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On the Calculation of the Additional Leakage Reactances of Salient-Pole Synchronous Machines

The damping winding and its behaviour in the longitudinal (direct) axis

A simple deduction, accepting more simplifications than in the paper, is given, answering the questions addressed by certain readers to the authors, during the years.

Let us assume that the magnetic induction is produced by a magnetic tension (and magnetomotive force, abbreviated m.m.f.) of the stator, having a sinusoidal distribution along the air-gap rotor circumference. The symbols are those of the paper.

Along the rotor circumference, for the longitudinal axis, the m.m.f. force will produce, generally, a distorted curve of the magnetic induction. The resultant m.m.f. will be produced by the stator armature current as well as by the electric currents in the damping winding bars (bars of the damper cage) of the rotor. We shall assume an even number of bars per pole. The numbering is that of Fig. 5 and Fig. 6 in the paper.

For the sake of simplicity, the magnetic saturation will not be considered.

For consistency, we shall consider that a part of the magnetic tension (and m.m.f.) produced by the currents of the damper cage will compensate the magnetic tension (and m.m.f.) of the stator armature having the mentioned distribution form. Then, for a uniform thickness of the air-gap, the distribution along the circumference above, of the resulted magnetic field strength and induction will be a sinusoid arc over the rotor pole-piece. The mentioned part of the magnetic field due to the damper cage, and the armature field will constitute, together, the useful or main magnetic field acting on the rotor, the distribution curve of the resultant magnetic field-strength in the air-gap keeping the mentioned form. The rest of the magnetic field produced by the damper cage of the rotor will be assumed as additional (or differential) leakage magnetic field.

Assuming a sinusoidal variation with time, the maximum electromotive force (e.m.f.) induced by the resultant magnetic field, with a sinusoidal distribution along the circumference, in any turn formed by two bars of ordinal numbers k, symmetric with respect to the longitudinal axis, is:

$$e_{k\,\max} = \omega B_M \frac{2}{\pi} \tau l_i \sin(k \cdot \alpha_c). \qquad (1)$$

The current intensity, maximum value, in the circuit formed by the two bars of ordinal number k, neglecting the influence of the cage end rings, will be:

$$i_{\max} = \sum_{j=1}^{k} \frac{1}{z_\sigma} \cdot \omega B_M \frac{2}{\pi} \tau l_i \sin\left(j \cdot \alpha_c\right), \qquad (2)$$

where z_σ represents the leakage impedance of the circuit constituted by the pair of bars, the influence of the end rings being assumed as negligible.

At its turn, this current will produce a magnetic field, at each point of the circumference, in the air-gap, in the various zones, outside the cage. In every space interval between the rotor bars $k-1$ and k, the magnetic field strength produced by the cage currents will have the amplitude:

$$H_k = \frac{1}{\delta} \cdot \frac{1}{z_\sigma} \cdot \omega B_M \, \frac{2}{\pi} \tau l_i \sum_{j=k}^{N} \sin\left(j \cdot \alpha_c\right). \tag{3}$$

This magnetic field strength amplitude has been calculated, assuming that the iron magnetic circuit saturation is neglected. Each arc length of the interval of the circumference between the bars $k-1$ and k, is given by the relation:

$$c(k)\,\tau = \frac{\alpha_c}{\pi} \tau. \tag{4}$$

Therefore, the magnetic energy of the air-gap portion of the half of the pole pitch, considering simultaneously all intervals, will be:

$$W_\delta = \frac{1}{2} \mu_0 \left(\frac{1}{\delta} \cdot \frac{1}{z_\sigma} \cdot \omega B_M \, \tau l_i \right)^2 \left(\frac{2}{\pi} \right)^2 \sum_{j=1}^{N} \left(c(j) \cdot \frac{\pi}{\alpha_c} \left(\sum_{k=j}^{N} \sin\left(k \cdot \alpha_c\right) \right)^2 \right) \alpha_c \frac{\tau}{\pi} l_i \, \delta. \tag{5}$$

The last three factors of formula above correspond to the considered volume.

