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COMPUTER AIDED DESIGN IN CASE OF THE LAMINATED COMPOSITE MATERIALS

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Abstract: *The work firstly shows the analytical calculus model used to compute the distribution of the strains and stresses over the thickness in case of the laminated composite plates subjected to bending. Then, it is briefly described the main steps (calculus procedures) of the Matlab program that is used in design of the laminated composite plate in order to compute the stresses and to draw the distributions of the both strains and stresses. Therefore, rectangular plates subjected to the uniformly distributed force, with different boundary conditions (all sides simply supported, two sides simple supported while the others sides are free end) may be designed by using this program. The program also reports: the stiffness matrix corresponding to the plate element of the laminated composite plate that is computed; the safety coefficients are also computed by using one of the failure theories (maximum stress criterion; Tsai-Hill's criterion, Tsai-Wu's criterion).*
Keywords: *composite, Matlab program, bending, stresses, strains, safety coefficients*

1. INTRODUCTION

The Matlab program used within this work is based on both the classical theory of the laminated composite plate [1-3] and the constitutive equation of the element of the laminated composite plate with linear variation of the temperature over the thickness [4].
Firstly, the theory is briefly described and then, the main steps of the calculus methodology, used in Matlab program is presented by using the logical scheme. Finally some particular cases are considered and the results obtained by using the Matlab program are comparatively analyzed.

2. WORK METHODS

2.1. Briefly theoretical aspects

It is considered the general case of a laminated composite plate element having n layers. The coordinates corresponding to the layer k are: z_{k-1} and z_k are the distances between the median plate surface and the upper or the bottom level of the layer k; $\bar{z}_k$ is the distance between the median plate surface and the median surface of the layer k.
It is also considered that the temperature linearly varies over the thickness of the laminated composite plate:

$$\Delta T = a + bz, \tag{1}$$

where the coefficients a and b are computed by using the following relations:

$$a = \frac{T_1 + T_2 - 2T_0}{2}; \qquad b = \frac{T_2 - T_1}{h}, \tag{2}$$

where T_1 is the final temperature to the uppermost surface of the plate corresponding to the coordinate $-z_0$; T_2 - final temperature at the bottom surface of the plate corresponding to the coordinate z_n; T_0 - initial temperature of the plate.

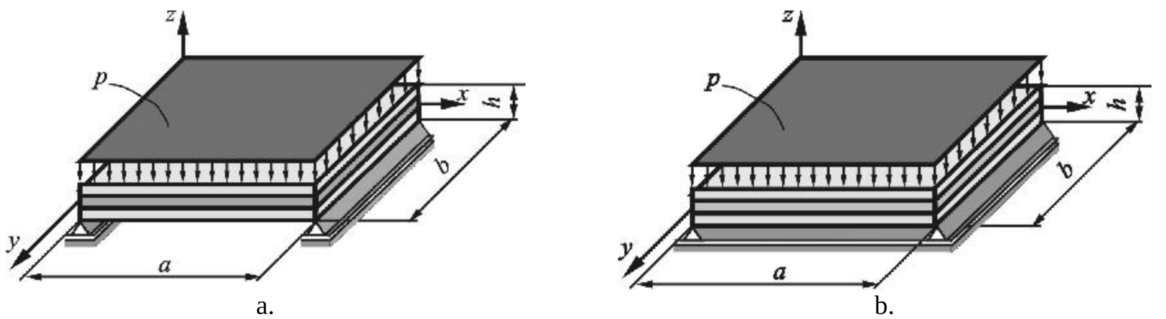
From the scientific literature in the field of the mechanics of the composite materials, it is already known the constitutive equation corresponding to the laminated composite plate element with linearly variation of the temperature over the thickness [2, 3, 5]:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} - \begin{bmatrix} [A^T] & [B^T] \\ [B^T] & [D^T] \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix}, \tag{3}$$

where:

$$\left.\begin{aligned} A_{ij} &= \sum_{k=1}^{N} \left[\overline{Q}_{ij}\right]_k (z_k - z_{k-1}); & A_{ij}^T &= \sum_{k=1}^{N} \left[\overline{Q}_{ij}\right]_k \cdot \{\alpha\}_k (z_k - z_{k-1}); \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{N} \left[\overline{Q}_{ij}\right]_k \left(z_k^2 - z_{k-1}^2\right); & B_{ij}^T &= \frac{1}{2}\sum_{k=1}^{N} \left[\overline{Q}_{ij}\right]_k \{\alpha\}_k \left(z_k^2 - z_{k-1}^2\right); \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{N} \left[\overline{Q}_{ij}\right]_k \cdot \left(z_k^3 - z_{k-1}^3\right); & D_{ij}^T &= \frac{1}{3}\sum_{k=1}^{N} \left[\overline{Q}_{ij}\right]_k \{\alpha\}_k \left(z_k^3 - z_{k-1}^3\right). \end{aligned}\right\} \tag{4}$$

