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Andrei NICOLAIDE • ELECTROMAGNETICS – General Theory of the Electromagnetic Field

ANDREI NICOLAIDE

# ELECTROMAGNETICS

GENERAL THEORY  
OF THE  
ELECTROMAGNETIC FIELD

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Domain:  
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Andrei Nicolaide

2nd edition

The author develops simultaneously the classical theory of electromagnetism and of electrodynamics within the special theory of relativity, as well as in the general theory of relativity. The general theory of the electromagnetic field is presented in four chapters: definitions and concepts; the study of the forces acting on the charge carriers at rest or in motion; and the energy of this field. Six appendices are presented: vector calculus; the expressions of differential operators in differential co-ordinates; general relations starting from the theory of special relativity; the equations of the electromagnetic field in the general theory of relativity, and tensor calculus.

The presentation of the general laws has been made in two manners as precised by the author. "Their direct introduction as a generalization of the experimental facts, and their derivation, starting from the Coulomb law (formula), and from certain relations of the special theory of relativity." The study of the phenomena in the framework of the general relativity has allowed the consideration of complex phenomena, like in the case of the Sagnac effect. Beyond a very complete bibliography, the corresponding French and German translations of certain terms are also precised.

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Domaine  
ÉLECTROMAGNÉTISME

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Electromagnetics:  
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Auteur :  
Andrei Nicolaïde

2ème édition

L'auteur développe tout à la fois la théorie classique de l'électromagnétisme et celle de l'électrodynamique dans la théorie de la relativité restreinte comme dans la théorie de la relativité générale. La théorie générale du champ électromagnétique est présentée en quatre chapitres : définitions des concepts, étude des forces agissant sur les porteurs de charge au repos ou en mouvement, lois du champ électromagnétique et énergie de ce champ. Six annexes sont présentées : calcul vectoriel ; expressions des opérateurs différentiels en coordonnées curvilignes ; relations générales en partant de la théorie de la relativité restreinte ; équations du champ électromagnétique dans la théorie de la relativité générale ; et calcul tensoriel.

La présentation des lois générales a été faite de deux manières, précise l'auteur. « Leur introduction directe, comme une généralisation des faits expérimentaux, et leur déduction en partant de la loi (formule) de Coulomb et des relations de la théorie de la relativité restreinte. » L'étude des phénomènes dans le cadre de la relativité générale a permis de considérer des phénomènes complexes, comme c'est le cas de l'effet de Sagnac. Outre une bibliographie très complète, les correspondants en français et allemand de certains termes sont précisés.

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**BRAȘOV, 2009**

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of Technical Sciences in Romania

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## **PREFACE TO THE SECOND EDITION**

The present edition represents an improved and augmented form of the first edition. Certain parts of the text and some figures have been remade and improved. The content has been augmented by adding three appendices devoted to the Special and General Theory of Relativity in connection with the Theory of the Electromagnetic field, based on several works including published contributions of the author.

The author should like to gratefully thank Dr. Phys. Jacques CURÉLY, from the Université Bordeaux 1 (France), for the attention paid to the review of the new form and content of the book, and in particular to the added parts.

Braşov, on the 22<sup>nd</sup> of July, 2009.

Andrei NICOLAIDE

## **PREFACE TO THE FIRST EDITION**

In the present work the physical fundamentals of electromagnetic phenomena are studied having in view their technical applications.

The book contains the general theory of the electromagnetic field necessary for the study of the principal applications in the following domains: Electrostatics, Electrokinetics, Electrodynamics and Magnetostatics.

The general theory contains the introduction (i.e., the definition) of fundamental concepts among which: field and substance, electric charge, electric current, state quantities of electric and magnetic fields, as well as the study of laws and energy of the electromagnetic field.

The general theory is presented in four chapters. Further, three appendices are added.

For practical applications, the consideration of electromagnetic phenomena at a macroscopic scale is of special interest. However, in many applications, it is necessary to know the phenomena at a microscopic scale.

At the same time, it is useful to have in view that the physical model is, in many cases, relatively simple in the case of a microscopic study. For this reason, in this work, the following procedure has been used: The various quantities and phenomena have been first examined at a microscopic scale, and then, by calculating the average values, the passage to macroscopic quantities describing the phenomena has been accomplished.

