ON THE COMPUTATION AND CONTROL OF A ROBOTIC SURGERY HYBRID SYSTEM

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Abstract : *The contribution of this paper consists in computation and control of a robotic surgery hybrid system using the differential dynamic logic (dL) algorithm. Hybrid systems contain discrete and continuous motions described by differential equations with discrete and continuous interactions. The discrete motions are instantly changing in the system, and the continuous transitions are subjected to restrictions raised from the discrete motions and control. The hybrid systems integrate three components: the continuous behavior of the first system, the discrete interventions of the second system and the control of the second system directed to the first system. In this paper, dL is applied to compute and control the cooperation between the surgeon and the robot during surgical procedures of the liver tumors.*
Keywords : *Differential logic control (dL), motion trajectories, surgical robots.*

1. INTRODUCTION

Hybrid systems are dynamic systems which exhibit interactions between discrete and continuous motions. For instance, the collision between rigid bodies, the cooperation between the surgeon and the robot in medical applications, the systems with stick and slip modes, backlash in gears, and dead zones in cog wheels are some examples captured by hybrid models [1-4].
The control of a hybrid system is provided by the dynamic differential logic algorithm (dL) [5-8]. dL is a logical algorithm presented in terms of its own syntax, semantics and axiomatics. The syntax defines what can be controlled and modified in the system behavior. The semantics gives meaning to the logic formulas and the ultimately interested in what is true in the behavior of the system.
This paper introduces the control the cooperation between the surgeon and the robot during surgical procedures of the liver tumors. A 2RR hemi-spheric mechanism is considered for simplify the theory. The control is concentrated on the motion of the tool-tip in order to avoid certain forbidden regions inside the working area. In surgical interventions, the surgeon and the robot are working with the same tool-tip, and the goal of the control is to stop crossing the critical boundaries in order to minimize the vascular damages and bleeding.

2. KINEMATICS OF THE ROBOT

In the robotic surgery hybrid system, the goal of the robot is to stop the tool-tip to cross the critical boundaries Γ in the working space Ω, helping the surgeon to resolve conflicting trajectories towards the final point.
The Γ contains, in the first place and near other forbidden areas, the hepatic veins (a, b) and c) and two accessory veins including right inferior hepatic vein (d) and caudate vein (e) (Figure 1). The surgeon manipulates freely the tool-tip in Ω without robotic interference, but, when the tool-tip located at the distance d to Γ, reaches its neighborhood to a distance $D < d$ from Γ the robot attenuates the speed of the tool-tip proportionally to D (Figure 2). Some *ab-initio* proposed trajectories of the tool-tip are shown by different colours.
A schematic trajectory starting from the initial position (red circle) of the tool-tip to the final task point (green circle) with bypassing the forbidden areas is shown in Figure 3.
Let us suppose that the working space Ω of the robot is defined by coordinates $r = (x, y, z)$. The tool-tip is simulated as a virtual joystick and it is located at the height h from the base of robot. A system of coordinates (X, Y, Z) is attached to the base of the system (Figure 4). We suppose that the position of the tool-tip is determined by three degree of freedom, i.e. the joint vector $q = (q_1, q_2, q_3)$ is defined as: q_1 is the rotation about

X -axis (pitch angle), q_2 is the rotation about Y -axis (roll angle) and q_3 the rotation about the axis Z which is common with the joystick axis (yaw angle). We suppose that q_3 does not influence the tool-tip position. The joystick can be controlled or in a rolling motion to the sides or in the pitched motion forward and backwards which result in the positioning of the tool-tip in the (x, y) plane. The elevation in z direction is controlled by the rotation of a knob at the top of the tool-tip [9]. The mechanism transformation matrix is given by [9, 10]

$$T_S^O = \begin{pmatrix} i_x / \sqrt{1-\cos^2(q_1)\cos^2(q_2)} & i_y & i_z / \sqrt{1-\cos^2(q_1)\cos^2(q_2)} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

where

$$i_x = \begin{pmatrix} \sin(q_2) \\ \cos(q_1)\sin(q_1)\cos(q_2) \\ \sin^2(q_1)\cos(q_2) \end{pmatrix}, \quad i_y = \begin{pmatrix} 0 \\ \sin(q_1) \\ -\cos(q_1) \end{pmatrix}, \quad i_z = \begin{pmatrix} -\sin(q_1)\cos(q_2) \\ \cos(q_1)\sin(q_2) \\ \sin(q_1)\sin(q_2) \end{pmatrix}. \tag{2}$$

