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ANALYTICAL MODEL FOR FRICTION FORCE BETWEEN A STEEL ROLLER AND A PLANE POLYMER SAMPLE IN SLIDING MOTION

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Abstract : *The friction force between a steel roller sliding on a plane polyurethane sample under low normal loads has been analytical evaluated. The analytical model is based on the Hertzian elastic deformation between the roller and a plane polymer operating at low normal loads. Based on the friction experiments realized with a Tribometer CETR UMT-2, were determined the friction forces and friction coefficients for various normal loads and sliding speeds. The elastic modulus of the polyurethane was experimentally determined by curve fitting of the indentation curve obtained also with the Tribometer CETR UMT-2. By using experimentally relations for variation of the friction coefficient with roller – polyurethane contact pressure and for variation of the elastic modulus of the polyurethane as function of normal load, a complex equation for the friction force in roller-polyurethane sliding contact has been developed. The analytical model has been verified by the experimental results.*
Keywords: *polyurethane, friction force, friction coefficient, indentation, Hertzian contact*

1. INTRODUCTION

The friction between steel and polymer is a complex problem including the molecular structure of the polymer, the elastic and viscoelastic behavior of the polymer, the contact pressure, the variation of the Young's modulus of the polymer as function of pressure, surface roughness, sliding speed and temperature [1,2,3,5]. Depending on the testing method (pin on disc, ball on disc, roller on plane) the friction force include two components: a component generated by adhesion between steel and polymer surface and a component generated by elastic deformation of the polymer in the sliding direction. These effects are observed on the friction coefficient also. It is difficult to establish a general relation for friction force or friction coefficient including all these parameters. In [3] are presented the dependence of the friction coefficient for a polymer as function of contact pressure under the general relation $« \propto p^{-\sim}$ where p is contact pressure and the exponent α <1. Generally, the experiments evidenced that the friction coefficient in dry steel – polymer contacts decreases by increases the contact pressure [1,2,3,5]. A theoretical model for the sliding friction between a polymide roller and a steel plane has been developed in [2] using the Hertzian line contact theory.

In the present paper the authors developed a new methodology to evaluate analytical and experimental the friction force and friction coefficient between a steel roller and a plane sample of polyurethane. The methodology is based on the constant of the elastic deformation and contact pressure between the roller and polyurethane during the sliding tests to avoid the elastic deformation of the polyurethane in the direction of the sliding (the elastic deformation of the polyurethane is only on applied load direction). The Hertzian model of the contact between a roller and a polyurethane sample has been used to evaluate the Young's modulus of the polyurethane and the local friction coefficient. Finally a general equation for the friction force on the deformed contact surface between roller and polyurethane in sliding motion has been developed and experimentally validated.

2. ANALYTICAL MODEL

The contact between a steel roller and the polyurethane plane sample is presented in Figure 1. By considering the viability of the Hertz equation used also in [2], the half width b of the contact between roller and polyurethane can be expressed by equation [6]:

$$b = 2 \cdot \sqrt{\frac{Fz \cdot R}{\pi \cdot L} \cdot \left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right]} \tag{1}$$

where Fz is normal load, R is radius of the roller, L is the length of the contact, E_1, E_2 are the Young's modulus of the steel and polyurethane, respectively and ν_1, ν_2 are the Poisson's coefficients for steel and for polyurethane, respectively.

The real contact area between the cylinder and the polyurethane sample are presented in Figure 1 as a slide from a circular surface having a length L and a circular segment lc as wide.

The circular segment lc can be determined by equation:

$$l_c = 2 \cdot R \cdot \arcsin\left(\frac{b}{R}\right) \tag{2}$$

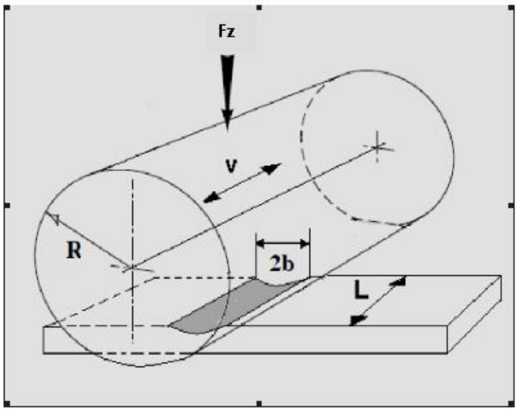

Figure 1: The surface contact between the roller and polyurethane sample

On the contact surface the Hertzian contact pressure have in any section following equation:

