Model Based Algorithm for the Study of the Vehicle Suspension

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Abstract. The design of a vehicle suspension system starts with very few input data. Simple models are used during initial simulations in order to ensure the wanted compromise between the comfort and dynamic performance qualities, at different vehicle speeds and loads. That stage leads to the setup on model of the needed suspension parameters, principally the stiffness of suspension spring and tire and the damping coefficient.

In an algorithmic way, this paper summarizes actual design recommendations existing in the field of vehicle suspensions. Based on the procedure in this article, a computer program was implemented in the MDesign software.

Introduction

The main functions of the vehicle's suspension system are: *to cushion* the shocks transmitted from the ground to the bodywork; *to dampen* the oscillations of the body and wheels; *to control* and limit the relative wheel-body positions, in order to optimize the wheel-ground interactions and to protect the parts that connect the rolling system with the drivetrain and body.

The functions of a passive suspension are realized by elastic elements, shock absorbers and guiding mechanisms. The cushioning function is done by the elastic elements (*suspension springs*, *tires*, anti-roll bars and bushings), able to store the energy of the mechanical shocks (induced by the ground unevenness) and to release that later, in a longer time interval. This way, the forces transmitted to the body and, consequently, its accelerations will be smaller.

The damping function of a passive suspension is realized mainly by the *shock absorbers* and, in a smaller measure, by the *tires* and bushings. These dampers generate frictional forces opposing to the relative body-wheels movements, consuming so the kinetic energy of the mechanical system and leading to the decrease of the oscillations amplitude.

The aim of this work is to give the main directions to be followed during the earliest design stages of passive suspensions, or, in other words, to indicate the main steps needed to be done when the design starts from scratch.

The main characteristics of a real suspension (firstly the stiffness of the spring and the damping coefficient of the shock absorber) will be adopted with respect to the results obtained after the simulation based on mathematical models.

Quarter-Car Model of Suspension

To understand the world, the humans imagined models. The models are representations, simplifications, abstractions, conceptualization and interpretations of reality (dictionary definition of model: "theoretic or material system which can be used to indirectly study the properties and transformations of another more complex system, that presents analogies with the first system"). The simulation is the process of models creation and logical manipulation in order to decide how the real world works.

The study of the low-frequency vertical oscillations can be realized with simple models, considering the vehicle as a system consisting in more undeformable bodies connected by massless springs and dampers [1], [2], [3].

Based on the principle that the best model is a simple one ensuring good results, the most often adopted model for the study of vehicle's ride behavior and quality is the so-called "quarter-car model" or "vehicle-corner model" [1], [2], [4], [3], [5], [6], [7]. This includes only two elements with concentrated inertial properties: one for a wheel and the other for the part of the body supported by that wheel. Because, generally, the up-and-down movement presents most interest, only the mass properties of those inertial elements will be considered (neglecting the moments of inertia involved in rotations) and only one DoF, the vertical translation, will be taken into account for each body (one renounce to the other five DoF).

The result is the model presented in Fig. 1, with a sprung mass (the vehicle body) and an unsprung mass (the wheel). The body is linked to the wheel through massless spring and damper, while the tire (represented here as a spring-damper combination) makes the connection between the wheel and the ground. This model is suited for frequencies up to 30 − 50 Hz, that are over the natural frequency of the unsprung mass.

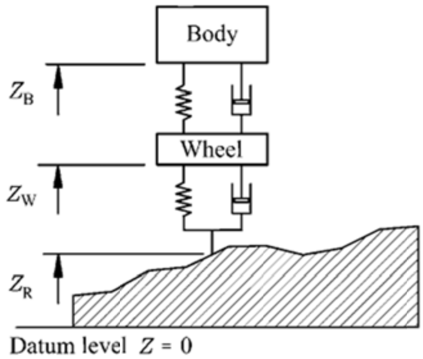

Fig. 1. Ride "quartet-car model" with two DoFs (vertical translation of wheel and body)

The excitation of the system is the force induced by the irregularities of the ground (road) surface. Thus, z_R is the elevation of the surface profile (considered as function of the distance traveled by the vehicle) and $\dot{z}_R$ represents the vertical velocity of the tire at the ground contact point (which is the slope of the profile multiplied by the forward speed of the vehicle).

