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AN ENERGETIC METHOD TO DETERMINE EXISTENCE OF THE RACEWAY CONTROL IN AN ANGULAR CONTACT BALL BEARING OPERATING AT LOW AXIAL LOAD

A. Popescu[1], M. I. Nazare[1], V. Carlescu[1], D. N. Olaru[1]
[1] Technical University "Gheorghe Asachi", Iasi, ROMANIA, popescu.andrei@tuiasi.ro

Abstract: *In an angular contact ball bearing the value of the angular speed of the ball and the angular position of the angular speed of the ball are usually determined by considering the control on the outer race or on the inner race, depending of the angular speed of the inner race and axial load. In a very low axial load there is a possibility that an oscillation to appear between the outer race control and inner race control. The authors developed an energetic methodology based on the analytical power losses in the both ball-races contacts as function of angular speed of the ball (ωb) and as function of angular position (β). Based on a computer program the values of the ωb and β have been determined by imposing that the total power loss generated by the friction in ball races contacts to have minimum value. For low axial load and low rotational speed, pivoting motion on both contact ellipses have been obtained without any race control.*
Keywords: *angular contact ball bearing, sliding speeds, contact ellipses, power losses, ball angular speed*

1. INTRODUCTION

In the study of the cinematics and dynamics of an angular contact ball bearing, multiple models can be used to evaluate the angular speed of the bearings balls and cage [1], [2].
Therefore, for an angular contact ball bearing loaded with an axial force in which the outer raceway is rotating with an angular speed of ω_0 and the inner raceway is fixed, the angular speed of the cage ω_c will be given in the hypothesis of the pure rolling balls on the raceway by the simplified relation [1], [2]:

$$\omega_c = \frac{\omega_0}{2} \cdot (1 + \gamma) \tag{1}$$

In the same hypothesis, the angular speed of the ball ω_b can be determined by relation:

$$\omega_b = \frac{\omega_0}{2} \cdot (1 - \gamma^2) \cdot \frac{dm}{2 \cdot db} \tag{2}$$

where:

$$\gamma = \frac{db \cdot cos\alpha}{dm} \tag{3}$$

α - the contact angle;
db - diameter of the ball;
The average diameter of the angular contact ball bearing *dm* is determined by relation:

$$dm = \frac{d + D}{2} \tag{4}$$

where *d* is inner race diameter and *D* is outer race diameter.
If the bearing is operating at a reduced or normal speed, relations (1) and (2) can be utilized for a first approximation. For a more realistic evaluation, especially for determining the power losses through friction on the contact ellipses, the hypothesis of the guidance of the balls on the raceway can be used [1]. Therefore, the vector position of the angular speed of the ball ω_b on the axial direction of the bearing, marked with the angle β (Figure 1), can be determined with different relations based on the type of guidance.
For guidance on the outer raceway:

$$\beta = arctg\left(\frac{sin\alpha}{cos\alpha + \gamma}\right) \tag{5}$$

For guidance on the inner raceway:

$$\beta = arctg\left(\frac{sin\alpha}{cos\alpha - \gamma}\right) \quad (6)$$

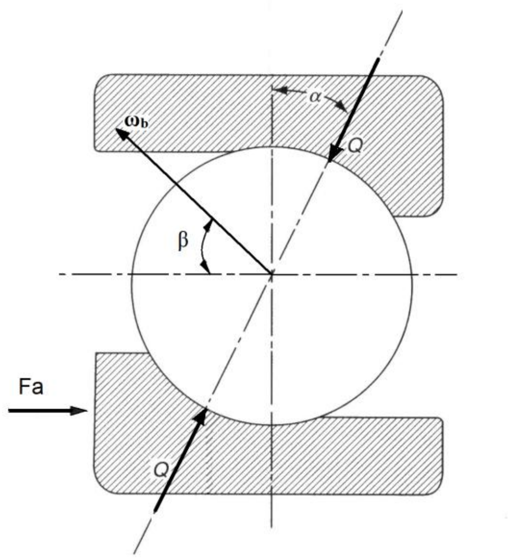

Figure 1: Angular contact ball bearing axial loaded

Equations (5) and (6) take into account that the bearing is operating at low and medium speeds and are based on the fact that the contact angle α exhibits a minimal change. To calculate the sliding speeds on the contact ellipse it is important to know the type of guidance and the value of the angle *β*.

Based on the papers published by [1] and [3] there is a possibility to establish the type of guidance for bearing operating at high speed when the contact angle of the two raceways change substantially. For low and medium operating speeds there is uncertainty regarding the type of guidance.