$$p(y) = p_{\max} \left[1 - \left(\frac{2 \cdot y}{lc} \right)^2 \right]^{0,5} \tag{3}$$

where y is the coordinate of a point in the interval between $-lc/2$ and $+\ lc/2$ and p_{max} is the maximum contact pressure in the center of the contact surface having following relation:

$$p_{\max} = \frac{2 \cdot Fz}{\pi \cdot b \cdot L} \tag{4}$$

When between the roller and polyurethane sample is realized a sliding motion, a surface shear stress $\tau(y)$ is developed in the direction of the motion and can be estimated by equation:

$$\tau(y) = \mu \cdot p(y) \tag{5}$$

where μ is the local friction coefficient.

The total friction force Ff is obtained by integration of the shear stresses on all contact surface. According to the above-mentioned equations, the total friction force has following analytical relation:

$$F_f = \int_{-lc/2}^{lc/2} p_{\max} \cdot L \cdot \mu \cdot \left[1 - \left(\frac{2y}{lc} \right)^2 \right]^{0.5} dy \tag{6}$$

To solve equation (6) it is necessary to have information about the variation of the polyurethane Young's modulus E_2 as function of the normal load Fz and, also to have information about the variation of the local friction coefficient μ as function of the contact pressure and sliding speed.

3. DETERMINATION OF THE YOUNG'S MODULUS FOR POLYURETHANE

Usually for the polymers the Young's modulus is determined by traction experiments according to the international standard ISO 37(3) [4,5]. During the traction tests the Young's modulus are changing values and as reference is considered only the value obtained for the first linear part from the stress-strain traction curve σ-ε.

In the contact problems of the polyurethane the Young's modulus have different values like in traction problems and to determine the variation of the Young's modulus as function on applied force the authors proposed a new experimental methodology. The proposed methodology is based on follows steps:

(i) indentation of the polyurethane sample with the steel roller to obtain the dependence curve between elastic deformation of polyurethane and the applied force Fz;

(ii) curve fitting the experimental curve with theoretical Hertzian's equation for elastic deformation δ in a contact between a roller and a plan surface [6]:

$$\delta = 0.39 \cdot \left[\frac{4 \cdot \left(1-\nu_1^2\right)}{E_1} + \frac{4 \cdot \left(1-\nu_2^2\right)}{E_2} \right]^{0.90} \cdot \frac{Fz^{0.90}}{L^{0.80}} \tag{7}$$

where E_1 and E_2 are expressed in Pa, Fz is expressed in N and L is expressed in m;

(iii) by imposing in the equation (7) the value of the elastic deformation δ obtained by experiment for a given applied force *Fz* and imposing for the Poisson's coefficient of the polyurethane a value of 0.4 [5] we obtained the Young's modulus E_2 in the vicinity of the given force *Fz*.

The indentation experiments were realized with Tribometer CETR UMT-2 from the Tribology laboratory as is presented in Figure 2. The steel roller is obtained from a roller bearing and have diameter of 7 mm, length of 14 mm and a surface roughness Ra = 0.06 μm. For steel roller E_1 = 210 GPa and ν_1 = 0.3. The roller is attached in the top of a cylindrical pin and the pin is mounted in the Tribometer's force sensor. The applied force *Fz* is realized by the Tribometer and varied between zero and 15 N. The polyurethane type P1716 B have a complex structure and is obtained by "Petru Poni" Macromolecular Research Institute from Iaşi, details of chemical structures being presented in [4]. The tested sample of polyurethane has a width L= 5 mm, a thickness of 0.7 mm and is attached by adhesive on a steel sample which is fixed by screws on the Tribometer table.

In Figure 3 is presented the deformation curve of the polyurethane P1716 B obtained by the soft of the Tribometer. The applied force *Fz* is indicated by negative values and elastic deformation δ of the polyurethane corresponds in the diagram with *Z* and have maximum value of 0.22 mm.

