6th International Conference
ðComputational Mechanics and Virtual Engineering ð
COMEC 2015
15-16 October 2015, Braşov, Romania

TIME-FREQUENCY ANALYSIS OF VRANCEA FAULT EARTHQUAKES

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***Abstract**: As the classical time or frequency Fourier analysis alone are insufficient to provide comprehensive information about the seismic signals who are known to be nonstationary, of ondulatory type and with time-frequency dependent energy power distribution, the more complex bi-dimensional time-frequency analysis methods are necessary to be employed. The studies of the seismic signal using time-frequency analysis are scarce. The authors' main goal is to evaluate the effectiveness of those methods relatively to Vrancea Fault generated earthquakes. The paper, for editorial space reasons, provides just a very brief description of the theoretical and numerical results of the time-frequency analysis conducted on 13 recorded strong ground motions generated by the Vrancea Fault.*
***Keywords:** Time-frequency analysis, seismic signals, energy density, S-Transform, Zhang-Sato distribution*

1. INTRODUCTION

Let's consider a seismic signal $x(t)$, defined as a continuous real functions of time $t \in [0,T]$, extended for mathematical reasons to $t \in \mathrm{R}$. The classical *time* or *frequency analysis,* employing the dual set of complex Fourier transforms, represented by the *Fourier transform (FT)*

$$X(\mu) = \frac{1}{\sqrt{2\text{®}}} \int_{-\infty}^{\infty} x(t) e^{-i\mu t} dt \qquad (1)$$

and the *inverse Fourier transform (IFT)*

$$x(t) = \frac{1}{\sqrt{2\text{®}}} \int_{-\infty}^{\infty} X(\mu) e^{i\mu t} d\mu \qquad (2)$$

alone, are insufficient to provide comprehensive information about this kind of *nonstationary signals,* of *ondulatory type* and with *time-frequency dependent energy power distribution*. In the above formulae, the variable $\mu \in \mathrm{R}$ represents physically the frequency. Must be mentioned that the complex Fourier transforms pair, extensively used by engineers in their analysis of the seismic signal, can still provide reliable information about the frequency range of the signal, (1), and can be used to generate ondulatory functions, (2), but is unable to offer any information about its time-dependent power spectrum. If information about the changes of the frequency content over time is required, the more sophisticated *bi-dimensional time-frequency analysis methods (TFAM)* are absolutely necessary to be employed.

The basic idea of time-frequency analysis is to devise a joint function of time t and frequency μ, named $P(t,\mu)$, that describes *the signal energy density simultaneously in time and frequency*, playing a similar role to that of probability density used in the probability theory. The description of the energy distribution of a signal in the time-frequency plane is obstructed by the lack of a basis of signals perfectly confined to a point of the time frequency plane, with respect to whom any signal could be represented as a superposition integral [6]. In the circumstances of missing such a basis, several time-frequency distributions have been proposed, in an attempt to have the least amount of energy spread in the time-frequency plane. Each one of these distributions has its advantages and disadvantages, [2], [3]. Therefore a considerable difference between a time-frequency energy distribution *(TFD)* of the signals and the probability densities is due to the fact that for a given signal the *(TFD) is not unique*.

Employing the modern, more complex, *bi-dimensional time-frequency analysis methods* require a more sophisticated mathematical formulation. The pure mathematical characterization of the seismic signal $x(t)$

provided by any *TFAM* must be verified to correctly reflect the physical properties of the phenomenon under scrutiny. Consequently, the mathematical formulae are additionally constrained by a number of relations generated by the physical characterization of the seismic signal. For the particular case of the seismic signal, the *energy distribution over the time-frequency plane* is the most important physical characteristic with effective repercussion in the dynamic behavior of civil and industrial structures.

