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SOLUTION OF PENDULUM EQUATION USING THE RITZ-GALERKIN METHOD

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Abstract: *Most of engineering problems, especially some oscillation equations are nonlinear, and in most cases, it is difficult to solve such equations, especially analytically. This paper considers the response of a nonlinear to random excitation. One of the well-known methods to solve nonlinear problems is perturbation method Ritz-Galerkin type methods, both continuous and discrete in time, are considered for approximating solutions of linear and nonlinear problems. Bounds reducing the estimation of the error to questions in approximation theory are derived for the several methods studied. These methods include procedures that lead to linear algebraic equations even for nonlinear problems. A number of computational questions related to these procedures are also discussed*.
Keywords: *Nonlinear vibration systems, natural frequency,equivalent nonlinear system.*

1 INTRODUCTION

Vibration is the motion of a particle or a body or system of connected bodies displaced from a position of equilibrium. Nonlinear problems are of interest to engineers, physicists and mathematicians and many other scientists because most systems are inherently nonlinear in nature. Most vibrations are undesirable in machines and structures because they produce increased stresses, energy losses, cause added wear, increase bearing loads, induce fatigue, create passenger discomfort in vehicles, and absorb energy from the system. Rotating machine parts need careful balancing in order to prevent damage from vibrations. When the displacement or its derivatives appear in the equation of motion with a power of two or more, the equation and the corresponding problem or system are said to be nonlinear. Whenever finite amplitudes of motion are encountered, nonlinear analysis becomes necessary. The superposition principle, which is very useful in linear analysis, does not hold true in the case of nonlinear analysis. Since mass, damper, and spring are the basic components of a vibratory system, nonlinearity may be introduced into the governing differential equation through any of these components.

2 SYSTEM MODEL

In the Ritz-Galerkin method, an approximate solution of the problem is found by satisfying the governing nonlinear equation in the average. To see how the method works [1,2,3], let the nonlinear differential equation be represented as

E[x(t)] = 0 **(1)**

An approximate solution of Eq. (1) is assumed as [2,3]

$$x(t)=\sum_{i=1}^{n} A_i \cdot {}_i(t)\,, \qquad \textbf{(2)}$$

where $\cdot_i(t)$ are prescribed functions of time and A_i are weighting factors to be determined.

If Eq. (2) is substituted in Eq. (1), we get a function E[x(t)].

The weighting factors A_i are determined by minimizing [4,5] the integral

$$\int_0^T E^2[x(t)]\,dt\ , \tag{3}$$

where T is the period of the motion.
Obtain

$$\int_0^T E[t]\frac{\partial E[t]}{\partial A_i}dt=0, \quad i=1,2...,n. \tag{4}$$

Equation (4) represents a system of *n* algebraic equations that can be solved to find the values of A_i .
We consider the formed from a lenght bar l with insignificant mass [5,6], caught in point O with the spring with the k elasticity constant on which hangs the body with the m mass.
The ordinary differential equation of the motion [6,7] can be written as:

$$ml^2\,\ddot{x}+kd^2\sin x\cos x+cd^2\cos^2 x\cdot\dot{x}-mgl\sin x=0\ , \tag{5}$$

or

$$\ddot{x}+\frac{cd^2}{ml^2}\cos^2 x\cdot\dot{x}+\frac{kd^2}{ml^2}\sin x\cos x-\frac{g}{l}\sin x=0 \quad . \tag{6}$$

If we consider the Tylor developments around point 0 for the function $\sin x$ and $\cos x$, form where we keep just the first two termens, we can write

$$\sin x=x-\frac{x^3}{6}\ ,\ \cos x=1-\frac{x^2}{2} \tag{7}$$

and equation of motion, while neglecting very small terms [6,7,8] we get

$$\ddot{x}+\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)x+\sim\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)x^3=0 \tag{8}$$

By using a one-term approximation [2,4] for *x*(*t*) as

$$x(t)=A_0\sin\mu t\ , \tag{9}$$

where

$$E[x(t)]=\ddot{x}+\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)x+\sim\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)x^3=0 \tag{10}$$

Substituting equation (9) into (10), obtain

$$E[x(t)]=-\mu^2A_0\sin\mu t+\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)A_0\sin\mu t+\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\sin^3\mu t \tag{11}$$

or

$$\begin{gathered}E[x(t)]=-\mu^2A_0\sin\mu t+\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)A_0\sin\mu t+\\ +\frac{3}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\sin\mu t-\frac{1}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\sin 3\mu t\end{gathered} \tag{12}$$

respectively

$$E[x(t)]=A_0\sin\mu t\left[\frac{kd^2}{ml^2}-\frac{g}{l}-\mu^2A_0+\frac{3}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\right]-\frac{1}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\sin 3\mu t \tag{13}$$