The numerical values for *Fz* and *Z* obtained from the Tribometer soft were used to determine the Young's modulus of the polyurethane in the vicinity of applied forces Fz = 1N, 5N, 10N and 15N. In Figure 4 is presented the curve fitted of the values of δ in the vicinity of the force Fz = 5N.

For the other applied forces *Fz* were obtained following values for E_2: 15MPa for Fz = 1N, 24MPa for Fz = 5N, 27MPa for Fz = 10 N and 28MPa for Fz = 15 N.

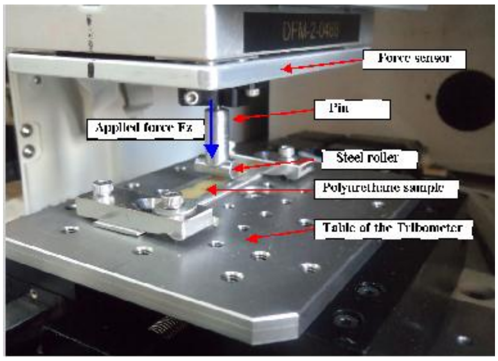

Figure 2: A general view of the indentation of the polyurethane P1716 B on Tribometer CETR UMT 2

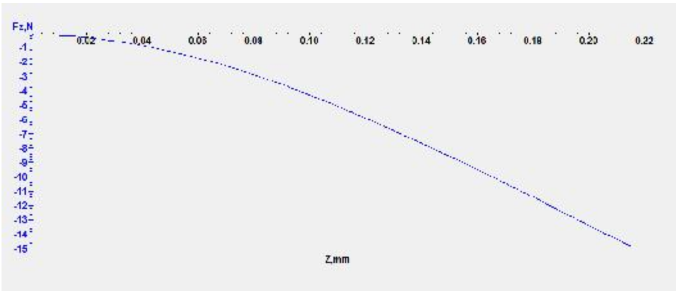

Figure 3: The experimental indentation diagram for polyurethane P1716 B

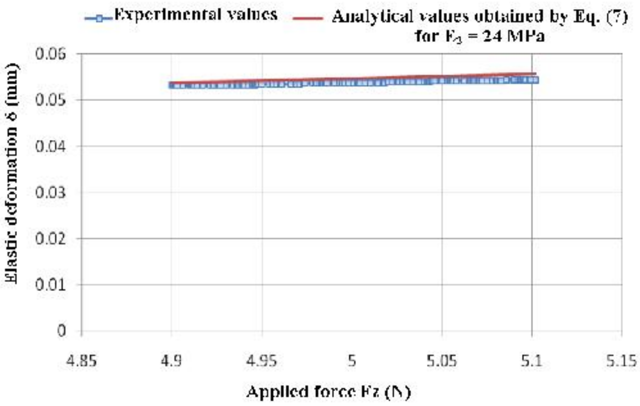

Figure 4: Curve fitting of the experimental elastic deformations with Eq. (7) in the vicinity of the force Fz =5N

Based on the values of Young's modulus E_2 determined for Fz = 1N, 5N, 10N and 15N an analytical relation by curve fitting the above-mentioned values has been obtained. The Young's modulus E_2 was approximated as function of applied force *Fz* by following relation:

$$E_2(Fz) = 29.35 \cdot \left[\frac{Fz}{Fz_{max}}\right]^{0.238} \tag{8}$$

where Fz_{max} = 15 N.

4. DETERMINATION OF THE VARIATION OF FRICTION COEFFICIENT ON CONTACT SURFACE

The experimental values for global friction coefficient obtained with Tribometer CETR UMT 2 for polyurethane P1716 B evidenced that increasing of the applied load *Fz* leads to decreasing of the global friction coefficient.
The measurement procedure for tangential force *Fx* and global friction coefficient COF between roller and polyurethane sample is presented in Figure 6. After the roller is pressed on the polyurethane sample with a force *Fz*, the table of the Tribometer realize linear motion in the direction of the roller's axis with a displacement of 4mm in both directions as is indicated in Figure 6. The Tribometer sensor indicated both the applied force *Fz* and tangential force *Fx*, last force having positive and negative values as function of the direction of the sliding. Also, by the soft of the Tribometer is indicated the variation of global friction coefficient COF (as a ratio between absolute value of *Fx* and absolute value of *Fz*) as function of the time.

