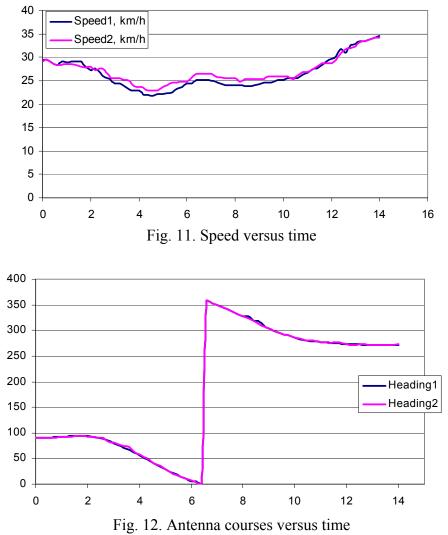


Fig. 10. Speed versus time diagram, with a detail

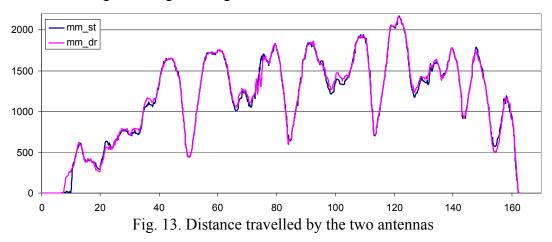
The speed values measured by the GPS receiver are more accurate than the coordinates of successive positions, so the traveled distance for each antenna can be calculated more precisely using the speed and time values. The differences in the speed measured indicate different distances traveled by antennas, meaning that the vehicle turns. The track can be represented in this case

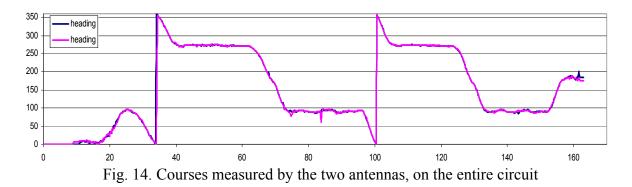
starting from a reference point chosen by the user and drawing points with polar coordinates, using speed and heading information from the two GPS receivers.

The speed and heading diagrams for these records are shown in Fig. 11 and Fig. 12. The differences in speed can be seen obviously when the vehicle turns.



Knowing the distances travelled by the two measuring points (the points where the two antennas are placed) (see Fig. 13) and the distance between these points it can be ascertained the turning radius, and using the speed and radius the yaw rate can be calculated. As for VBOX, a calibration is necessary at the beginning of the test, in order to calculate the yaw angle of the vehicle for successive positions. This calibration can be done also using the logged data, extracting the information for a straight driving road segment.





A diagram of the heading variation during the move on the entire circuit is given in Fig. 14, where can be noticed some obvious deviations of the curve from the expected values. Differences between the two curves are normal, since the vehicle is not moving in straight line, but the peaks that appears for a single antenna are obviously caused by some measuring errors. These errors should be filtered out.

Also based on speed and heading values, the variation of X and Y coordinates can be calculated, and using Eq.5 and Eq.6 it is possible to find the slip angle for the respective point.

However, the distances measured between the equivalent points (considering the time of each record) on the two tracks recorded with two GPS receivers is not constant (as it is in reality). The differences are caused by cumulated errors of speed and heading measurements. The measurement resolutions of speed (0.01 km/h) and heading (0.1 degrees) lead to a deviation of about 2 cm for two successive positions. However, the errors given by signal propagation and receiving conditions (like number of satellites in view, differential signal availability) may be higher.

#### Conclusion

A direct comparison between the results presented in the paper, obtained with the dual antenna system (VB20SL) and with two independent receivers, is not appropriate, since the tests where conducted in different locations, at different times and with different driving conditions. A comparative test between these two types of logging systems will be made as a future work.

The results obtained for particular cases lead us to the conclusion that it is possible to estimate the slip angle, yaw rate, radius of turns and other derived parameters, using logged data from two independent GPS receivers. Even when using very accurate devices, like VB20SL and VBOX, the results are influenced by the reception conditions and different results can be obtained for different places, considering the same behavior of the vehicle. For critical applications of automated vehicle control it is mandatory to use additional sources of information, which can be IMU devices or the vehicle CAN Bus (taking data like wheel speeds and/or steering wheel turning angle). When the goal of the test is, for example, to analyze the vehicle behavior in different driving conditions but in the same location, the dual antenna GPS systems, and also the systems composed by two identical independent GPS receivers, are appropriate.

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# References

[1] J.H. Yoon, H. Peng, Vehicle sideslip angle estimation using two single-antenna GPS receivers. In ASME 2010 Dynamic Systems and Control Conference, pp. 863-870. ASME (2010)

[2] M.S. Grewal, L.R. Weill, A.P. Andrews, Global positioning systems, inertial navigation, and integration. John Wiley & Sons, ISBN 0-471-20071-9 (2007)

[3] J. Ryu, E.J. Rossetter, J.C. Gerdes, Vehicle sideslip and roll parameter estimation using GPS. In Proceedings of the International Symposium on Advanced Vehicle Control (AVEC), Hiroshima, Japan, vol. 2, pp. 373-380 (2002)

[4] O. Mezentsev, Y. Lu, G. Lachapelle, R. Klukas, Vehicular navigation in urban canyons using a high sensitivity GPS receiver augmented with a low cost rate gyro. In ION GPS, vol. 2002, pp. 24-26 (2002)

[5] D.M. Bevly, J. Ryu., J.C. Gerdes, Integrating INS sensors with GPS measurements for continuous estimation of vehicle sideslip, roll, and tire cornering stiffness. Intelligent Transportation Systems, IEEE Transactions on 7, no. 4, p: 483-493 (2006)

[6] R. Anderson, D.M. Bevly, Estimation of slip angles using a model based estimator and GPS. In American Control Conference, 2004. Proceedings of the 2004, vol. 3, pp. 2122-2127. IEEE (2004)

[7] P. Phondeenana, R. Thitipatanapong, S. Klongnaivai, N. Noomwongs, et al. Driver Behavior Detection based On PPP-GNSS Technology, SAE Technical Paper 2014-01-2006, (2014) doi:10.4271/2014-01-2006.

[8] R. Rajamani, Vehicle dynamics and control. ISBN 0-387-26396-9, Springer, 2006.

[9] I. Preda, Gh. Ciolan, Vehicle mathematical model for the study of cornering. In Annals of the Oradea University. Fascicle of Management and Technological Engineering, Volume XI (XXI), 2012, Nr.2, pp. 1.22-1.32 (2012)

[10] I. Preda, V. Vulpe, V. Enache, Simulation of driver behaviour in cornering manoeuvre. In Proceedings of International Congress on Automotive and Transport Engineering, CONAT 2004, Braşov (2004)

[11] Racelogic, VB20SL User Guide, on http://www.racelogic.co.uk.

[12] Racelogic, Slip Angle Explained, Vbox App. Notes, on http://www.racelogic.co.uk.

[13] Racelogic, VBOXTools Software Manual, v. 1.11, on http://www.racelogic.co.uk.

[14] Garmin Intl., GPS 18x Technical Spec., Rev. B (2008), on http://www.garmin.com.

[15] Garmin Intl., Garmin Proprietary NMEA 0183 Sentence Technical Spec. (2006)

[16] D. Covaciu, Study of the dynamic and in-traffic vehicle behaviour using GPS and CAD applications, Ph.D. Thesis, Transilvania University of Brasov, 2010.