Under the circumstances of missing unicity, the general approach consists in a two steps procedure. In step (1) the bi-dimensional energy distribution $P(t,\mu)$ is obtained either by: *(a)* atomic decomposition methods or *(b)* Cohen class representations. The first type methods employ a complex transformation of the signal $x(t)$ to express the $P(t,\mu)$, while the second type methods are proposing an analytical expression for $P(t,\mu)$. The physical truthfulness of the previously obtained $P(t,\mu)$, by either method, is evaluated in step (2) by imposing a *set of mathematical and physical constraints* relevant for the class of the seismic signals. The most important constraints ensuring that a time-frequency distribution $P(t,\mu)$ can be interpreted as a joint energy density, are:

a. Non-negativity:

$$P(t,\mu) \ge 0 \tag{3}$$

This property allows also the interpretation of $P(t,\mu)$ as a true probability density function (if properly normalized).

b. Correctness of the energy marginal densities in time $\left|x(t)\right|^2$ and frequency $\left|X(\mu)\right|^2$, respectively:

1. $$P_t(t) = \int_{-\infty}^{\infty} P(t,\mu)d\mu = \left|x(t)\right|^2 \tag{4}$$

2. $$P_\mu(\mu) = \int_{-\infty}^{\infty} P(t,\mu)dt = \left|X(\mu)\right|^2 \tag{5}$$

c. Conservation of the total energy of the signal $x(t)$:

$$E_{total} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(t,\mu)dtd\mu < \infty \tag{6}$$

Flandrin [5] and Cohen [2] had described several additional constraints for the distribution $P(t,\mu)$ evaluation, but those do not have such an important physical significance for the case of the seismic signal.

Remark 1: Relation *(a)* is in general violated by the classical Cohen class distributions $P(t,\mu)$ and, for practical applications, it is replaced by a more relaxed constrained. It is imposed that the distribution must be at least a real function.

Remark 2: Relation *(c)* indicates the existence of a limited total energy of the seismic signal.

Remark 3: It must also be mentioned that the energy is conserved if *(b1)* and *(b2)* are satisfied [1].

Theoretically, according to Rao [9], the one-dimensional marginal densities, in this case either the *time energy density* $\left|x(t)\right|^2$, $t \in \mathrm{R}$ or the *frequency energy density* $\left|X(\mu)\right|^2, \mu \in \mathrm{R}$, are uniquely defined by the corresponding moments up to order *n.* For practical applications, only the first four moments of the energy marginal densities or combination of them are used. The first moment of time energy density, the *mean time*:

$$t_m = \int_{-\infty}^{\infty} t \cdot \left|x(t)\right|^2 \cdot dt \tag{7}$$

indicates the time value where the time energy density is concentrated. The concentration of the density around the mean time is measured by the standard deviation defined as:

$$\pm_t = \left[\int_{-\infty}^{\infty} (t - t_m)^2 \cdot \left|x(t)\right|^2 \cdot dt\right]^{1/2} \tag{8}$$

Similarly, the first moment of the frequency energy density, the *mean frequency:*

$$\mu_m = \int_{-\infty}^{\infty} \mu \cdot \left|X(\mu)\right|^2 \cdot d\mu \tag{9}$$

indicates where the frequency value energy density is concentrated. The spread of the spectral density around the mean frequency, known as *bandwidth*, is measured by the standard deviation defined as:

$$\sigma_{\omega} = \left[\int_{-\infty}^{\infty} (\omega - \omega_m)^2 \cdot |X(\omega)|^2 \cdot d\omega \right]^{1/2} \tag{10}$$

The third and fourth moments are used to calculate the *skewness* and *kurtosis coefficients* The skewness coefficient indicates the degree of asymmetry of the energy distribution around its mean, while the kurtosis coefficient measures the relative flatness of a distribution relative to a normal distribution. In time, these moment are defined as:

$$skew_t = \frac{\int_{-\infty}^{\infty} (t - t_m)^3 \cdot |x(t)|^2 \cdot dt}{\sigma_t^3}; \; kurt_t = \frac{\int_{-\infty}^{\infty} (t - t_m)^4 \cdot |x(t)|^2 \cdot dt}{\sigma_t^4} \tag{11}$$

and, in a similar manner, in frequency:

$$skew_{\omega} = \frac{\int_{-\infty}^{\infty} (\omega - \omega_m)^3 \cdot |X(\omega)|^2 d\omega}{\sigma_{\omega}^3}; \; kurt_{\omega} = \frac{\int_{-\infty}^{\infty} (\omega - \omega_m)^4 \cdot |X(\omega)|^2 d\omega}{\sigma_{\omega}^4} \tag{12}$$

