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DYNAMIC RESPONSE AND ISOLATION DEGREE OF STATIONARY FORCED VIBRATION IN THE CASE OF VIBRATION SUPPORTS WITH COMPLEX VISCOELASTIC BEHAVIOR

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Abstract: *Through this paper work it seeks to define the dynamic response and the isolation degree of stationary forced vibrations in the case of vibration isolators supports with the complex viscoelastic behavior. For this purpose, it will analyze the following complex viscoelastic models: the viscoelastic model Voigt-Kelvin, Maxwell viscoelastic model and Hooke - Maxwell viscoelastic model (Zener). It is considered that these systems are excited from the outside with a harmonic force. Following determination of differential equations of motion for each model and identify solutions systems of equations can be defined the dynamic response and the isolation degree of stationary forced vibrations.*
Keywords: *dynamic response, isolation degree, stationary forced vibration, complex viscoelastic behavior*

1. GENERAL NOTIONS

Stationary vibration is the oscillating motion that, throughout the considered period of time, is not interrupted. It is considered that the vibration is forced (maintained) if, from the outside, is acting on a structural system with a disruptive force to maintain motion (introduce energy in the oscillating system). The effect achieved, in this case, is contrary to the natural tendency of friction damping of the free vibration.
In the following there is provided a forced vibration classification [3]:
a) According to the damping:
- damped: viscous, dry, hysteretic and ordinary;
- without damping.
b) By the nature of disruptive force
- determinist: periodical (harmonic and ordinary), impulse and ordinary;
- random.
The dynamic response of the vibration represent the dynamic evaluation of vibration where the dynamic excitation point is the headquarters of disruptive force F(t) and the reception point is the headquarters of the transmitted force F_T [1]. The isolation degree of the transmitted vibrations is denoted by I and express the percent reduction of the vibration [1]:

$$I = [1 - T]100 \quad [\%] \qquad (1.1)$$

where: T - vibration transmissibility or the transmission degree of vibrations and represents the ratio between the response parameter (like physical magnitude transmitted) and the excitatory parameter (as physical input).
The vibration supports with complex viscoelastic behavior are support systems which are composed of both resilient components and viscous damping elements, so the elastic properties combined with the viscous.

2. THE DYNAMIC RESPONSE AND THE ISOLATION DEGREE OF VIBRATION. THE CASE OF VIBRATION ISOLATORS SUPPORTS WITH THE COMPLEX VISCOELASTIC BEHAVIOR.

For exemplification considers a system with one degree of freedom of mass m, viscoelastic supported (figure 2.1).

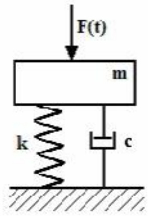


Figure 2.1: Dynamic model with one degree of freedom, viscoelastic supported [4]

In this case (the system with one degree of freedom or single mass) forced vibration equation is:

$$F_i + F_a + F_e = F(t) \quad (2.1)$$

We have:

$F_i = m \cdot a$ *– force of inertia;*
$F_a = c \cdot v$ *– force of dumpping;*
$F_e = k \cdot x$ *– elastic force.*

By replacing relation above in equation (2.1) we get:

$$m \cdot a + c \cdot v + k \cdot x = F(t) \quad (2.2)$$

Ideally, when disruptive force F (t) is deterministic, periodic and harmonic, relation for it is:

$$F(t) = F_0 \cdot \sin \omega \cdot t \quad (2.3)$$

where:
m – viscoelastic supported mass;
c – viscous damping coefficient;
k – support system rigidity (elasticity constant)
a – acceleration;
v - velocity;
x - movement;
F(t) – disruptive force
F_0 – disruptive force amplitude of the vibration source;
ω – its pulsation;
t – time.

If we use the notations $a = \dot{v} = \ddot{x}$; $v = \dot{x}$ in equation (2.2) then it may be written:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = F(t) \quad (2.4)$$

Using the folowing notations, after dividing the equation (2.4) to *m*:

$\frac{c}{m} = 2 \cdot n$ – damping factor;

$p = \sqrt{\frac{k}{m}}$ – own pulsation of elastic system

and by replacing the disturbance force F(t) with relation (2.3), equation (2.4) becomes

$$\ddot{x} + 2 \cdot n \cdot \dot{x} + p^2 \cdot x = \frac{F_0}{m} \cdot sin\omega \cdot t \quad (2.5)$$

The solution of (2.5) consists of two terms, one representing free vibration damped - which stops after a short time - and another representing maintained vibration [5],

$$x_1 = \frac{F}{k} \cdot \frac{1}{\sqrt{\left(1-\frac{\omega^2}{p^2}\right)^2 + \left(\frac{2 \cdot n}{p}\right)^2 \cdot \left(\frac{\omega}{p}\right)^2}} \cdot \sin(\omega \cdot t - \alpha) \quad (2.6)$$

Viscoelastic material behavior can be described using mechanical designs like those in Figure 2.2, consisting of springs, dampers and dry friction elements, unlocks when the tensile force exceeds a threshold value [8].