Concerning the presentation of the general laws, there are, in principle, two possibilities:

- a. The introduction of these laws directly, as a generalization of experimental facts;
- b. The derivation of these laws starting from the Coulomb law and the Special Theory of Relativity. The Special Theory of Relativity has been used because it permits the



derivation of the equations of the theory of electromagnetic field starting from a small number of general equations.

Appendix 3, which contains the main formulae of the Special Theory of Relativity, and the derivation of certain relations between forces, given by the author, facilitates to follow the calculations of Chapters 2 and 3.

Also, some relatively recent considerations on the theory of relativity have been mentioned in Introduction.

The text has been elaborated so that all references to the special theory of relativity may be omitted; however, in this case, the number of basic general equations that are not derived from more general relations is greater.

The study of the mentioned domains, namely Electrostatics, Electrokinetics, Electrodynamics, Magnetostatics, can be carried out by using the general laws of electromagnetic field for these various cases. Certain important problems concerning the mentioned domains are analysed in the present work.

A more detailed study of the mentioned domains can be found in several works devoted to these subjects, including the works of the author, mentioned in Bibliography.

The system of units used in this work is the International System of Units (SI) and all formulae are written in this rationalized system.

This work differs to some extent from many other usual textbooks and works by the attention paid to certain subjects like the passage from the microscopic theory to the macroscopic one, the way of using the Special Theory of Relativity, and the simplicity of the presentation.

Certain parts of this work, especially those related to the Theory of Relativity, represent the content of the lectures of an extra-course given by the author at the Université Bordeaux 1 (France) in the summer semester of 2001.

The author thanks especially Doctors of Physics: Jean-Claude GIANDUZZO, Head of the Centre of Electrical and Electronic Resources, and Jacques CURÉLY, both from the Université Bordeaux 1 (France), for their support for the presentation of these lectures and for their valuable comments.

At the same time, the author wishes to thank Professor Florin Teodor TĂNĂSESCU, from the Polytechnica University of Bucharest, secretary general of the Academy of Technical Sciences in Romania, for his valuable support and suggestions.

Further, the author wishes to thank Professor Dan BIDIAN, from the *Transilvania* University of Braşov, for having read the manuscript and for his useful comments.

The present work is devoted to the students in Electrical Engineering and Computers and also to all those interested in an introduction in Electromagnetics.

Finally, the author should like to gratefully thank Dr. Phys. Jacques CURÉLY, from the Université Bordeaux 1, for the attention paid to the review of the manuscript and for his valuable comments and suggestions.

Andrei NICOLAIDE

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## LIST OF SYMBOLS

$\mathbf{a}$	–	vector (p. 205).
$A$	–	linear current density, also called linear current sheet (p. 72).
$\mathbf{A}$	–	vector potential (magnetic) in the reference frame $K$ (p. 129, 153).
$\mathbf{A}'$	–	vector potential in the reference frame $K'$ in motion relatively to the reference frame $K$ (p. 153).
$A_0, A_1$	–	vector potentials in the reference frames $K_0$ and $K_1$ (p. 268).
$\mathbf{B}$	–	magnetic induction, also called magnetic flux density (p. 41, 108-110); magnetic induction in any reference frame $K$ (p. 151, 152).
$\mathbf{B}'$	–	magnetic induction in any reference frame $K'$ in motion relatively to the reference frame $K$ (p. 152, 153).
$\mathbf{B}_i = \mathbf{M}_j$	–	intrinsic magnetic induction (p. 78).
$B_{n1}, B_{n2}$	–	normal components at two points, very near, situated on both sides of the separation surface of two media, in the same reference frame (p. 160).
$B_{oP}$	–	magnetic induction at point $P$ in the reference frame $K_0$ (p. 114).
$\mathbf{B}_1, \mathbf{B}_2$	–	vector quantities at two points, very near, situated on both sides of the separation surface of two media, in the same reference frame (p. 160).
$c$	–	velocity of light in empty space, i.e., in vacuo (p. 102).
$C_q$	–	curve with an electric charge distribution (p. 48).
$d\mathbf{l}$	–	line element (p. 32).
$d\mathbf{S}$	–	surface element (p. 35, 214).
$dV$	–	volume element (p. 48).
$dV_0, dV_1$	–	volume element in the reference frame $K_0$ and $K_1$ , respectively, (p.103).
$\mathbf{D}$	–	electric displacement (p. 41), also called electric flux density (p. 92) and electric induction (p. 123), in any reference frame $K$ (p. 152).
$\mathbf{D}'$	–	electric displacement in any reference frame $K'$ in motion relatively to the reference frame $K$ (p. 152).
$D_{n1}, D_{n2}$	–	normal components at two points, very near, situated on both sides of the separation surface of two media, in the same reference frame (p. 156, 157).
$\mathbf{D}_0$	–	electric displacement in the reference frame $K_0$ (p. 108).
$\mathbf{D}_1, \mathbf{D}_2$	–	vector quantities at two points, very near, situated on both sides of the separation surface of two media, in the same reference frame (p. 156).
$e$	–	electric charge of electron in absolute value (p. 38).
$\mathcal{E}$	–	electromotive tension or electromotive force (p. 55).
$\mathbf{E}$	–	electric field strength, also called electric field intensity (p. 53); electric field strength, macroscopic value (p. 50); electric field strength in any reference frame $K$ (p. 151, 153).
$\mathbf{E}'$	–	electric field strength in any reference frame $K'$ in motion relatively to the reference frame $K$ (p. 151, 153).
$E_c$	–	Coulombian component of the electric field strength (p. 53).