In the relation (3), $[A]$ is called rigidity matrix in plane because it links the vector of strains $\{\varepsilon_x^0 \quad \varepsilon_y^0 \quad \gamma_{xy}^0\}^T$ developed within the median surface of the plate and the vector of forces $\{N_x \quad N_y \quad N_{xy}\}^T$ developed at the level of the same surface; $[D]$ - bending rigidity matrix because it links the vector of the curvatures $\{\kappa_x^0 \quad \kappa_y^0 \quad \kappa_{xy}^0\}^T$ and the vector of moments $\{M_x \quad M_y \quad M_{xy}\}^T$; $[D]$ - bending-tensile coupling matrix.
The term $\left[\overline{Q}_{ij}\right]_k$ from relations (4) is the component of the rigidity matrix $\left[\overline{Q}\right]_k$ corresponding to the layer k with respect to the global coordinate system *xOy*. The matrix $\left[\overline{Q}\right]_k$ depends on the both the orientation angle θ of the reinforcement fibers and the rigidity matrix $[Q]$ of that layer k with respect to the local coordinate system *12* (material coordinate system) whose *1* axis is parallel to the fiber direction while the *2* axis is perpendicular on the fiber direction.



a. b.

Figure 1: The cases of loading analyzed: a. The *I*[st] *case* of loading; b. The *II*[nd] *case* of loading

In case of the rectangular plate made of laminated composite material that is simply supported along the edges having the dimension b while the others two edges are free (fig. 1, a), the strain component ε_x is non-zero [2]:

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - \frac{\partial^2 w_0}{\partial x^2} \cdot z = \frac{A_{11} z - B_{11}}{A_{11} D_{11} - B_{11}^2}, \tag{5}$$

and the normal stress σ_x developed at the level of layer k is computed with the following relation [2, 3, 5]:

$$(\sigma_x)_k = (Q_{11})_k \cdot \varepsilon_x \,. \tag{6}$$

The relationship between the thermal coefficients corresponding to the two coordinate systems (global and local) is the following [2]:

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \frac{1}{2}\alpha_{xy} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2-n^2 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix}, \tag{7}$$

where $m = \cos\theta$; $n = \sin\theta$; θ is the orientation angle of the reinforcement fibers.