The reference system is usually adopted in two variants: having a single origin (displacement z=0) for ground, wheel and body or having separate origins for the ground and masses. In that second case (Fig. 1), the positions z_i=0 will correspond to the equilibrium positions, that must be previously calculated. This way, the gravitational forces of wheel and body will not appear explicitly in the movement equation.

By applying the Newton's second law to the sprung and unsprung masses, the system's equations of motion can be obtained:

$$m_B \cdot \ddot{z}_B = F_{1S} + F_D - W_B \qquad (1)$$
$$m_W \cdot \ddot{z}_W = F_{1T} - (F_{1S} + F_D) - W_W$$

in the case of the absolute coordinate system, respectively

$$m_B \cdot \ddot{z}_B = F_S + F_D \qquad (2)$$
$$m_W \cdot \ddot{z}_W = F_T - (F_S + F_D)$$

in the case of the relative coordinate systems, with origins at the static equilibrium positions.

Here, the indices $_W$, $_B$, $_S$, $_D$ and $_T$ stand respectively for wheel, body, spring, damper and tire; so, m_W and m_B are the lumped masses of the wheel and body; W_W and W_B are the weights of the wheel and body; F is a force produced by spring or damper or tire. From the comparative study of the equations (1) and (2) it results:

$$F_S = F_{1S} - W_B \quad \text{and} \quad F_T = F_{1T} - W_W - W_B = F_{1T} - W_{tot}, \qquad (3)$$

where $W_{tot} = W_W + W_B$ is the total weight of the vehicle "quarter".

Special attention must be payed to this aspect, because else confusions may exist and wrong results may be obtained.

The forces F_T, F_S and F_D depend on displacements z, on velocities $\dot{z}$ and, possibly, on time t. Normally, these functions are (highly) unlinear. In these conditions, the solving of the equations system (2) may represent a difficult task and can be realized in the general case only by approximate numerical methods [3].

Exact solutions for the equations system (1) or (2) were obtained only for linear functions F. For that reason, the assumption of linear equations systems is generally considered, in order to simplify the study of vehicle vibrations [1].

To approximate the actual non-linear equations system with a linear one, the spring characteristics (force vs. deflection) are approximated by a straight line in the range of operation, while the damping, which contains both hydraulic and dry (Coulombian) friction components, will be approximated with a viscous damping. More precisely, one considers that: the spring force F_S is proportional with the spring deflection $z_W - z_B$; the damper force F_D is proportional with the relative speed of the damper ends $\dot{z}_W - \dot{z}_B$; the tire force F_T has an elastic component, proportional to the tire deflection $z_R - z_W$, and a viscous friction component, proportional to the speed of the tire deflection $\dot{z}_R - \dot{z}_W$.

With these explanations, the system of equations (2) takes the well-known shape [1], [2], [3], [5], [6], [7]:

$$\begin{aligned} m_B \cdot \ddot{z}_B &= k_S \cdot (z_W - z_B) + c_D \cdot (\dot{z}_W - \dot{z}_B) \\ m_W \cdot \ddot{z}_W &= k_T \cdot (z_R - z_W) + c_T \cdot (\dot{z}_R - \dot{z}_W) - k_S \cdot (z_W - z_B) - c_D \cdot (\dot{z}_W - \dot{z}_B) \end{aligned} \tag{4}$$

where the constant values k_S and k_T are the stiffness coefficients of the model's spring and tire, whille c_D and c_T are the viscous damping coefficients of the model's damper and tire.

It must be noted that the study of the oscillations based on simple models, including only lumped masses (rigid bodies) and linear springs and dampers, can be made only for small amplitudes, because only in this case a linear approximation with sufficient precision can be achieved for those nonlinear elements. Also, it must be remembered that the tire-ground force is a contact force, which means it is absolutely necessary that $F_{1T} \geq 0$ or $F_T \geq -W_{tot}$ (if the tire leaves the ground, no force will act on the tire tread).