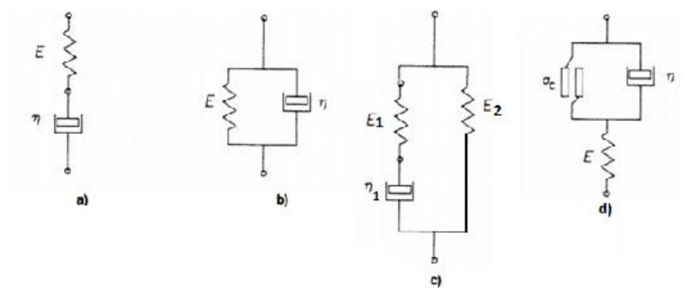
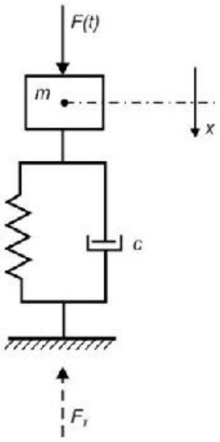
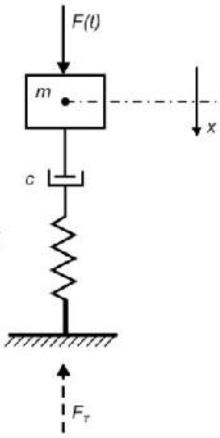
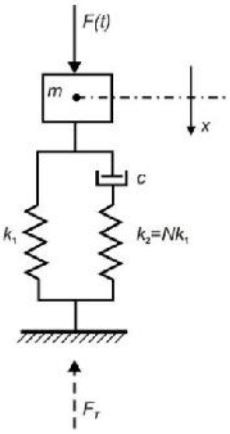


Figure 2.2: Complex rheological models without mass: a) The Maxwell b) The Kelvin-Voigt c) The Hooke-Maxwell (Zener), d) The viscoelastic

Table 2.1 describes the dynamic response and isolation grades of vibration for the linear viscoelastic systems with mass excited from the outside with a harmonic force in both cases of damping:

- Elastomeric structural damping: $\delta = \frac{c\omega}{k}$
- Viscous damping discreet: $\varsigma = c/(2mp)$

Table 2.1: The dynamic response and isolation degree of vibration for the linear viscoelastic systems with mass excited from the outside with a harmonic force