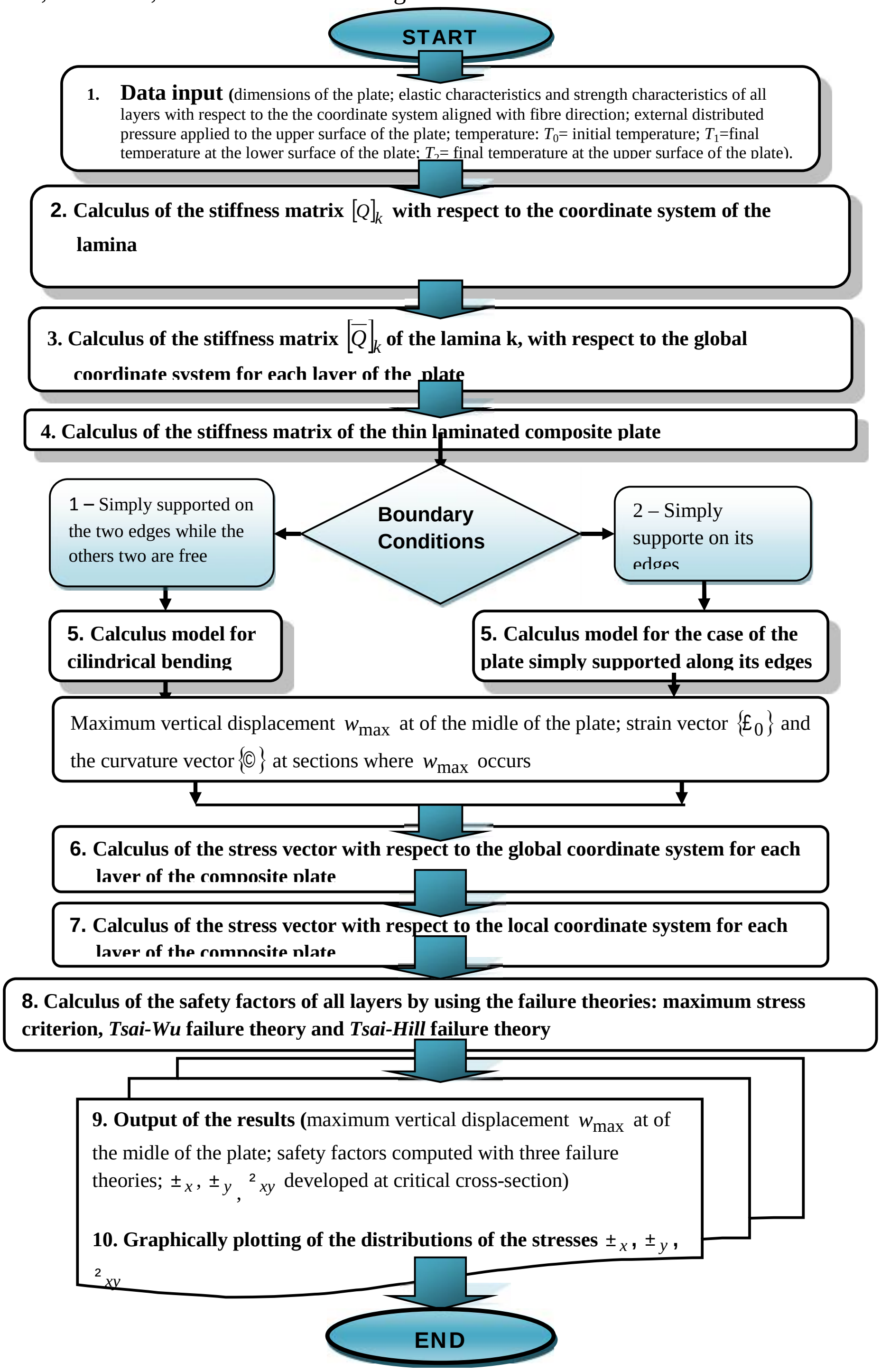


Figure 2: Logical scheme of the application programme used to compute the thin laminated plate made of composite material

Assuming that the temperature change ΔT varies linearly on the thickness of the plate, the application programme considers two cases of boundary conditions:

- case 1 of loading, rectangular plate is simply supported along the edges having the dimension b while the others two edges are free (fig. 1, a).
- case 2 of loading, plate is simply supported along all edges (fig. 1, b).

2.2. Application program

It was created an application program by using Matlab soft, based on the analytical calculus model. This application programe may be easily used to compute a thin laminated plate made of composite materials subjected to an uniformly distributed pressure p while the temperature ΔT linearly varies on the thickness of the plate.
To easily understanding of the methodology used by the application program, the logical scheme of the Matlab program is shown in the figure 2. The calculus relations are from the classical theory of the thin laminated plates made of composite material [1-3] with linearly temperature change on the thickness [2, 5].
Finally, the application programe graphically draws the distribution of the stresses over the thickness of the laminated at the level of the critical cross-section. It also computes maximum deflection $w_{0\max}$ at the midpoint of the plate and the safety coefficient c of the laminated composite plate.
Moreover, the program give us the information about the number of the layer that firstly failures. The program computes the safety coefficient c by using the three failure theories: maximum normal stress theory, *Tsai-Wu*'s failure theory or *Tsai-Hill*'s theory. The last two failure theories are dedicated for composite strength calculus.

2.3. Particular cases analyzed

Herein, it is studied the mechanically behavior of a laminated composite plate [0/90/0] having the dimensions $a = 800\,mm$, $b = 400\,mm$ (fig. 3) while the total thickness of the plate is $h = 6\,mm$. All layers have the same thickness. Each lamina is made of epoxy resin unidirectional reinforced with continuous E-glass fibres. The characteristics of the lamina are: modulus of elasticity $E_1 = 1.4 \cdot 10^5\,MPa$ in the fiber direction; modulus of elasticity $E_2 = 5 \cdot 10^3\,MPa$ in the direction perpendicular on fibers; transversal modulus of elasticity $G_{12} = 5 \cdot 10^3\,MPa$ in reinforcing plane 12; tensile strength $\pm_{1t} = 120\,MPa$ in fiber direction; compressive strength $\pm_{1c} = 100\,MPa$ in fiber direction; tensile and compressive limit stresses $\pm_{2t} = 50\,MPa$ and $\pm_{2c} = 120\,MPa$ in the direction of fibers and in the direction perpendicular on fibers; thermal expansion coefficients $\sim_1 = 0.63 \cdot 10^{-5}\,grd^{-1}$ and $\sim_2 = 2.052 \cdot 10^{-5}\,grd^{-1}$ in the direction of fibers and in the direction perpendicular on fibers, respectively.
The composite laminated plate is subjected to an uniformly distributed pressure $p = 3 \cdot 10^{-3}\,N/mm^2$ that is applied perpendicular to the plate. The initial temperature of the composite laminated plate is $T_0 = 20\,^\circ C$. The final temperatures at the upper and lower surfaces of the late are $T_1 = 60\,^\circ C$ and $T_2 = 100\,^\circ C$ due to a heat flow.