Estimation of the Quarter-Car Model Parameters

In the predesign stage of the suspension, the number of known input data is very small. Fortunately, few parameters are needed to solve the linear equation system (4).

A possibility for the initial approximation of the needed data it is presented further.

Knowing the mass m_{tot} sustained by the wheel, and adopting statistical proportions, the masses m_W and m_B can be approximated. Books as [1], [2], [5] or OEMs statistics as [8] indicate that the mass of the wheel m_W represents 10…15% of the total mass m_{tot} (smaller percentages for cars with non-driving wheels and independent suspensions and larger percentages for heavy vehicles with driving and rigid axles).

In the same books it is also mentioned that the tire's stiffness is about 10 times bigger (or more) as the spring's stiffness ($k_T \cong 10 \cdot k_S$), while the damping coefficient ratio of the tire and damper has an inversed value ($c_T \cong 0.1 \cdot c_D$). More than that, in some works (as [2] and [5]) it is indicated that the tire's damping is small and its influence can be neglected ($c_T \cong 0$), mainly in the early stages of the suspension design.

Taking the movement equation of the body in the system (1) and considering the vehicle at rest, it results:

$$F_{1S} = k_S \cdot d_{Sst} = W_B = m_B \cdot g \tag{5}$$

which permits to calculate the model's spring stiffness k_S if one knows the static deflection of the spring d_{Sst}. In [2] it is recommended for cars a mean value d_{Sst}=254 mm.

These method is in fact equivalent with the one in which the natural frequency of the body,

$$f_{Bn} = \frac{\omega_{Bn}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_S}{m_B}}, \tag{6}$$

it is adopted, because this also allows to calculate the model's spring stiffness:

$$k_S = m_B \cdot (2\pi\, f_{Bn})^2. \tag{7}$$

For street cars, the recommended value of the natural frequency f_{Bn} (in the absence of damping) is about 1…1.5 Hz [1], [2], [5]. This corresponds to the walking frequency, at which the humans resist better in standing position. More stiff suspensions are used for race cars, and the natural frequency can be in the range 2…2.5 Hz [5] or even can reach 3 Hz [9]. That means, if compared with a street car, the springs of a race car may be 9 times stiffer!

Replacing k_S/m_B from the equations (5) in the equations (6) it results the relationship between the static deflection of the spring d_{Sst} and the natural frequency of the body f_{Bn}:

$$d_{Sst} = \frac{g}{(2\pi f_{Bn})^2}. \tag{8}$$

The natural frequency of the wheel, also called *wheel hop frequency*, will be influenced both by the stiffness of the spring and tire (k_S and k_T), because the two springs are working in parallel:

$$f_{Wn} = \frac{\omega_{Wn}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_S+k_T}{m_W}}. \tag{9}$$

If one notes with $i_k=k_T/k_S$ the ratio of the tire and spring stiffness and with $i_m=m_W/m_B$ the ratio of wheel and body masses, then, using equations (6) and (9), the wheel hop frequency can be calculated versus the body's natural frequency:

$$f_{Wn} = f_{Bn}\cdot\sqrt{\frac{1+i_k}{i_m}}. \tag{10}$$

For example, if i_k=10 and i_m=1/9, then the wheel's natural frequency will be 9.95 times bigger as the body's natural frequency.

The last input data necessary to completely define the quarter-car model is the coefficient of damping c_D. This value can be estimated on the base of the *damping ratio*, defined as

$$\xi_D = \frac{c_D}{2\sqrt{k_S \cdot m_B}}. \tag{11}$$

The damping ratio equal with the unit, $\xi_D = 1$, it is named *critical damping ratio*. Smaller values of the damping ratio, $\xi_D < 1$, will still permit relative body-wheel oscillations, while bigger values, $\xi_D > 1$, will cancel the oscillation (after excitation, the body will approach asymptotically the static equilibrium position).

It should be mentioned that the suspension's damping ratio (provided mainly by the shock absorbers) is usually in the range of 0.2…0.4 for street passenger cars [1], [2], [5], while values of 0.8…0.9 are frequently found at the race cars [9].