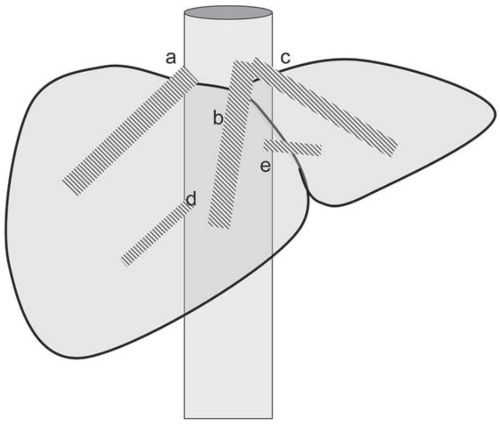

Figure 1: Schematic view of hepatic vein: three hepatic veins (a, b and c) and two veins including right inferior hepatic vein (d) and caudate vein (e) [11].

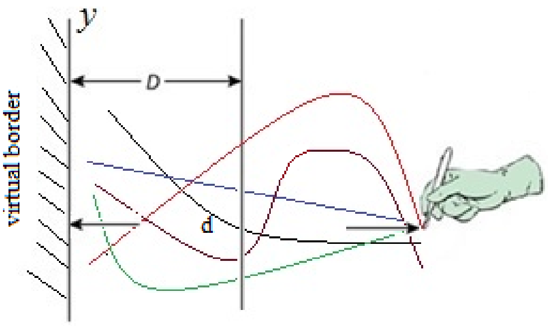

Figure 2: Cooperatively control to restrict the tool-tip to cross Γ.

The transformation matrix of roll angle and the position of the tool-tip, respectively, are given by

$$T_J^S = \begin{pmatrix} \cos(q_2) & -\sin(q_2) & 0 & 0 \\ \sin(q_2) & \cos(q_2) & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

$$T_J^O = T_S^O T_J^S . \tag{4}$$

The Jacobean matrix which transform the joint vector q into working space Ω is

$$J_L = \begin{pmatrix} -h\cos(q_1)\cos(q_2)\sin^2(q_2)/\alpha & h\sin(q_1)\sin(q_2)/\alpha & 0 \\ -h\sin(q_1)\sin(q_2)/\alpha & h\cos(q_1)\cos(q_2)\sin^2(q_1)/\alpha & 0 \\ h\cos(q_1)\sin^3(q_2)/\alpha & h\sin^3(q_1)\cos(q_2)/\alpha & 0 \end{pmatrix}, \tag{5}$$

with $\alpha = \sqrt{(1-\cos^2(q_1)\cos^2(q_2))^3}$. The inverse kinematics of the robot is given by

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \arctan 2(i_z, i_y) \\ \arctan 2(i_x, i_z) \\ \arctan 2\{-\sin(q_2)\sin(q_3)/\alpha, \sin(q_2)\cos(q_3)/\alpha)\} \end{pmatrix}. \tag{6}$$