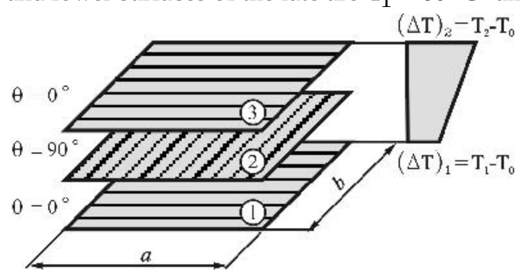
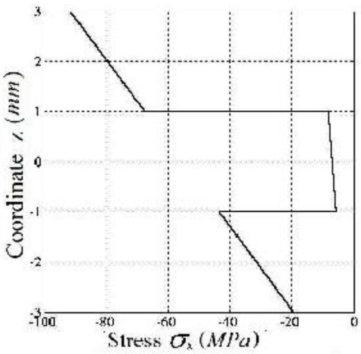


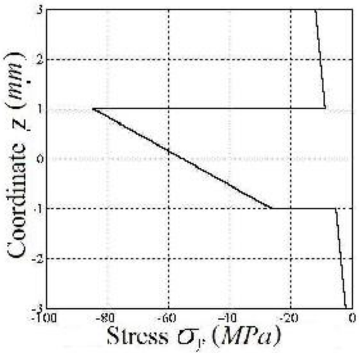
Figure 3: The composite laminated plate analyzed

3. RESULTS

By running persistently of the application program it may obtain the distributions of the both $\pm_x, \pm_y$ normal stresses shown in the figures 4-8 and the results presented in the Table 1 concerning: maximum deflection $w_{0\max}$; safety coefficient c; number of the ply that firstly failures.



a.



b.

Figure 4: Results obtained by using application program in *the IInd case* of loading.
(T0 = 20 °C ; T1 = 60 °C; T2 = 100 °C) at the level of the critical cross-section:
a. Distribution of $\pm_x$ normal stresses; b. Distribution of $\pm_y$ normal stresses

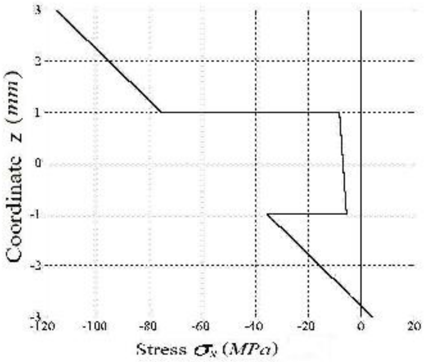


Figure 5: Distribution of the $\pm_x$ normal stresses over the critical cross-section, obtained by using application program in the *Ist case* of loading (T_0 = 20 °C; T_1 = 60 °C; T_2 = 100 °C)

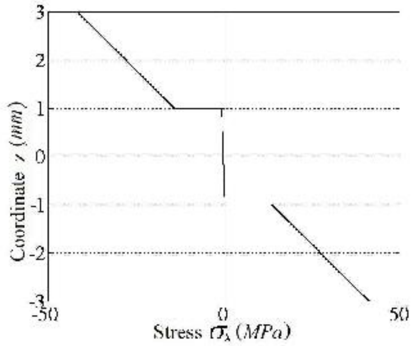


Figure 6: Distribution of the $\pm_x$ normal stresses over the critical cross-section, obtained by using application program in the *Ist case* of loading with T0 = T1 = T2 = 20 °C