Remark 4: It must be emphasized that when dealing with seismic signals, the four moments described above, which are obtained directly from the original signal or its frequency energy density, can be compared with the similar results obtained from the time-frequency approach.

2. TIME-FREQUENCY DISTRIBUTIONS

For better understanding of the bi-dimensional energy distributions obtained, the investigation is conducted using two types of time-frequency analyses: *(a)* atomic decompositions (*Short Time Fourier Transform* and the *S-transform*) and *(b)* the classic Cohen class representations (*Wigner-Ville*, *Zhang-Sato* and *Choi-Williams* distributions).

2.1. Atomic Decomposition Methods

The first approach, known as the *Atomic Decomposition Methods*, is to write the signal as the superposition of time-frequency functions (atoms) derived from translating, modulating and scaling a basis function, having a definite time and frequency localization. For a real signal $x(t)$, this kind of time-frequency representation is given by:

$$x(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} TF(\tau, \omega) \cdot g_{\tau,\omega}(t) \cdot d\tau \, d\omega \tag{13}$$

where

$$TF(t, \omega) = \int_{-\infty}^{\infty} x(\tau) \cdot \overline{g_{t,\omega}(\tau)} \cdot d\tau \tag{14}$$

and $g_{t,\omega}$ are the time-frequency atoms, assumed to have finite energy. A normal transition between those atomic decompositions, which are linear transformations, and their corresponding time-frequency energy distributions is made by taking $P(t,\omega) = |TF(t,\omega)|^2$. The two methods of atomic decomposition used in this paper are the classical *Short Time Fourier Transform (STFT)* which leads to the distribution known as the *spectrogram (SP)* and the more recent *S-Transform (ST)* [10] and its corresponding distribution known as *ST-spectrogram (ST-SP)*. The form of the time-frequency atoms used in both methods is presented in Table 1.

Table 1: Form of time-frequency atoms

Decomposition Method	**STFT**	**S-Transform**
$g_{t,\omega}(\tau)$	$g(\tau - t)e^{-i\tau\omega}$	$g(\tau - t; \omega)e^{-i\tau\omega}$

It must be mentioned that while for the STFT, the function $g(t)$ can be any complex window with small time support, for the S-Transform, it is a Gaussian window $g(t;\omega) = \frac{1}{\sigma(\omega)\sqrt{2\pi}} \exp\left(-\frac{t^2}{2\sigma(\omega)^2}\right)$ with $\sigma(\omega) = \frac{2\pi}{|\omega|}$ a scale factor, meant to change the width of the window.

2.2. Cohen Class Representations

In 1966 Cohen, [4], provided and demonstrated the existence of a general formula to create a time-frequency distribution with desirable physical properties:

$$P(t,\omega) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{x\left(u - \frac{\tau}{2}\right)} \cdot x\left(u + \frac{\tau}{2}\right) \cdot \phi(\theta,\tau;x) \cdot e^{-i\theta t - i\tau\omega + i\theta u} \cdot du\, d\theta\, d\tau \qquad (15)$$

where $\phi(\theta,\tau;x)$ is the *kernel of the distribution,* a function playing *a determinant role in the development of the distribution* properties. If the kernel is independent of the signal, $\phi(\theta,\tau)$ then the distribution is called *bilinear distribution*. The problem of those distributions (15) is that, in general, the constraint (a) is violated, [2], and this can cause problems with their physical interpretation. Therefore, it is agreed that if the constraint (a) is not satisfied, at least they should be real valued.