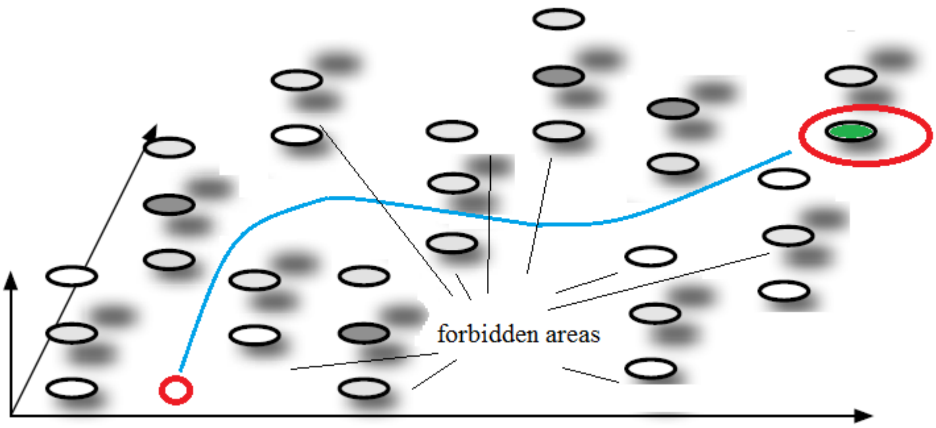

Figure 3: Safe trajectory starting from the initial position of the tool-tip (circle red) to the final task point (green circle) with bypassing the forbidden areas.

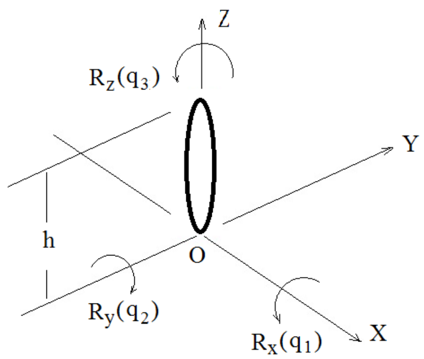

Figure 4: System of coordinates attached to the system.

3. dL SHORT DESCRIPTION

The hybrid language dL permits description of the interaction between a discrete evolution and the continuous action of the tip-top. To represent these interactions, dL includes the non-determinism concept and the ability to describe the continued evolution of the tool-tip. The operations in dL are: logical and $a \wedge b$; logical or $a \vee b$; negation $\neg a$; existential and universal quantifications in R, $\exists x P(x)$ and $\forall x P(y)$, respectively; all running of a satisfying the condition ψ (box mode) $[a]\psi$ satisfying ψ $<a>\psi$. The language of modelling contains the statement in the continuous evolution is written as $\dot{x}_1 = \varphi_1, \dot{x}_2 = \varphi_2, \ldots \dot{x}_n = \varphi_n \,\&\, \psi$; an assumption is expressed as $?\psi$; the assignment is $x_i := \varphi_i$; the non-deterministic assignment of any value $x_i := *$; the sequentially running a and b, $a;b$; non-deterministic choice $a \cup b$; non-deterministic loop $a*$. Details on dL can be found in [3-8]. The state variables can be discrete and continuous. A sequence of dL program that contains an arbitrary input of a non-deterministic value to f, followed by three non-deterministic choices [12]

$$\begin{aligned} &\mathrm{ctrl} = (f := *; \\ &(\dot{r} = gf \;\&\; f >= 0) \cup \\ &(\dot{r} = gf \;\&\; (f <= 0) \wedge (r >= D)) \cup \\ &(\dot{r} = g(r/D)f \;\&\; (f <= 0) \wedge (r <= D)))*, \end{aligned} \tag{7}$$

where r is the tool's position on the x axis, f is the force exerted by the surgeon on the tool in the direction of the x axis, and g a constant. The goal of the algorithm requires that SI starting from a location $(r >= 0)$ continues to stay in a safe location in Ω at every moment of time for any input conditions. The safety property is described by $(r >= 0) \rightarrow [\mathrm{ctrl}(r >= 0)$. KeYmaera is an instrument that can check the safety property of the algorithm [7]. The constraints are modelled in linear or nonlinear inequalities over Boolean-valued variables

$$\begin{aligned} &formula ::= \{clause \wedge\} * clause \\ &clause ::= linear\_consta\operatorname{int}s \mid boolean\_var \rightarrow linear\_constraint \cup \\ &clause ::= nonlinear\_consta\operatorname{int}s \mid boolean\_var \rightarrow nonlinear\_constraint \end{aligned} \tag{8}$$

The algorithm permits the introduction of a large number of constraints by simple syntactic statements, and the surgeon decides to give up or not to a few of these constraints, or to solve any possible conflict between them. The performance of dL is measured by verification of all safety conditions of SI trajectories in Ω and the degree of performing the control task. The algorithm pays attention to causal relationships between variables by a full compatibility between continuous and discrete actions.