This paper proposes a model to calculate the angular speed of the ball (ω_b) and the vectors position of the angular speed of the ball (β) utilizing the power losses throw sliding friction on the two contact ellipses (ball-outer raceway and ball-inner raceway).

Therefore, the sum of the power losses due to friction on the two contact ellipses expressed as function of the two unknown parameters ω_b and β will be minimized and the real values for ω_b and β are determined.

2. POWER LOSSES ON THE CONCACT ELLIPSES

On a contact ellipse with the semi-axes *a* and *b* (Figure 2) the sliding speed is present in two directions: the speed v_s on the rolling direction (OY) and the sliding speed v_p caused by the pivoting motion of the ball (not represented in Figure 2). For the contact ellipse presented in Figure 2, the power loss for a slice of the ellipse generated by the sliding speed in rolling direction is calculated based on the formula [4], [5]:

$$dP = \left| vs \cdot dFs \right| \quad (7)$$

where dFs is the elementary friction force and v_s is the sliding speed in the Y direction.

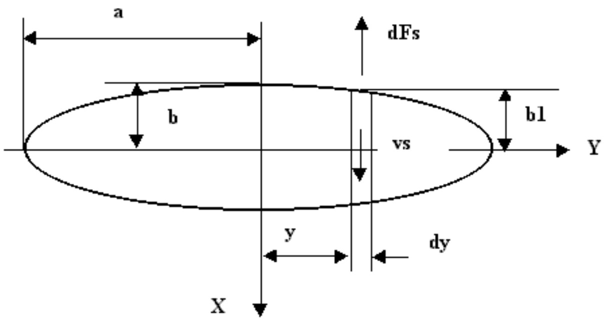

Figure 2: The sliding speed and the friction force on the contact ellipse [5]

2.1 Estimation of the friction forces

The friction force is defined by the following relation [4]:

$$dFs = \int_{-b1}^{b1} (\tau \cdot dx) \cdot dy \tag{8}$$

The tangential tension present in the contact ellipse τ is given by the relation:

$$\tau = \mu \cdot p_H \cdot \left(1 - \frac{x^2}{b^2} - \frac{y^2}{a^2}\right)^{\frac{1}{2}} \tag{9}$$

where μ is the friction coefficient on contact ellipse considered as a constant value on all surface.
According to the Huppert's model [4], the contact pressure can be expressed in the following manner:

$$p_H \cdot \left(1 - \frac{x^2}{b^2} - \frac{y^2}{a^2}\right)^{\frac{1}{2}} = p_H \cdot \left(1 - \frac{x^2}{b1^2}\right)^{\frac{1}{2}} \cdot \left(1 - \frac{y^2}{a^2}\right)^{\frac{1}{2}} \tag{10}$$

were b1 is the contact ellipse semi-axes of the slice (Figure 2), p_H is the maximum Hertzian contact pressure in the centre of the contact ellipse, *a* and *b* are the semi-major and semi-minor axes of the ellipse.
The semi-axes of the slice *b1* can be determined by [4]:

$$b1 = p_H \cdot \left(1 - \frac{y^2}{a^2}\right)^{\frac{1}{2}} \tag{11}$$

The semi-major and semi-minor axes of the contact ellipse are determined based on the following formula [4]:

$$a_{i,e} \approx 1.1552 \cdot R_{Xi,e} \cdot k_{i,e}{}^{0.4676} \cdot \left(\frac{Q}{E \cdot R_{Xi,e}{}^2}\right)^{\frac{1}{3}} \tag{12}$$

$$bi,e \approx 1.1502 \cdot R_{Xi,e} \cdot k_{i,e}{}^{-0.1876} \cdot \left(\frac{Q}{E \cdot R_{Xi,e}{}^2}\right)^{\frac{1}{3}} \tag{13}$$

where E is Young's modulus for the materials in contact, Q is the normal load, $R_{Xi,e}$ is the equivalent radius in the rolling direction determined by [4]:

$$R_{Xi} = \frac{db}{2} \cdot \left(1 - \frac{db \cdot \cos\alpha}{dm}\right) \tag{14}$$

$$R_{Xe} = \frac{db}{2} \cdot \left(1 + \frac{db \cdot \cos\alpha}{dm}\right) \tag{15}$$

and k is the radius ratio:

$$k_{i,e} = \frac{R_{Yi,e}}{R_{Xi,e}} \tag{16}$$

$R_{Yi,e}$ is the transverse radius for a ball - race contact.