Adopting such a value for the damping ratio, the coefficient of damping c_D can be calculated based on the equation (11) and now the "quarter-car" model is fully defined.

Starting for the system of movement equations (4), the transmissibility functions of the model (which are also the transfer functions, because the mathematical model of the "quarter-car" is linear) can be now written (as in [10] or [4]), calculated and graphically represented (as in the Figs. 2 and 3). Based on the transmissibility functions, the input data of the model can be changed so that the ride behavior to correspond better to the requirements (comfort, road holding or a combination of both).

The ground-body transmissibility of the accelerations (the ratio of the maximum acceleration of the body and the maximum acceleration of the ground [2]) is given by the equation

$$Ta_{\mathrm{rb}}(\omega) = \frac{k_1 k_2 + i\, k_1\, c\, \omega}{\chi\, \omega^4 - [k_1 + k_2(1+\chi)]\omega^2 + k_1 k_2 + i\, [k_1 c\, \omega - (1+\chi)\, c\, \omega^3]} \tag{12}$$

where $\chi = m_{\mathrm{W}}/m_{\mathrm{B}}$ is the ratio of the unsprung and sprung masses; $k_1 = k_{\mathrm{T}}/m_{\mathrm{B}}$, $k_2 = k_{\mathrm{S}}/m_{\mathrm{B}}$ and $c = c_{\mathrm{D}}/m_{\mathrm{B}}$ are the tire stiffness, spring stiffness and damper coefficient divided by the body mass.

That function is often used to characterize the suspension, because it indicates how the ground irregularities are felt into the vehicle body. Generally, for typical passenger cars the peak value of this amplitude ratio is in the range of 1.5…3 (in Fig. 2 a ratio of 2.2 can be observed), while for typical heavy trucks the peak value is dependent on the road and operating conditions, but in the worst case may reach levels as high as 5 or 6 [2].

The ground-body transmissibility of the accelerations is very sensitive to the damping level and so it can be controlled through a well-choose damping ratio ξ_{S}.

For a comfortable ride, the acceleration of the sprung mass must be kept to a minimum. Because that, a reasonable way to optimize the suspension is to choose a value of damping ratio that, on the ground-body accelerations transmissibility curve, generates a relative maximum (or at least a stationary point) for different damping ratios at the frequency $\frac{1}{2\pi}\sqrt{\frac{2k_S}{m_B}} = \sqrt{2} \cdot f_{\mathrm{Bn}}$ [7]. On the 2 DoF "quarter car" model, that leads to the optimal damping ratio for comfort:

$$\xi_{\mathrm{Dopt}} = \frac{1}{2\sqrt{2}}\sqrt{\frac{k_T+2k_S}{k_T}} = 0.354 \cdot \sqrt{\frac{k_T+2k_S}{k_T}}, \qquad \text{or} \qquad c_{\mathrm{Dopt}} = \sqrt{\frac{k_S \cdot m_B}{2}}\sqrt{\frac{k_T+2k_S}{k_T}} \tag{13}$$

where the influence of the tire's stiffness k_{T} is relatively small due to its value, which is bigger with about one order of magnitude as the stiffness k_{T} of the spring.

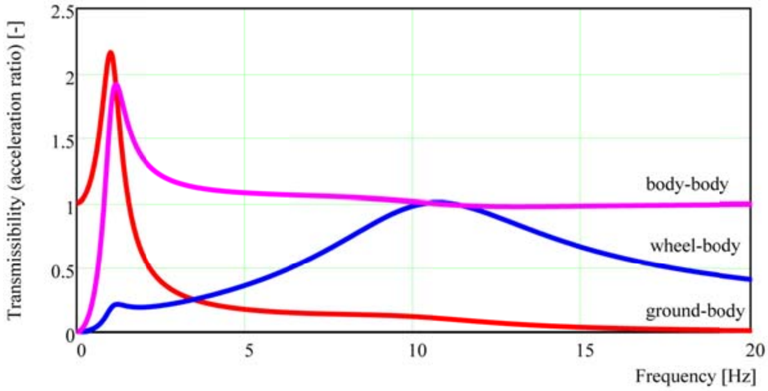

Fig. 2. Plotted example of accelerations transmissibility functions for the "quartet-car model"; all have the response at the body, while the excitations are from: ground irregularities (red); wheel imbalance (blue); imbalance on the body (magenta)