4. CONTROL

When the surgeon applies a force f to the tool-tip, the robot allows it according to [5,6]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = G(f), \tag{9}$$

in the 1D case. Here, r is the tool-tip's position on x axis, and G is a constant multiple of f. The trajectories of the tool-tip and Γ can be described by applying the Greenwood and Novikov results [13-15]. The Ω is bounded by a constant boundary $-g$, $g \geq 0$, and the motion is described by

$$s_n = s_0 + \sum_{p=1}^{n} x_p, \tag{10}$$

where $x_p, p \geq 1$ are state variables. We suppose that tool-tip reaches Γ at the time t_g

$$t_g := \min\{n \geq 1 : s_n < -g\}. \tag{11}$$

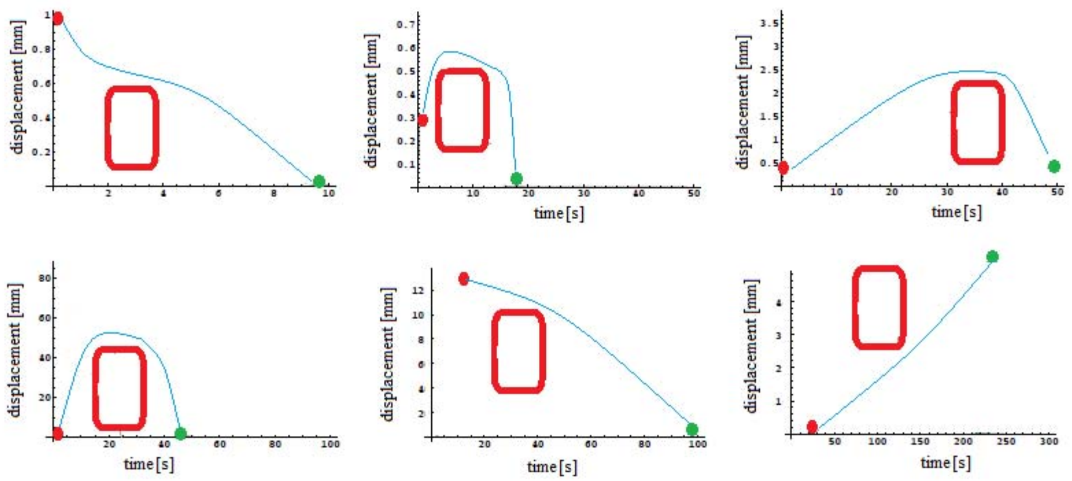

Figure 5: Different safe tool-tip trajectories in Ω .

The contact area of the tool-tip and the tissue in the vicinity of Γ is identified by checking the minimum distance between the tool-tip and Γ

$$\min\left(\frac{1}{2}(r_1 - r_2)^T (r_1 - r_2)\right), \tag{12}$$

where r_1 and r_2 are the position of the tool-tip and Γ , respectively. The class $M \in R^2$ of given motions of the tool-tip is defined as

$$M := \{0 < \mu < 1; |\nu| < 1\} \cup \{0 < \mu < 2; |\nu| \leq 1\} \cup \{\mu = 1, \nu = 0\} \cup \{\mu = 2, \nu = 0\}, \tag{13}$$

where μ and ν are positive arbitrary constants. These motions can be generated by a genetic algorithm [15] or by the modified Kronecker sequences implemented into quasi-Monte Carlo [16]. The overall speed $\dot{r}_1$ is given by the control law [12]