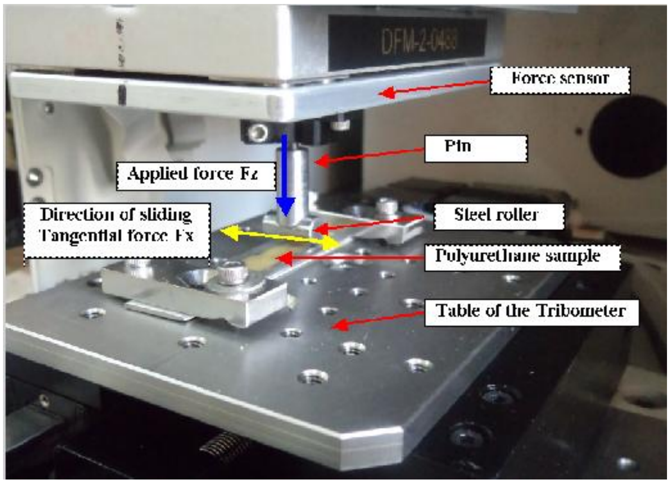

Figure 6: General view of the Tribometer CETR UMT 2 for friction experiments of polyurethane P1716 B

In Figure 7 is presented the variation with time of the global friction coefficient COF for polyurethane P1716 B obtained for a linear speed v = 10 mm/s and for Fz = 1N, 5N, 10N and 15N. The split of the diagrams corresponds to the change of the sliding direction. It can be observe that by increasing of the applied load Fz from 1N to 15N the global friction coefficient decreases from average values of 0.22 to 0.09. Similar variations of the global friction coefficient COF with normal load have been obtained for the linear speed of 1 mm/s and 5 mm/s.

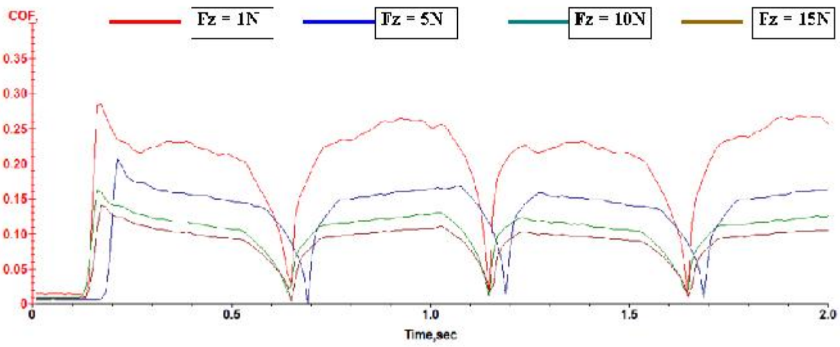

Figure 7: Variation of the global friction coefficient COF for polyurethane P1716 B at a linear speed v = 10mm/s

Based on the experimental values of the global friction coefficient COF obtained at 10mm/s was proposed the following equation for variation of the global friction coefficient as function of the ratio Fz/Fz_{max}:

$$COF(Fz) = 0.105 \cdot \left[\frac{Fz}{Fz_{max}} \right]^{-0.282} \tag{9}$$

A similar dependence between local friction coefficient and contact pressure *p(Fz,y)* has been proposed:

$$\mu(Fz,y) = 0.105 \cdot \left[\frac{p_{\max}(Fz)}{p_{\max}(Fz_{\max})} \cdot \sqrt{1-\left(\frac{2y}{lc(Fz)}\right)^2} \right]^{-0.282} \tag{10}$$

By considering Young's modulus of the polyurethane E_2 as function of applied force Fz (Eq.8) and the local friction coefficient μ as function of contact pressure and local position y (Eq. 10), the final equation of the friction force F_f on contact surface is derived from Eq. 6 and have following general relation:

$$F_f(Fz) = \int_{-lc(Fz)/2}^{lc(Fz)/2} p_{\max}(Fz) \cdot L \cdot \mu(Fz,y) \cdot \left[1-\left(\frac{2y}{lc(Fz)}\right)^2\right]^{0.5} dy \tag{11}$$

5. VALIDATION OF THE ANALYTICAL MODEL

In Figure 8 is presented the variation of the tangential force *Fx* (which correspond in absolute value with friction force *Ff*) for linear speed of 10mm/s and for applied force Fz = 1N, 5N, 10N and 15N. The positive and negative values correspond to the direction of the linear speed.