The minimization [2,4,9] of the function of Eq. (13) requires

$$\int_0^T E\frac{\partial E}{\partial A_0}dt=0\ . \tag{14}$$

Obtain

$$\begin{gathered}\int_0^T E[x(t)]\frac{\partial E[x(t)]}{\partial A_0}dt=\int_0^T\left\{A_0\left[\frac{kd^2}{ml^2}-\frac{g}{l}-\mu^2A_0+\frac{3}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\right]\sin\mu t-\frac{1}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\sin 3\mu t\right\}\times\\ \times\left\{\left[\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)-2\mu^2A_0+3A_0^3\sim\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\right]\sin\mu t-\frac{3}{4}\sim A_0^2\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\sin 3\mu t\right\}dt\end{gathered} \tag{15}$$

We have [2,4,8,9]

$$\int_0^T E[x(t)]\frac{\partial E[x(t)]}{\partial A_0}dt=A_0\left[\frac{kd^2}{ml^2}-\frac{g}{l}-\mu^2A_0+\frac{3}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\right]\left[\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)-2\mu^2A_0+3A_0^3\sim\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\right]\times$$

$$\times\int_0^T \sin^2\mu t\,dt-\frac{3}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\left[\frac{kd^2}{ml^2}-\frac{g}{l}-\mu^2A_0+\frac{3}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\right]\times$$

$$\times\int_0^T \sin\mu t\sin 3\mu t\,dt-\frac{1}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\left[\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)-2\mu^2A_0+3A_0^3\sim\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\right]\times$$

$$\times\int_0^T \sin\mu t\sin 3\mu t\,dt+\frac{3}{16}\sim^2A_0^5\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)^2\times\int_0^T \sin^2 3\mu t\,dt \qquad \textbf{(16)}$$

We obtain

$$A_0\left\{\left[\frac{kd^2}{ml^2}-\frac{g}{l}-\mu^2A_0+\frac{3}{4}\sim A_0^3\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\right]\times\left[\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)-2\mu^2A_0+3A_0^3\sim\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\right]+\frac{3}{16}\sim^2A_0^6\right\}=0 \qquad \textbf{(17)}$$

Solving this the quadratic equation in μ^2 gives

$$\mu^2=\frac{3}{4A_0}\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)+\frac{3}{4}\sim A_0^2\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)+\frac{\sqrt{m}}{4A_0^2} \qquad \textbf{(18)}$$

where

$$m=\left[3\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)A_0+3\sim A_0^4\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)\right]^2-8A_0^2\left[\frac{15}{4}\sim A_0^2\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)-\right.$$
$$\left.-\frac{3}{2}\sim A_0^4\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)+\frac{9}{4}\sim^2A_0^6\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)^2+\frac{3}{16}\sim^2A_0^6+\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)^2\right] \qquad \textbf{(19)}$$

Therefore, the response for the nonlinear system is

$$x(t)=A_0\sin t\sqrt{\frac{3}{4A_0}\left(\frac{kd^2}{ml^2}-\frac{g}{l}\right)+\frac{3}{4}\sim A_0^2\left(\frac{g}{6l}-\frac{2kd}{3ml^2}\right)+\frac{\sqrt{m}}{4A_0^2}} \qquad \textbf{(20)}$$

3 NUMERICAL RESULTS

The pendul formed from a lenght bar $l=0,5m$, with insignificant mass, caught in point O, with the elasticity constant $k=36\frac{N}{m}$, in which hangs the body with the $m=1kg$ mass and $c=1\frac{N\cdot s}{m}$, $d=0,4m, \sim=5rad^{-2}$.

Obtain

$$\mu^2=3,42(1-0,126768A_0^2) \qquad \textbf{(21)}$$

respectively for the nonlinear response

$$x(t)=A_0\sin 1,8493t\sqrt{1-0,126768A_0^2}\ . \qquad \textbf{(22)}$$

4. CONCLUSION

The object of the paper is to offer a brief survey of numerical simulations in nonlinear vibration systems. The development of computational methods provided an opportunity to create various kinds of software which are able to analyse the properties of the above mentioned systems. The possibilities of several kinds of software are introduced with the aid of a numerical example. Specific solutions found using these techniques include the free-vibration response of single-degree-of-freedom oscillators and two-degree-of-freedom systems

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