$$\dot{r}_1 = \dot{r} - \left(1 - \frac{d}{D}\right)(\dot{r} \cdot n_1) n_1 , \tag{14}$$

where d is the distance from the tool-tip to Γ with the normal n_1

The unsafe region is represented in Figure 5 as a red region. In this region, the tool-tip can collide Γ , The tool-tip trajectory is monitored in a coordinate system fixed to the tool-tip and centred at its initial position. Different trajectories in Ω generated from (13) are displayed in Figure 3, where the tool-tip is drawn as a red circle and the final point as a green circle. The modified Kronecker sequence is applied to generate from (13) the possible tool-tip trajectories, by generalizing of the golden ratio by a metallic ratio $\varphi = \frac{a + \sqrt{a^2 + 4}}{2}$ with a a positive integer.

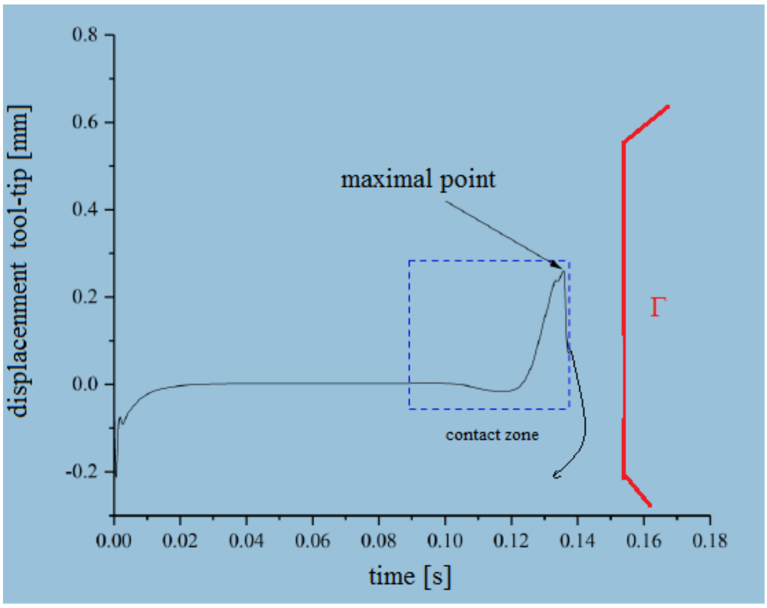

Figure 6: Trace control of a given trajectory.

The Γ is the end of Ω marked with red colour. When the tool-tip approaches Γ, the normal speed component to Γ is attenuated and slowly cancelled. It is the case of the regions noted by A and B.

The trace control of a given trajectory is shown in Figure 6. The contact area of the tool-tip and the tissue in the vicinity of Γ (enclosed in an interrupted line square) is determined by (12). This zone has a maximum point from which the tool-tip reduces its speed in proportion to the distance to Γ in order to bypass the zone and to continue its task.

5. CONCLUSION

The differential dynamic logic dL is used to control a hybrid system consisted by a cooperative system surgeon-robot. The hybrid system is coupling the continuous motions of the surgeon and discrete motions of the robot in a single routine in which the computation, physical aspects and control are interacting.

The novelty of the work consists in extension of the dL to the trace control of trajectories, the stability of actions and safe movements in order to minimize the vascular damages and bleeding. The algorithm corrects the movement of the tool-tip through hybrid discrete and continuous logics and prevent its collision to forbidden areas. This article augments differential dynamic logic dL with new ways for the combined continuous dynamics of a differential equation in continuous time with adverse and discrete interactions of the robot. The dL is able to provide valuable control, especially when the position of the tumor is ambiguous. The results of simulations provide a guarantee of the safe control algorithm for different input conditions and tumor locations.

Acknowledgement: This work was supported by a grant of the Romanian ministry of Research and Innovation, CCCDI – UEFISCDI, project number PN-III-P1-1.2-PCCDI-2017-0221 / 59PCCDI/2018 (IMPROVE), within PNCDI III.

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