Designation	Viscoelastic system Voigt-Kelvin	Viscoelastic system Maxwell	Viscoelastic system Hooke-Maxwell (Zener)
Damping case	Elastomeric structural damping	Elastomeric structural damping	Elastomeric structural damping
	Viscous damping discreet	Viscous damping discreet	Viscous damping discreet
Schematic representation of the model			
Equation of motion	$m\cdot\ddot{x} + c\cdot\dot{x} + k\cdot x = F_0\cdot\sin\omega\cdot t$	$\begin{cases} m\cdot\ddot{x} + c\cdot(\dot{x}-\dot{y}) = F_0\cdot e^{j\omega t} \\ c\cdot(\dot{x}-\dot{y}) = k\cdot y\end{cases}$	$\begin{cases} m\cdot\ddot{x} + c\cdot(\dot{x}-\dot{y}) + kx = F_0\cdot e^{j\omega t} \\ c\cdot(\dot{x}-\dot{y}) = N\cdot k\cdot y\end{cases}$
Amplitude A	$A = \frac{F_0}{k}\cdot\frac{1}{\sqrt{(1-\Omega^2)^2+\delta^2}}$	$A^0 = \frac{F_0}{k}\cdot\sqrt{\frac{1+\delta^2}{\Omega^4+\delta^2(1-\Omega^2)^2}}$	$A(\Omega,\delta) = \frac{F_0}{k}\cdot\sqrt{\frac{N^2+\delta^2}{N^2(1-\Omega^2)^2+\delta^2(N+1-\Omega^2)^2}}$
	$A = \frac{F_0}{k}\cdot\frac{1}{\sqrt{(1-\Omega^2)^2+4\varsigma^2\Omega^2}}$	$A^0 = \frac{F_0}{k}\cdot\sqrt{\frac{1+4\varsigma^2\Omega^2}{\Omega^4+4\varsigma^2\Omega^2(1-\Omega^2)^2}}$	$A(\Omega,\varsigma) = \frac{F_0}{k}\cdot\sqrt{\frac{N^2+4\varsigma^2\Omega^2}{N^2(1-\Omega^2)^2+4\varsigma^2\Omega^2(N+1-\Omega^2)^2}}$
Amplitude of the transmited force	$Q^0 = F^0\sqrt{\frac{1+\delta^2}{(1-\Omega^2)^2+\delta^2}}$	$Q^0 = F^0\sqrt{\frac{\delta^2}{\Omega^4+\delta^2(1-\Omega^2)^2}}$	$Q^0(\Omega,\delta) = F_0\sqrt{\frac{N^2+\delta^2(N+1)^2}{N^2(1-\Omega^2)^2+\delta^2(N+1-\Omega^2)^2}}$
	$Q^0 = F^0\sqrt{\frac{1+4\varsigma^2\Omega^2}{(1-\Omega^2)^2+4\varsigma^2\Omega^2}}$	$Q^0 = F^0\sqrt{\frac{4\varsigma^2\Omega^2}{\Omega^4+4\varsigma^2\Omega^2(1-\Omega^2)^2}}$	$Q^0(\Omega,\varsigma) = F_0\sqrt{\frac{N^2+4\varsigma^2\Omega^2(N+1)^2}{N^2(1-\Omega^2)^2+4\varsigma^2\Omega^2(N+1-\Omega^2)^2}}$
The transmissibility	$T = \sqrt{\frac{1+\delta^2}{(1-\Omega^2)^2+\delta^2}}$	$T(\Omega,\delta) = \frac{\delta}{\sqrt{\Omega^4+\delta^2(1-\Omega^2)^2}}$	$T(\Omega,\delta) = \sqrt{\frac{N^2+\delta^2(N+1)^2}{N^2(1-\Omega^2)^2+\delta^2(N+1-\Omega^2)^2}}$
	$T = \sqrt{\frac{1+4\varsigma^2\Omega^2}{(1-\Omega^2)^2+4\varsigma^2\Omega^2}}$	$T(\Omega,\varsigma) = \frac{2\varsigma\Omega}{\sqrt{\Omega^4+4\varsigma^2\Omega^2(1-\Omega^2)^2}}$	$T(\Omega,\varsigma) = \sqrt{\frac{N^2+4\varsigma^2\Omega^2(N+1)^2}{N^2(1-\Omega^2)^2+4\varsigma^2\Omega^2(N+1-\Omega^2)^2}}$

3. CONCLUSION

Maxwell model can simulate the behavior of a material that has an elastic response followed by a viscous flow constant when applied at baseline t0, a force which then remains constant (figure 3.1 a).
Kelvin-Voigt model is used to model material which, at the discharge, are returning to the reference state, but not through the same path as the load (figure 3.1 b).
This behavior, characterized by hysteresis, it is associated with a energy dissipation.

The Hooke-Maxwell (Zener) is rheological model of standard linear solid deformable this means that this type of system models elastomeric materials with structural damping $\delta = c\omega / k$ and materials with $\varsigma = c/(2mp)$. Viscoelastic model should be used when you know elastic properties, viscous and hardening of the material.
The model is used to characterize the behavior of glass and vitreous materials.
In this model also includes the effects of temperature variations, which enables simulation of heating or cooling (see figure 3.1 c) [8].

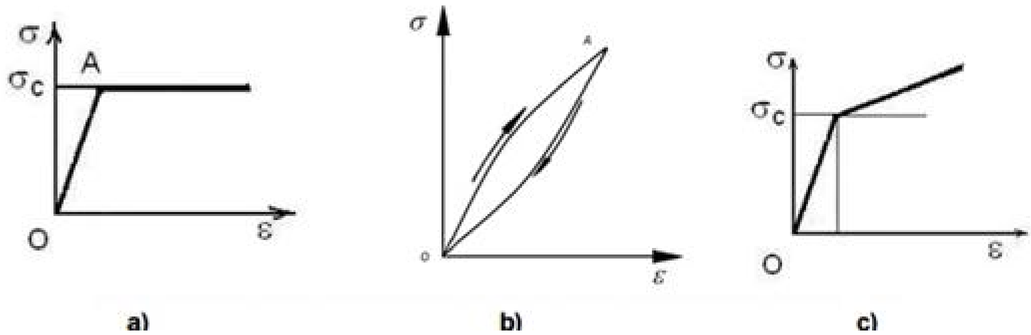


Figure 3. Dynamic response of complex models

Table 3.1: Variation of amplitude A for structural damping elastomer (a) or discrete viscous damping (b) [1]