In the present paper only three classic distributions from the Cohen's class (the *Wigner-Ville*, *Choi-Williams* and *Zhang-Sato* distributions) are selected and presented, based on their popularity in the technical literature and generality. From absence of publishing space, herein, for these three distributions the kernels are presented in *Table 2*.

Table 2: Kernels of the selected time-frequency distributions

Distribution	Wigner-Ville	Choi-Williams	Zhang-Sato
Kernel $\phi(\theta,\tau)$	1	$\exp\left(-\frac{\theta^2\tau^2}{\sigma}\right)$	$\exp\left(-\frac{\theta^2\tau^2}{\sigma}\right)\cos(2\pi\ \tau)$

The verification of the general constraints *(a) - (c)* for all five time-frequency distributions discussed above are summarized in Table 3, see [2],[7],[8],[11].

Table 3: Properties of the time-frequency distributions

Property	Constraint	WVD	CWD	ZSD	SP	ST-SP
Non-negativity	(a)	-	-	-	√	√
Time Marginal	(b1)	√	√	√	-	-
Frequency Marginal	(b2)	√	√	-	-	-
Energy	(c)	√	√	√	√	-
Reality		√	√	√	√	√

3. TIME-FREQUENCY ANALYSIS OF VRANCEA FAULT EARTHQUAKE RECORDS

All time-frequency methods mentioned before were used in the analyses of the 13 recorded strong ground motions, generated by the Vrancea Fault and reported on an extended study [12]. Herein, *for editorial space reasons only a very brief description of the theoretical and numerical results of this study is presented.* Precisely, only the *TDF* analyses of NS component of Incerc 1977 earthquake accelerogram is offered. The numerical computation was carried employing the Matlab code [13] and a number of modified functions from [14].

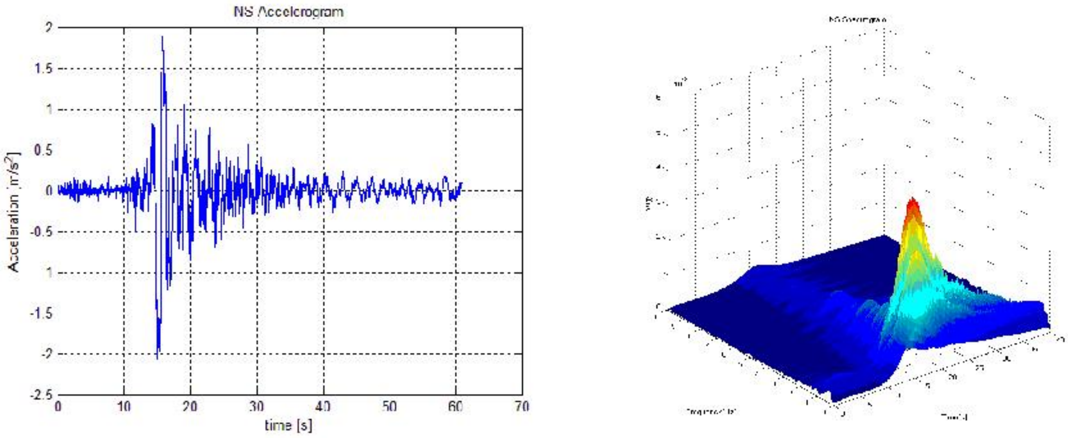
In *Figure 1a* the NS component of the accelerogram registered at Incerc in 1977, used as numerical investigation base, is shown. The 3D representation of the corresponding Spectrogram is pictured in *Figure 1b*. The five *TFDs* of NS horizontal component are shown in *Figures* 2a through *2e*. On the left side of each bi-dimensional representation the frequency marginal superimposed with the normalized cumulative frequency energy are plotted. Below , the normalized cumulative energy variation in time, together with the time marginal are shown.