2.2 Estimation of the sliding speeds on the outer and inner contact ellipses

The sliding speeds for a point from the inner (v_{si}) and outer (v_{so}) raceway ball-race contact ellipses are based on the following formula [1]:

$$v_{si,o}(x, ne, \beta, \omega_b) = v_{i,o}(x, ne) - v_{bi,o}(x, \beta, \omega_b) \tag{17}$$

where *ne* is the rotational speed of the outer race.
The tangential speeds of a point from the inner and outer race $v_{i,o}$ are determined by equations [1]:

$$v_{i,o}(x, ne) = \frac{-dm}{2} \cdot \omega_{i,o}(ne) - A_{i,o}(x) \cdot \omega_{i,o}(ne) \cdot cos(\alpha) \tag{18}$$

where the angular speed on the inner and outer raceway, and parameters $A_{i,o}(x)$ are, respectively:

$$\omega_i(ne) = 0 - \omega_e(ne) \tag{19}$$

$$\omega_o(ne) = \pi \cdot \frac{ne}{30} - \omega_c(ne) \tag{20}$$

$$A_{i,o}(x)=\left\{\left(Ra_{i,o}^{2}-x^{2}\right)^{0.5}-\left(Ra_{i,o}^{2}-a_{i,o}^{2}\right)^{0.5}+\left[(0.5\cdot db)^{2}-a_{i,o}^{2}\right]^{0.5}\right\} \quad (21)$$

where $Ra_{i,o}$ are the radius of the deformed ball-race contact surfaces determined by relations [1]:

$$Ra_{i,o}=d\cdot 2\cdot\frac{f_{i,o}}{(2\cdot f_{i,o}+1)} \quad (22)$$

where $f_{i,o}$ are the ball- race conformities (usually $f_{i,o}$= *0.515 – 0.525*) [2]

As are presented in equations (17) and (18) the sliding speeds are functions of the position on the contact ellipse (variable x).

2.3 Evaluation of the power losses on contact ellipses on rolling direction

The power loss on the inner and outer raceway in the rolling direction can be calculated using following equations [5]:

$$P_i(ne,\beta,\omega_b)=\int_{-a0}^{a0}\left|v_{si}(x,ne,\beta,\omega_b)\right|\cdot dF_{Si}(x)dx \quad (23)$$

$$P_o(ne,\beta,\omega_b)=\int_{-a0}^{a0}\left|v_{so}(x,ne,\beta,\omega_b)\right|\cdot dF_{So}(x)dx \quad (24)$$

Thus, the total power loss on the rolling direction is expressed by:

$$P(ne,\beta,\omega_b)=P_o(ne,\beta,\omega_b)+P_i(ne,\beta,\omega_b) \quad (25)$$

2.4 Evaluation of the power losses on contact ellipses caused by pivoting motion

Considering that on the two ball-race contact ellipses are developed pivoting motion, the pivoting torque on a ball-race contact ellipse is determined by following equation [1], [4]:

$$MP_{o,i}=\frac{3}{8}\cdot(\mu\cdot Q)\cdot a_{o,i} \quad (26)$$

where μ is the friction coefficient which is considered constant on all the contact ellipse surface.

The total power loss generated by sliding in pivoting motion on the two contact ellipses are given by equation:

$$P_s(ne,\beta,\omega_b)=MP_o\cdot\left|\omega_{so}(ne,\beta,\omega_b)\right|+MP_i\cdot\left|\omega_{si}(ne,\beta,\omega_b)\right| \quad (27)$$

The angular pivoting speeds ω_{so} and ω_{si} are determined by the equations [1]:

$$\omega_{so}(ne,\beta,\omega_b)=-\omega_o(ne)\cdot sin(\alpha)+\omega_b\cdot sin(\alpha-\beta) \quad (28)$$

$$\omega_{si}(ne,\beta,\omega_b)=\omega_i(ne)\cdot sin(\alpha)+\omega_b\cdot sin(\beta-\alpha) \quad (29)$$

2.5 Evaluation of the total power losses on contact ellipses

The total power loss on the two contact ellipses generated by the ball motion is determined as a sum of the total power loss generated in the rolling direction and the total power loss generated by the pivoting motion:

$$P_{Total}(ne,\beta,\omega_b)=P(ne,\beta,\omega_b)+P_s(ne,\beta,\omega_b) \quad (30)$$

3. SIMULATION OF THE TOTAL POWER LOSSES IN 7205 BALL BEARING

The total power loss obtained with the equation (30) is a function of two parameters, ω_b and β. For a given geometry, a given load and speed, we can determine the values of ω_b and β such that the total dissipated power by friction, given by the equation (30) to be minimal. A program has been developed to determine the minimum power on the 7205B angular contact ball bearing with the following characteristics: *d= 25 mm; D=52 mm; db=7,928 mm; fi=0,515;fe=0,522; Q=3,94 N; α= 25 degrees.*

For the friction coefficient, the limit value was considered $\mu=0.1$ (limiting lubrication conditions to ensure a constant coefficient of friction).