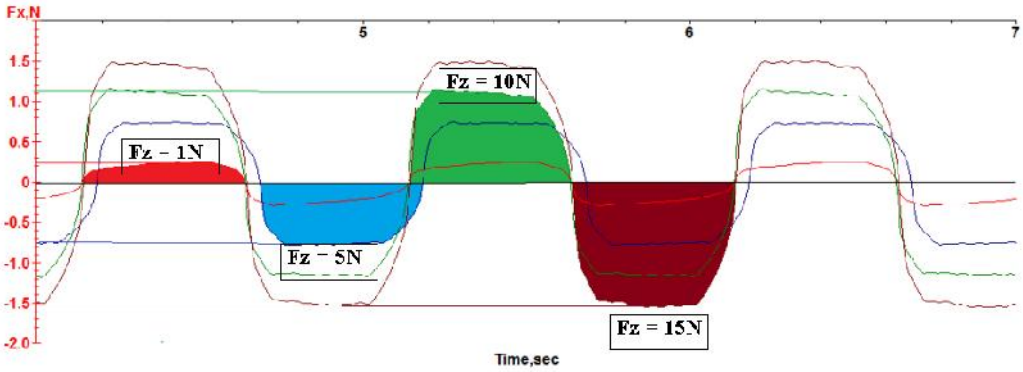

Figure 8: Variation of the friction force with applied load *Fz* for polyurethane P1716 B at a linear speed v = 10mm/s

Eq. 11 has been solved for the linear speed of 10mm/s and the results have been compared with the experimental values of friction forces presented in Figure 8. In Figure 9 are presented the correlations between the analytical values of the friction force F_f obtained by Eq. 11 and experimental values. It can be observed good correlation between proposed model and experiments.

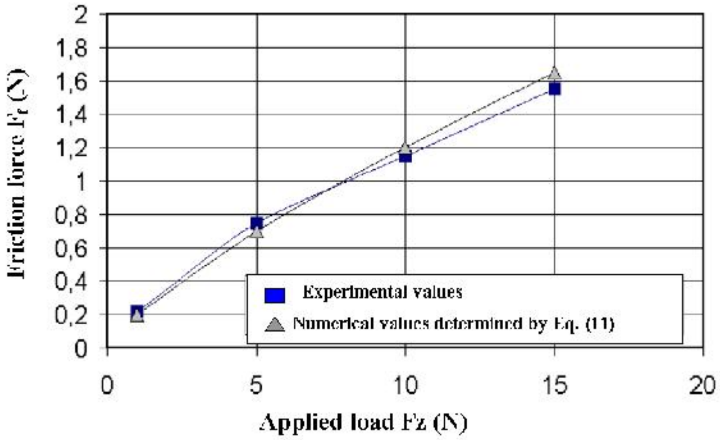

Figure 9: Correlation between analytical and experimental friction force as function of applied load Fz for polyurethane P1716 B at a linear speed v = 10mm/s

6. CONCLUSIONS

A complex experimental and analytical procedure has been developed by authors to obtain an analytical equation for the friction force in a sliding contact realized by a steel roller pressed on a polyurethane plane sample.
The experimental methodology to determine the friction force and friction coefficient between a steel roller and a plane polyurethane with sliding motion in the direction of the roller's axis is an original one. By this methodology, the deformed contact and the contact pressure between the roller and the polyurethane is maintained constant during the experiments, so, only the adhesion component of the friction force has been determined.
Because the Young's modulus of the polyurethane is not constant under various applied loads, the indentation diagram between applied force and elastic deformation has been realized. Based on Hertzian model for elastic deformation and by curve fitting of the indentation diagram, the Young's modulus of the polyurethane in the vicinity of four applied forces has been determined. Also, a relation between Young's modulus and applied force has been obtained.
Based on the experimental variation of the global friction coefficient as function of applied force an equation for variation of the local friction coefficient as function of contact pressure has been proposed.
A general equation for friction force on contact surface including the variation of the Young's modulus of the polyurethane as function of applied force and variation of the local friction coefficient as function of contact pressure has been developed. The friction forces obtained by general equation and by experiments have been compared and a good correlation has been obtained.

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