$E_i$	–	impressed component of the electric field strength (p. 53).
$E_i$	–	electric field strength produced at any point by a point-like electric charge with the ordinal number $i$ (p. 86).
$E_l$	–	electric field strength in the large sense (p. 52).
$E_{\text{macro}}$	–	macroscopic value of the electric field strength (p. 50).
$E_{\text{micro}}(\mathbf{r}, t)$	–	microscopic value of the electric field strength at a point at any moment (p. 50).
$E_{\text{micro}}$	–	microscopic value of the electric field strength (p. 50).
$E_n$	–	non-Coulombian electric field strength (p. 53).
$E_o$	–	electric field strength at a point at rest in the reference frame $K_o$ (p. 97, 107).
$E_{ox}, E_{oy},$ $E_{oz}$	–	component of the electric field strength at a point, along the $O_o x_o,$ $O_o y_o,$ $O_o z_o$ axes, in the reference frame $K_o$ (p. 107).
$E_r$	–	induced electric field strength component (rotational, solenoidal or curl component) (p. 53).
$E_{t1}, E_{t2}$	–	tangential components at two points, very near, situated on both sides of the separation surface of two media, in the same reference frame (p. 159).
$E_0$	–	electric field intensity produced by external causes (p. 65).
$E_1$	–	electric field strength at a point at rest in the reference frame $K_1$ (p. 98).
$E_1, E_2$	–	electric field strengths at two points, very near, situated on both sides of the separation surface, in the same reference frame (p. 158).
$E_{1x}, E_{1y},$ $E_{1z}$	–	component of the electric field strength at a point, along the $O_1 x_1,$ $O_1 y_1,$ $O_1 z_1$ axes, in the reference frame $K_1$ (p. 106).
$E_{21}$	–	electric field strength at any point 2 produced by a point-like charge with index 1 (p. 86).
$F$	–	force in general, and force acting upon a point-like electric charge (p. 49).
$F_{\text{el}}$	–	force of electric nature acting on a point-like charge (p. 52).
$F_E$	–	force acting upon an electrically polarized small body (p. 68).
$F_i$	–	force acting upon any point-like electric charge $q$ , and due to a point-like electric charge $q_i$ (p. 86).
$F_{\text{mag}}$	–	force of magnetic nature acting upon a moving point-like electric charge (p. 110).
$F_{\text{non-el}}$	–	force of non-electric nature acting on a point-like charge (p. 52).
$F_o$	–	force exerted upon a point-like charge $q$ , at rest in the reference frame $K_o$ (p. 97).
$F_{ox}, F_{oy},$ $F_{oz}$	–	components of the force along the $O_o x_o,$ $O_o y_o,$ $O_o z_o$ axes, of the force exerted upon a point-like charge $q$ , at rest in the reference frame $K_o$ (p. 106).
$F_1$	–	force exerted upon a point-like charge $q$ , at rest in the reference frame $K_1$ (p. 98).