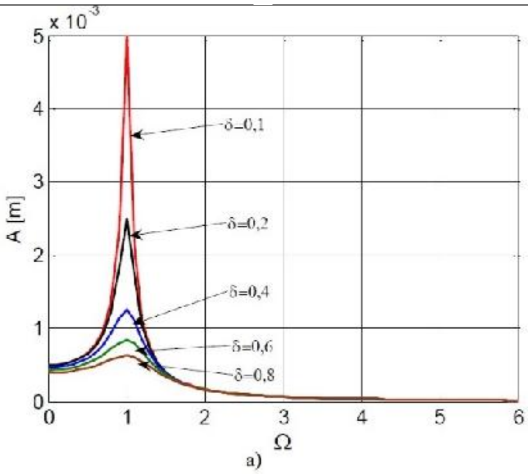
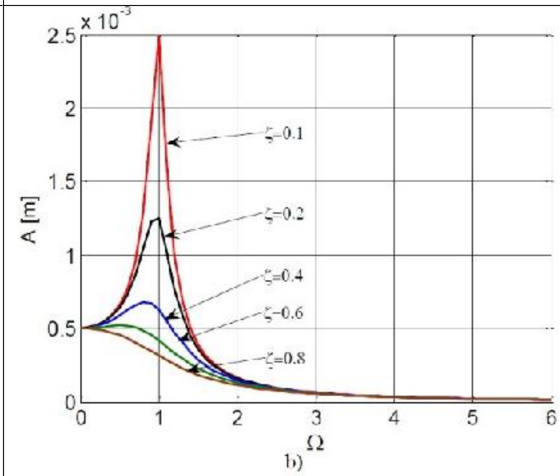
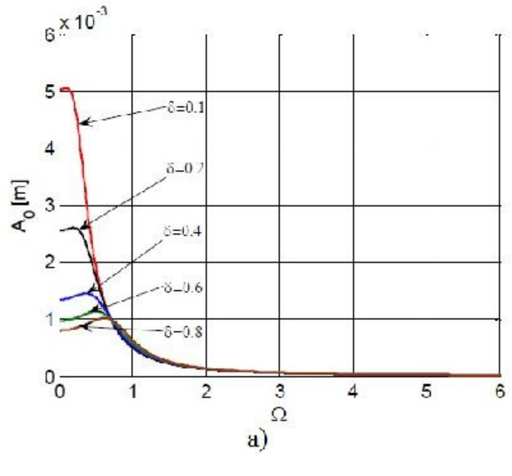
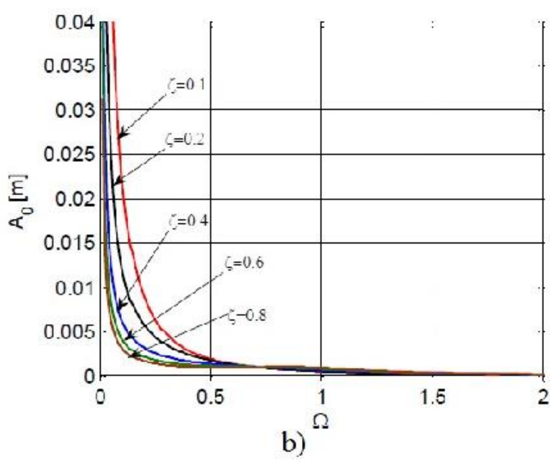
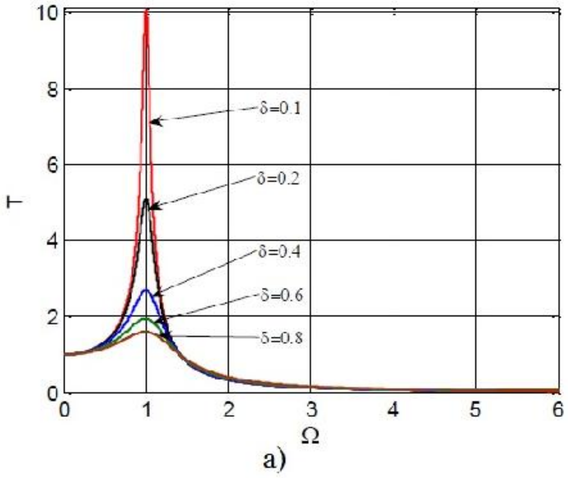
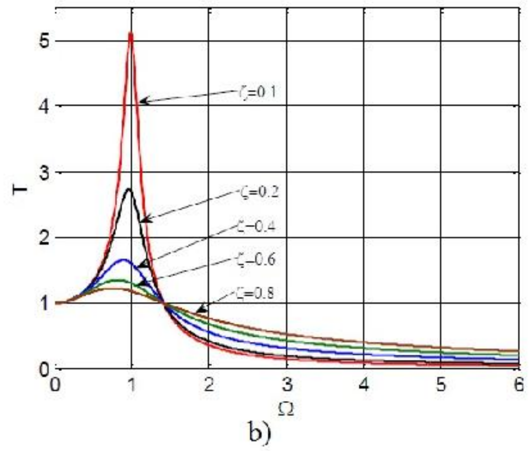
Designation	Variation of amplitude A depending on δ and Ω	Variation of amplitude A depending on Ω and ς
Viscoelastic system Voigt-Kelvin		
Viscoelastic system Maxwell		