The numerical global characteristics of those distributions considered are summarized in Table 4. From the frequency domain representations, the right subplot of *Figures 2a – 2e,* the energy concentration is evident at 0.42, 0.62, 0.73, 0.81 and 1.75 Hz. Of these frequency components, the dominant is located at 0.62 Hz. Beyond 3 Hz the spectral amplitudes become insignificant. For the same component, the time domain representation indicates that the maximum amplitude is located at around 15 s.

Table 4: Numerical characteristics of the time-frequency distributions for NS component of Incerc 1977 earthquake accelerogram

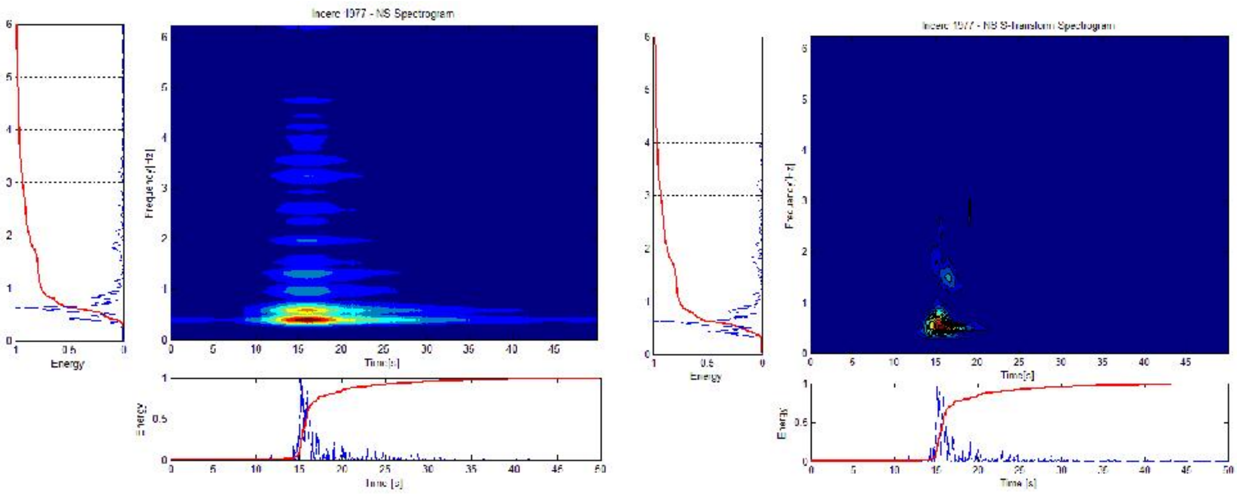
Time-frequency distribution	E_{total}	t_m	$\pm_t$	$skew_t$	$kurt_t$	μ_m	$\pm_\mu$	$skew_\mu$	$kurt_\mu$
SP	1096.8	17.7317	5.6767	2.5507	11.3080	1.0608	2.9907	7.0734	65.1241
ST - SP	2029	19.0379	6.5305	1.6134	6.8097	4.4990	5.4043	1.6335	4.7482
WVD	1098.0	17.7575	5.4403	3.0756	13.8424	1.3013	2.4518	6.9548	59.9822
CWD	1098.0	17.7575	5.4403	3.0756	13.8424	1.3013	2.4518	6.9548	59.9822
ZSD	1186.0	17.65	5.3796	3.2559	14.94	2.4552	3.0245	2.0139	7.482

As it can be seen from Table 4, the numerical results obtained by our analysis are similar for all distributions, in accordance with their theoretical characterization. A notable exception is the ST-spectrogram, for whom the numerical characteristics are quite different from the others. This can restricts its use for the characterization of the energy distribution.