The quantity $c(j)$ represents, as explained, the ratio between the length of the arc of circumference between the bars $k-1$ and k, and the pole pitch arc.

Now, we shall calculate the energy of the main magnetic field of the same volume. All intervals of the magnetic tension (magnetomotive force) contribute for producing a sinusoidal main harmonic of the magnetic tension or m.m.f.. Similarly, the main harmonic produced by all damper bars, in the longitudinal (direct) axis will be calculated. In the case of a uniform air-gap we shall obtain:

$$H_{Md\delta} = \frac{1}{\delta} \cdot \frac{1}{z_\sigma} \cdot \omega B_M \, \tau l_i \sum_{k=1}^{N} B_M \, \frac{2}{\pi} \sin(k \cdot \alpha_c) \cdot \frac{4}{\pi} \sin(k \cdot \alpha_c), \tag{6}$$

where the first factor before the sine intervenes when calculating the induced electromotive force in the circuit constituted by one pair of bars, while the second intervenes when calculating the main harmonic of the magnetic field strength produced by the electric current of the same pair of bars.

The corresponding magnetic energy in the air-gap of the half of a pole pitch, assuming an equal distance between neighbour bars, will be:

$$W_1 = \frac{1}{2} \mu_0 \left(\frac{1}{\delta} \cdot \frac{1}{z_\sigma} \cdot \omega B_M \, \tau l_i \right)^2 \left(\sum_{k=1}^{N} B_M \, \frac{2}{\pi} \sin(k \cdot \alpha_c) \cdot \frac{4}{\pi} \sin(k \cdot \alpha_c) \right)^2 \frac{1}{2} \cdot \frac{1}{2} \cdot \tau l_i \, \delta. \tag{7}$$

The factors 1/2 of formula (7) above occurs twice, once, from the calculation of the integral of the energy in the case of a sinusoidal distribution of the magnetic field strength along the rotor circumference, and twice from the integration limits along the half of a pole pitch.

The ratio between the energy expression W_δ and W_1 will be:

$$\sigma_d = 1 + \tau_{Dd\delta} = \frac{\left(\frac{2}{\pi} \right)^2 \sum_{j=1}^{N} \left(c(j) \cdot \frac{\pi}{\alpha_c} \left(\sum_{k=j}^{N} \sin\left(k \cdot \alpha_c\right) \right)^2 \right) \alpha_c \frac{\tau}{\pi}}{\left(\frac{8}{\pi^2} \right)^2 \sum_{j=1}^{N} \left(\sin(j \cdot \alpha_c) \, \sin(j \cdot \alpha_c) \right)^2 \frac{\tau}{4}}. \tag{8}$$

Hence:

$$\sigma_d = 1 + \tau_{Dd\delta} = \frac{\pi^2}{4} \cdot \frac{\sum_{j=1}^{N} \left(c(j) \cdot \frac{\pi}{\alpha_c} \left(\sum_{k=j}^{N} \sin(k \cdot \alpha_c) \right)^2 \right)}{\sum_{j=1}^{N} (\sin(j \cdot \alpha_c))^2} \cdot \frac{\alpha_c}{\pi}. \tag{9}$$

If using an adequate tool, like Maple soft, the double sum of the numerator, occurring in several formulae of the paper, may be written in a computer program in a simple form, because this software allows the usage of double summation.

In the case in which the non-uniformity of the air-gap thickness is taken into consideration, the calculations are modified as shown in the paper.

The calculation of the leakage damper reactance, for the quadratic (transversal) axis, has been carried out, in the paper, in a similar manner.

The present paper may be of interest still nowadays, despite various progresses, e.g., the finite element method, especially for the many cases in which simplified diagrams are required for to be introduced in certain complex networks.

The mistakes existing in formulae (6), (37 b), (52) of the paper, at the time of publication, have been corrected, avoiding the necessity of an erratum.

The paper has also been translated from German into Russian in the publication Ekspress Informacija, Elektricheskie Mashiny i Apparaty, No. 2, Moskva, 1970.