The calculations result in the values of the contact ellipse semi axes as being $a_O=1{,}96\cdot 10^{-4}$ m and $b_o=0{,}5\cdot 10^{-4}$ m.

Figure 3 shows the graphs of the total power variation P_{Total}, according to the angle β, for 4 values of the angular velocity ωb: 97,8 rad/s; 98rad/s; 98,033rad/s; 98,2rad/s.

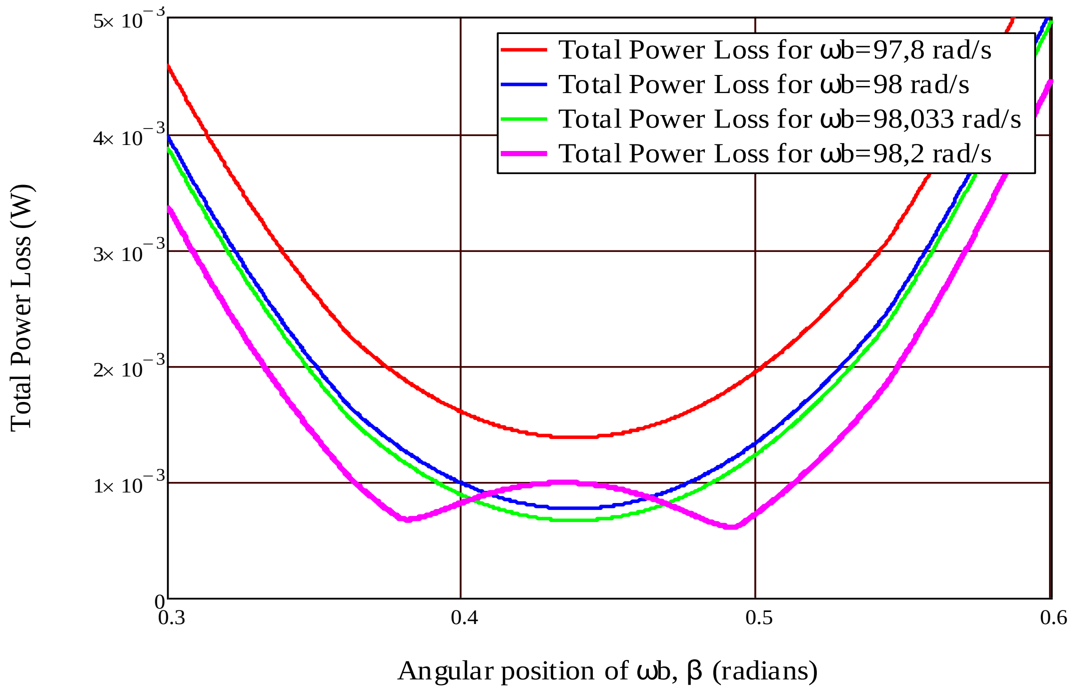

Figure 3: Total power losses as function of the contact angle β

For a given speed of the bearing (*ne=400 rpm*) the minimum of the total power loss corresponds to a single point (where β is 0.4386). This value for angle β does not correspond to any value obtained by the race control. The angular value of $\omega_b=98,033$ is obtained using Equation 2. Passed a certain value of the angular position, namely 98.033 rad/s, the contact angle β exhibits an oscillation that has two points of minimum power, which implies that the ball has guidance on the inner or outer raceway (Figure 3).
For the value of $\beta=0.4386$ and $\omega_b=98,033$ *rad/s*, it was calculated that the total sliding speed values on the inner and outer contact ellipses, v_i and v_o are given by the following equations:

$$v_i(x,ne,\beta,\omega_b)=v_{si}(x,ne,\beta,\omega_b)-\omega_{si}(ne,\beta,\omega_b)\cdot x \quad (31)$$

$$v_o(x,ne,\beta,\omega_b)=v_o(x,ne,\beta,\omega_b)-\omega_o(ne,\beta,\omega_b)\cdot x \quad (32)$$

Figures 4 and 5 presents the sliding speed distribution on inner and outer race respectively.