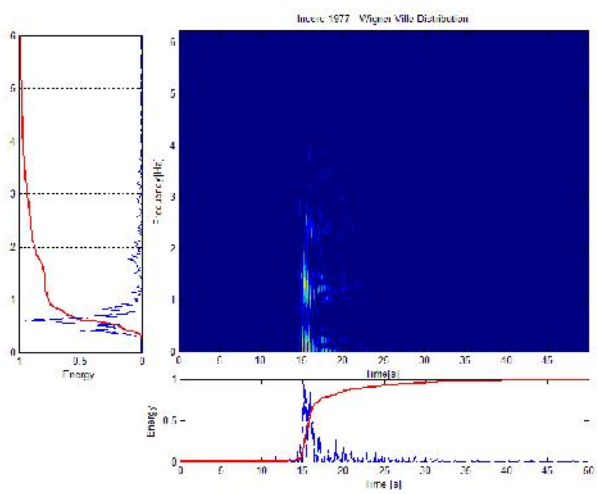


a) Time history b) Spectrogram

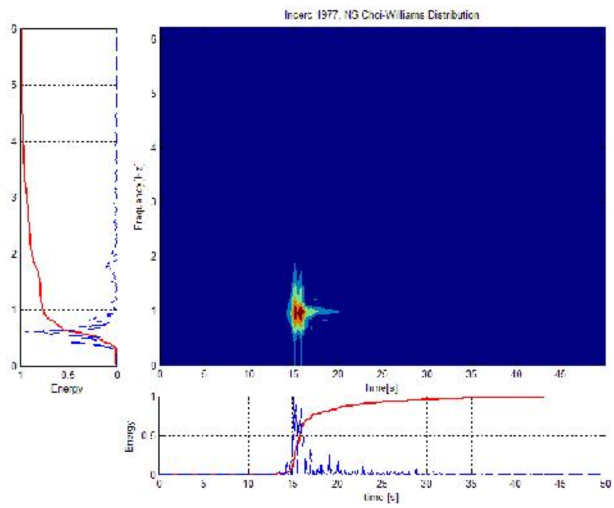
Figure 1: NS accelerogram at INCERC 1977



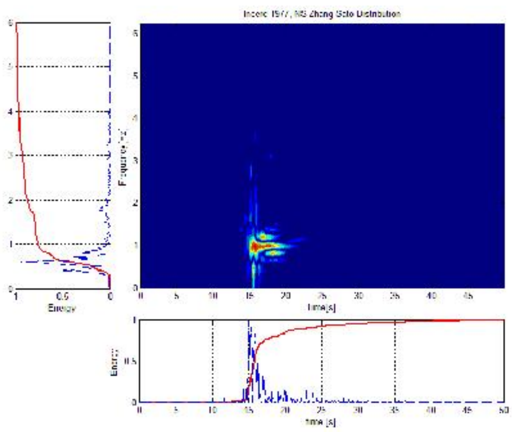
a) Spectrogram b) ST - Spectrogram



c) Wigner – Ville Distribution



d) Choi – Williams Distribution



e) Zhang – Sato Distribution

Figure 2: Time-frequency energy densities for NS component of Incerc 1977 earthquake

4. CONCLUSION

A number of general conclusions can be drawn from the above numerical results presented:

- For the global characterization of the frequency content and energy distribution, the *spectrogram* is a well suited method. It emphasizes the dominant frequencies contained in the seismic signals, but the time resolution of these frequencies it not that fine.
- The time resolution of the *ST-spectrogram* is better for low frequencies, but for higher ones is limited, as we could saw from the study of other earthquake records [12]. Also, the fact that this distribution doesn't preserve the energy and the marginals can restricts its use in the description of time-frequency energy density.
- Even though the *WVD* has significant better time-frequency resolution, the information that this *TFD* carries is blurred by the existence of cross-terms. This makes the realistic interpretation of the time-frequency characteristics of the earthquake signals difficult. Therefore *WVD* is not suited for the characterization of the seismic signal energy density.
- The *CWD* suppresses much of the cross-term interference. However some of the details present in the *WVD* are lost.
- The *ZSD* facilitates a better identification of time-frequency components of the seismic signal then the other *TFDs,* even though it can suffer from small spurious concentrations of energy.

Further research is necessary and a number of adaptive and optimized methods for *TFDs* will be considered. Also, the physical interpretation of the others *TFDs* characteristics will be further studied, in order to have a characterization as complete as possible of the seismic accelerograms.

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