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THEORETICAL CONSIDERATIONS CONCERNING THE LONG JUMP

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Abstract: *The aim of the present paper is to present some theoretical aspects concerning the possibility to calculate the length of the jump in case of long jump trial. There are presented some relations found in literature and the data that are needed to be use. There are mentioned the methods to calculate the solutions of the motion equation of the jumpers and the difficulties that appear from mathematical point of view. It is highlight that a realistic relation is very difficult to be found considering the complexity of the athlete behaviour.*
Keywords: *jump trajectory, long jump, long-jump models.*

1. INTRODUCTION

The long jump is one of the most important athletic trials known since the first Olympic games. The main phases of the long jump are: run-up, take-off, aerial and landing. The performances of a long jumper are directly depends on some important aspects as: the qualities as sprinter, the force developed in legs at the take-off moment, flight and landing.
Considering the above mentioned dependences it is clear that any jumper must have at the end of the run-up a high horizontal velocity with the take-off placed as close as is possible on the take-off board. A very good result can be obtained if the athlete obtains a maximum horizontal velocity that is used in the take-off moment. After the take off moment a very important development refers to the flight and trajectory. Some theoretical aspects concerning the optimal trajectory and the length of the jump are presented in the following.

2. ASPECTS CONCERNING THE TRAJECTORY AND BODY FORCES

The motion of the long jump athlete can be well described by a vector equation. Neville De Mestre [6] presented an equation taking into consideration mechanical characteristics of the athlete and the environmental data:

$$m\ddot{\vec{r}} = m\vec{g} - \frac{1}{2}\rho A\vec{v}^{2}C_D\vec{\tau} \tag{1}$$

where m represents the athlete mass, r is the position vector, $\ddot{\vec{r}}$ represents the acceleration, $m\vec{g}$ is the gravity force, ρ is the air density, A is the cross section area of the athlete in normal direction to the velocity, $\vec{v} = (\dot{\vec{r}})$ is the velocity, C_D represents the friction coefficient with the air, and $\vec{\tau}$ is the versor of the velocity.

The value $\frac{1}{2}AC_D$ can be considered constant for the same athlete. The take off condition, at the initial moment of the take-off, $(t = 0)$, are given by:

$$\begin{cases} \vec{v} = (v\cos\alpha, v\sin\alpha), \\ \vec{r} = 0. \end{cases} \tag{2}$$

The equation (1) can be splited in two components, one tangential and the other one normal to the trajectory (figure 1) [6]:

$$m\dot{\vec{v}} = -mg\sin\cdot \ -\frac{1}{2}\circ AC_D\vec{v}^2 \tag{3}$$

and

$$m\vec{v}\,\dot{\cdot} = -mg\cos\cdot \tag{4}$$

where, at the moment $t=0$ results $\vec{v}=V$, $\cdot\ =\sim$, $x=0$, $y=0$, and $\cdot$ is the angle made by the tangent to trajectory vs. Ox axis (horizontal axis).

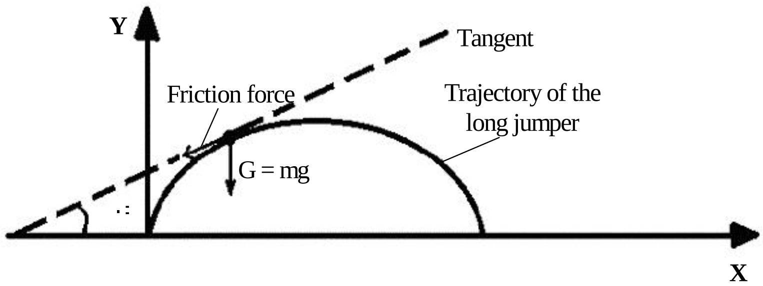

Figure 1. Force description in long jump [6]