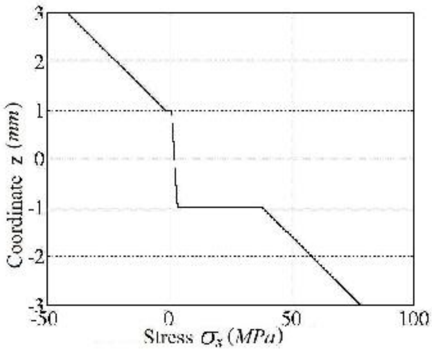
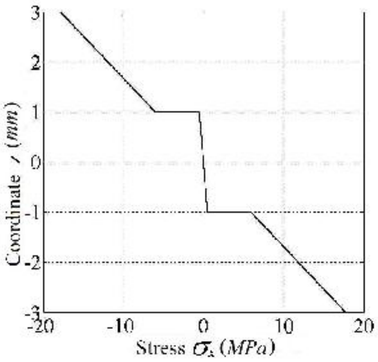
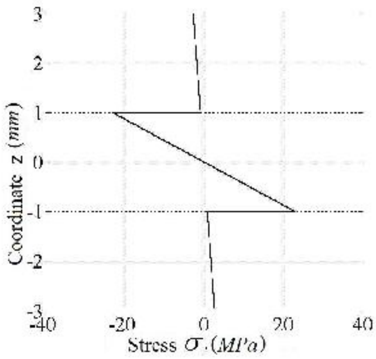


Figure 7: Distribution of the $\pm_x$ normal stresses over the critical cross-section, obtained by using application program in the *Ist case* of loading with T0 = 20 °C ; T1 = - 20 °C; T2 = 20 °C)



a.



b.

Figure 8: Results obtained by using application program in *the IInd case* of loading
(T0 = T1 = T2 = 20 °C) at the level of the critical cross-section:
a. Distribution of $\pm_x$ normal stresses; b. Distribution of $\pm_y$ normal stresses

Table 1: Results obtained by using the application programme in some cases of loading

No.	Case of loading	Temperature			Deflection $w_{0\max}$ (mm)	Safety coefficient c of the plate			The first ply ruptured	Fig.
		T_0 (°C)	T_1 (°C)	T_2 (°C)		Max. normal stress theory	*Tsai-Hill*'s failure theory	*Tsai-Wu*'s failure theory		
1	The IInd case	20	60	100	2.8519	10.178	7.681	8,272	Ply 3	4
2	The Ist case	20	60	100	6.556	8.6778	-	-	Ply 3	5
3	The Ist case	20	20	20	6.556	24.1071	-	-	Ply 3	6
4	The IInd case	20	20	20	2.8519	18.7929	18.2034	18.4747	Ply 1	8
5	The Ist case	20	-20	20	6.556	15.3141	-	-	Ply 1	7

It may be observed that, in case of the using of the *Tsai-Hill*'s failure theory or Tsai-Wu's failure theory, the value of safety coefficient c is smaller than in case of the failure theory of the maximum normal stress (fig. 4,a and 5). This means that it may recommend as being better to use one of the two specific failure theories in case of the laminated plates made of composite materials.
Analysing of the values computed for the maximum deflection w_{0max}, it may observe that w_{0max} is greater in case of the cylindrical bending than in case of the simply supported along all edges.

It may easily analyze another particular cases of loading: $T_0 = T_1 = T_2 = const. = 20\,^{\circ}C$ (fig. 6 and 8) and $T_0 = 20\,^{\circ}C$; $T_1 = -20\,^{\circ}C$; $T_2 = 20\,^{\circ}C$ (fig. 7). It may observe that if the composite laminated plate is subjected to the external pressure, by cooling only the lower layer subjected to the tensile normal stresses, this layer will be the first layer failure (fig. 7). In the same manner, the heating of the layer subjected to the compressive normal stresses (Fig. 4 and 5) only leads to the failure of this layer that will be the first ruptured layer.

4. CONCLUSIONS

The using of the different failure theories of the layer, accentuated the accuracy of the specific failure criteria described in the literature for the composite structures.
Application program developed facilitates the understanding of the aspects concerning the effects of the temperature changes on the state of stresses in case of the thin laminated plates made of composite materials.
The paper shows that the numerical applications based on the theoretical calculus model leads to interesting remarks concerning the comparatively mechanical behavior of the composite laminated plate considered when it is subjected to the external pressure and to different temperature changes at the both upper and lower surfaces of the plate, respectively.
The application program used in this paper is friendly-user and useful to analyze the mechanical behavior of a thin laminated plate made of composite material, mechanically loaded with linearly change of the temperature on its thickness. It rapidly computes the safety coefficient c of the laminated plate made of composite material, the values of the stresses developed at the level of the critical cross-section.
Finally, it graphically shows the distribution of one component of the stress vectors depending of the choosing of the user, at the level of the critical cross-section. The major advantage of the application program consists in the fact that it may be easily used in case of any symmetrical structure of a composite limited plate (different number of layers, different orientation angles of the layers, layers made of different materials etc.)

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