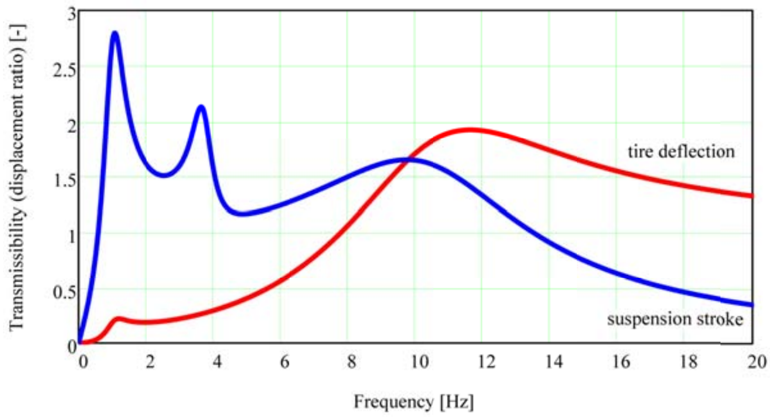

Fig. 3. Plotted example of tire deflection function (red) and suspension stroke function (blue) as these result from the "quartet-car model"

Even this value of damping is effective in keeping the acceleration low in a wide frequency range (exceeding the natural frequency of the wheel), that is not adopted always because the passive suspension must to realize a compromise of the ride performances with the dynamical ones.

It will be mentioned here also that the peak value of the ground-body transmissibility must be verified both for the loaded and unloaded vehicle, where the rear axle put the biggest problems due to a larger change of the mass.

When damping is present, as it is the case of any vehicle suspension, the resonance occurs at the true damped natural frequency

$$f_{\mathrm{d}} = \frac{\omega_d}{2\pi}\sqrt{1-\xi_D^2} = \frac{1}{2\pi}\sqrt{\frac{k_{ech}}{m_B+m_W}}\sqrt{1-\xi_D^2}\,, \tag{14}$$

where $k_{\mathrm{ech}} = k_{\mathrm{T}} \cdot k_{\mathrm{S}} / (k_{\mathrm{T}} + k_{\mathrm{S}})$ is the equivalent stiffness of the suspension (two springs in series). Due to the relative small values of the damping ratio ξ_{D}, the natural frequency with damping of the street cars is a little bit bigger as the one without damping.

They are also of a big interest the tire deflection, z_{R}-z_{W}, and the suspension stroke, z_{W}-z_{B} [8]. Both can be characterized by the absolute values $Td_{\mathrm{def}}=|(z_{\mathrm{R}}-z_{\mathrm{W}})/z_{\mathrm{R}}|=1-|z_{\mathrm{W}}/z_{\mathrm{R}}|$ and $Td_{\mathrm{str}}=|(z_{\mathrm{W}}-z_{\mathrm{B}})/z_{\mathrm{R}}|=|z_{\mathrm{W}}/z_{\mathrm{R}}-z_{\mathrm{B}}/z_{\mathrm{R}}|$ that indicate haw the amplitude of the road irregularities are producing tire deflection and suspension stroke, Fig. 3. Bigger values as 1 will indicate amplification, while smaller values will mean attenuation.

Conclusions

For the study of the motor-vehicle independent suspensions there are used simple dynamic and mathematic models, mainly in the first stages of the design process. For the primary suspension set-up, the 2 DoF "quartet-car model" is the most used, due to its good quality result-complexity ratio.

This article summarizes main aspects involved in the model simulation of vehicle suspension behavior. Based on the algorithm presented here, a computer model was implemented in the MDesign software [11], which may be used by automotive engineers or students to realize the initial steps in the design of vehicle suspension.

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