The position of the athlete in any moment is given by the coordinates on both axis Ox and Oy and it can be found from the equations (3) and (4). Equation (4) can be multiply with $\cos\cdot$ and can be rewritten as:

$$mv\ d(\sin\cdot\ )/dt = mg(\sin^2\cdot\ -1) \tag{5}$$

denoting $\Psi = \sin\cdot$, the equation that represents the athlete motion during the long jump becomes [6]:

$$v' = -\Psi - £v^2 \tag{6}$$

$$v\Psi' = \Psi^2 - 1 \tag{7}$$

where $£ = C_D\circ AV^2/(2mg)$, and for $t=0$ results

$$\begin{cases} v=1, \\ \Psi = \sin\sim\ , \\ x=0, \\ y=0. \end{cases} \tag{8}$$

In the present case £ represents the contribution of the friction force comparing with the gravity force. For an athlete who is doing a long jump trial the value of £ is small and smaller than value 1. Both equation (6) and (7) with the initial conditions (8) have an unique solution but this solution can not be found exactly analytically.
The solution can be found considering some methods [6]:

a) Runge-Kutta standard method of fourth order with initial data for v and Ψ;
b) Approximation of the jumper with an projectile assumption that offer the possibility to find values for the parameter £ ;
c) The use of Newton-Raphson algorithm being obtained an transcendent function (David Burghes et al. [3], A.J. Ward-Smith [9] and Horace Lamb [4]);
d) A method similar with the "c" one developed by M. N. Brearley [2]considering an approximation of $\ddot{y} = -\vec{g}$;
e) Perturbation method for $£ << 1$.

Some of the values of £ , obtained by the above described methods, are presented in table 1.

Table 1: Coefficients £ obtained by different methos [6]

· (degrees)	£ values - method “c”	£ values - method “d”	£ values - method “e”
0	0	0	0
10	-0,06	-0,08	-0,08
20	-0,21	-0,28	-0,28
30	-0,37	-0,50	-0,53
40	-0,48	-0,65	-0,73
45	-0,50	-0,67	-0,78
50	-0,48	-0,65	-0,80
60	-0,38	-0,50	-0,72
70	-0,21	-0,28	-0,53
80	-0,06	-0,08	-0,27

As is presented in table 1 there are different values of £ for angle · . A complete and interesting study about all there values is presented by Neville De Mestre [6].
In some studies the athlete body is considered as a projectile or as a particle with the whole mass concentrated in a point (e.g. mass centre). So in [7] based on [1] it is presented a ration that approximate the trajectory of the athlete:

$$y(x) = x\left(\frac{v_i}{u_i} + \frac{g}{2\sim u_i^2}\right) - \frac{g}{4\sim^2 u_i^2} e^{(2\sim x)-1}, \tag{9}$$

where u_i and v_i are the launching velocities (initial velocities in moment of take-off), ~ is the drag coefficient per unit mass and x is the distance from the take-of place and a current position of the athlete along the jump.

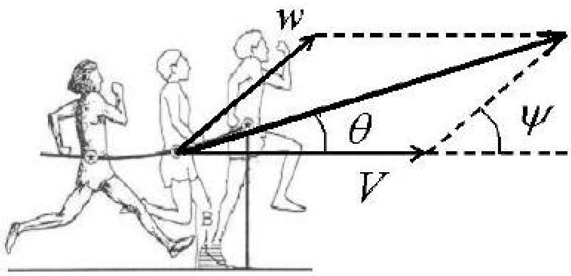

Figure 2 The model of the take-off [8]

The value of the takeoff angle · (figure 2) can be found as solution of the cubic equation [7]:

$$2\,wV\cos^3 \cdot + \left(2w^2 + V^2 + 2gh\right)\cos^2 \cdot - \left(w^2 + 2gh\right) = 0\,, \tag{10}$$

where V is the horizontal velocity of the mass centre at the take-off, w is the take-off velocity generated by the jumper leg push.
In [5] it is given an equation of the jump length L as a correlation between jump velocity and jump length:

$$L = \frac{v^2 \sin 2\cdot}{2g}\left[1 + \sqrt{1 + \frac{2gh}{v^2 \sin^2 \cdot}}\right], \tag{11}$$

where v is the take-off velocity, and h is the difference between the take-off height of mass centre and the landing height of the same mass centre.

3. CONCLUSION

The good results of a long jump athlete are influenced by some elements as the run-up velocity, the force in legs in the moment of takeoff, the behaviour during the flight, etc. A difficult problem is to find a relation that can

offer the possibility to calculate the length of the jump. The presented data in literature are mainly focused on considering the athlete as a projectile, in fact the point of the centre of the mass.
A formula that can put together all influences of the jumper behaviour is very difficult to be done. Relations (9) and (11) are good to be used in case of world-class jumpers.

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