Table 3.2: Variation of transmissibility T for structural damping elastomer (a) or discrete viscous damping (b) [1]

<table>
<tr><th>Designation</th><th>Variation of transmissibility T depending on Ω and δ</th><th>Variation of transmissibility T depending on Ω and ζ</th></tr>
<tr><td>Viscoelastic system Voigt-Kelvin</td><td>
</td><td>
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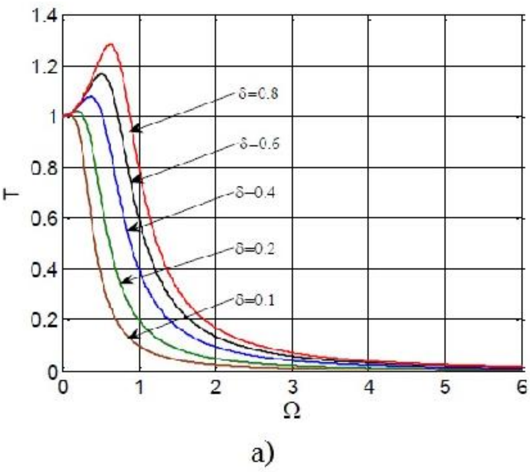
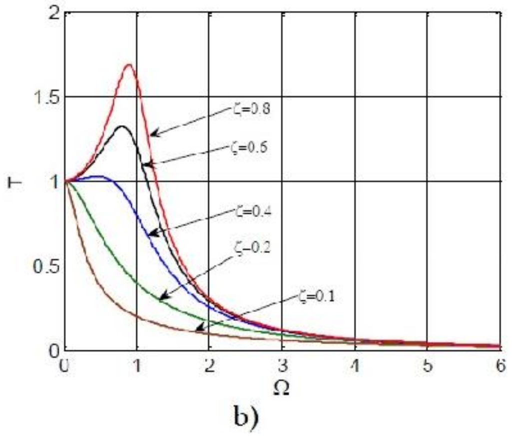
<tr><td>Viscoelastic system Maxwell</td><td>
</td><td>
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REFERENCES

[1] Bratu, Polidor. Analiza structurilor elastice. Comportarea la acțiuni statice și dinamice, Editura IMPULS, București, 2011

[2] Bratu, Polidor. Vibrații Mecanice. Teorie. Aplicații Tehnice. Editura IMPULS, București, 1998

[3] Buzdugan Gh., Fetcu L., Radeș M.. Vibrații mecanice. Editura Didactică și Pedagogică, București, 1982

[4] Ene Gheorghe, Pavel Cristian. Introducere în tehnica izolării vibraţiilor şi a zgomotului, Editura Matrix Rom, București, 2012

[5] Buzdugan Gh. Măsurarea vibrațiilor mecanice. Editura Tehnică, București, 1964

[6] Bratu, Polidor. Curs de Vibrații Neliniare și Aleatorii

[7] Bratu, Polidor Curs de Analiza dinamică a mașinilor cu acțiune vibrantă și prin șoc

[8] http://www.resist.pub.ro/Cursuri_master/CNS/Cap3_IV.pdf