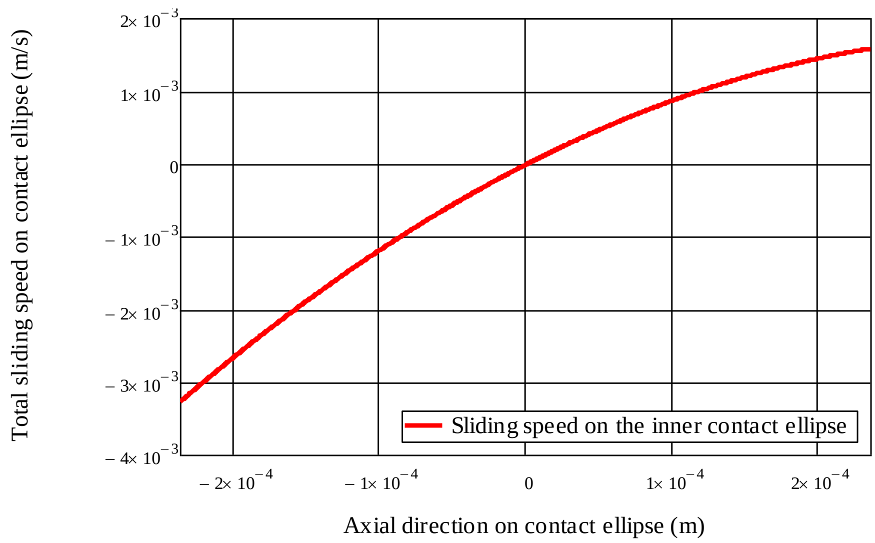

Figure 4: Sliding speed on the inner contact ellipse for $\omega_b=98.033$ *rad/s* and $\beta=0.4386$

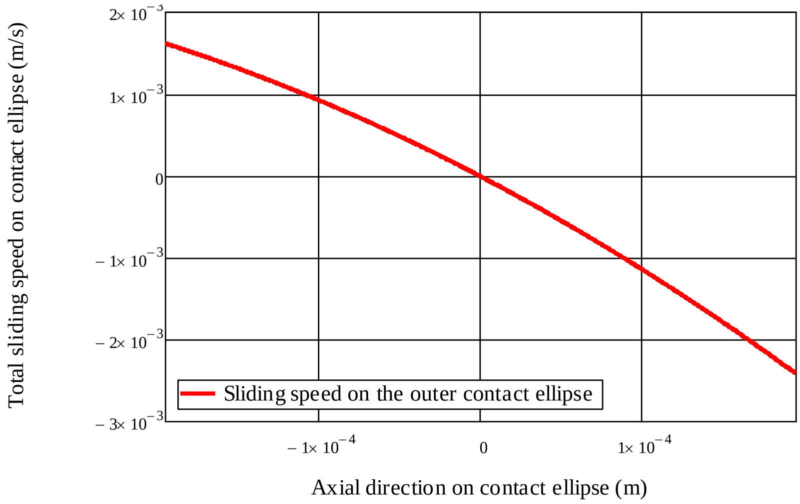

Figure 5: Sliding speed on the outer contact ellipse for ω_b *=98.033 rad/s and* β*=0.4386*

As it can be seen from Figure 4 and Figure 5, there is a minimum value of the total power loss that doesn't correspond to the theory of guidance on one of the raceways [1], [3]. Also, from the value of the angle β the vector of the angular speed of the ball is not perpendicular on the contact line, that means existence of the pivoting motion on both outer and inner contact ellipses. In the high speed conditions, as result of centrifugal forces acting on the balls, the contact angle α decreases on outer race contact and increases on inner race contact and a raceway control of the ball can appears.

4. CONCLUSION

1. The paper presents a methodology for determining the kinematics parameters of the ball (ω_b, β) based on the power dissipated by friction on the contact ellipses between the ball and the raceways.
2. Starting from the classical relations given in the literature for the sliding speed on the contact ellipses in an angular contact ball bearing, the power losses for the sliding friction were calculated, both for the rolling direction and for the ball's pivoting motion. A constant friction coefficient was considered for the two contact ellipses.
3. The total power consumed by friction was considered to be a function of the two cinematic parameters of the ball, ω_b and β, based on a program in Mathcad, the value of the angle β was determined at which the power dissipated by friction is minimal when the angular velocity of the ball varies around the value established in the literature.
4. The results obtained for the 7205B bearing with a load on the ball of 3.94 N and a speed at the outer ring of *400 rpm* indicate that the bearing works with pivotal movement on both contact ellipses without any guidance on any of the raceways.
5. Simulation results are to be validated by experimental testing.

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