STUDY OF CRACK PROPAGATION IN ORTOTROPIC (COMPOSITE) MATERIALS BY USING THE BOUNDARY ELEMENT METHOD

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Abstract*: The application of the Boundary Elemeny Method (BEM) to the computation of stress intensity factor (SIF) and the crack propagation angle in orthotropic materials is the aim of this paper*
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1.INTRODUCTION

The increasing requirements in the design of machanical elements imply the necessity to include in the analysis differernt aspects that traditionally have only been approximated, if considered at all. This is the case of the contact tractions between solids and the stress and strains that appear inside them due to this effect.

It is true that, in most case, these stress are reduced to a very small region in the neighbourhood of the contact zon, and they do not effect the behavior of this structure. However, in order cases, the contact stresses are either the most important ort else they modify substantialy the reponse oThis is the case of joining elements, tribology or crack closure effects, among many others.

Over the last few years important advancements have been made in the inclusion of contact formulations into standard finite element,[5,16] or boundary element programs.[3,10] This last method seems to have proved advantageous in treating the linear contact problem, taht is the contact between lineart elastic solids with small displacements and strains, as occurs for instance along the crack lips of elastic bodies.

The formulation of the BEM is primarily incuded for completeness, so are the formulation and algoritma used to solve the contact problems between two solids. Finally several exemples are explained in detail, specially the study of contact traction in bolted joints in composite laminates.

The increasing use of materials with an important degree of anisotropy, due to its macroscopic structure like concrete, wood or fiber composites, or due to the production process like in certain rolling milling or pulling processes, has made necessary the application of the general theories of Fracture Mechanics to these types of materials. In the particular case of Linear Elastic Fracture Mechanics (LEFM) the prediction of the direction and rate of propagation of cracks under monotononic or alternating loads is one of the most important fields of study for these kinds of materials.

On the other hand, it is well-known that the most important effort in LEFM for isotropic materials has been led to the study of crack propagation in one node problems, while for isotropic materials the consideration of mixed modes is the standard situation. The problem is then considerably more complex than the one corresponding to isotropic materials, even if only orthotropic materials are considered, because it is also needed the computation of the predicted angle of propagation of the crack with respect to the principal directions of orthotropy and also with respect to the applied load.

Many classical works study the description of the behavior of orthotropic material {1}, wile in the specific context of the LEFM it is possible to name refs [2] and [3] among many others. The formulae of the stress distribution in the neighbourhood of the tips of a crack inside an orthotropic and also with respect to the applied load.

In LEFM, the most used parameter to determine the rate of propagation under monotonic loads or the prediction of life in fatigue situations is, no doubt, the stress intensity factor (SIF). Many books are only dedicated to the presentation of the SIF values for a great variety of situations, but except a few closed solutions for very simple case, the application of numerical techniques is a common feature in these computations. The F.E.M. has been widely used for these kinds of applications, for isotropic and orthotropic materials [6-9]. However, the important advantages of the B.E.M. have become evident in these last years for stress concentration problems and more specifically in problems with singularities like the ones appearing in the LEFM field. This fact has implied an increasing application of this method in the present caser, being the one used in this paper.

A complete review of the obtained in this field by using the B.E.M. may be found in ref.[10] with a complete set of references. However, the application of this method to orthotropic materials has not been considered enough. The aim of this work is the computation of the relevant parameters in LEFM, specially the SIF for plane orthotropic problems under any kind of boundary load.

First of all, a brief review of the concepts of t5he theory of crack propagation in orthotropic materials is presented, with additional references. After this, the formulation of the B.E.M. in multidomain problems is also briefly reviewed. Also the different methods that have been compared to the computational of the SIFs are detailed, with different examples comparing the results with several others obtained by means of the F.E.M. Finally, a specific problem of crack propagation in an orthotropic material is also discussed.

2. STRESS FIELD IN THE NEIGHRHOOD OF THE TIP OF A CRACK AND CHARACTERIZATION OF FRACTURE TOUGHNESS IN AN ORTHOTROPIC MATERIAL

As it is known, the constitutive relation in two dimensions an orthotropic material may be written as

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12} & 0\\ A_{12} & A_{22} & 0\\ 0 & 0 & A_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}\qquad\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}=\begin{bmatrix}B_{11} & B_{12} & 0\\ B_{12} & B_{22} & 0\\ 0 & 0 & B_{33}\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \gamma_{xy}\end{bmatrix}\tag{1}$$

with the coefficients A_{ij} given by

$$A_{11}=\frac{E_x}{1-\nu_x\nu_y}\qquad A_{12}=\frac{\nu_y E_x}{1-\nu_x\nu_y}\qquad A_{22}=\frac{E_y}{1-\nu_x\nu_y}\qquad A_{33}=G_{xy}\tag{2}$$

that also fulfill

$$\nu_x E_y=\nu_y E_x\qquad\qquad \frac{1}{G_{xy}}=\frac{4}{E_{45}}-\frac{1-2\nu_x}{E_x}-\frac{1}{E_y}\tag{3}$$

with E_{45} the elastic modulus along 45^0 of the principal axis of orthotropy, that have been considered here as the coordinate system x,y. Coefficients B_y can be computed very easily by inverting the first expression in (3.1).

The expression of the stress field around a tip of a crack in a general anisotropic material may be found for example in ref. [24] which in the case of symmetric or antisymmetric loads with respect to the plane of the crack and for orthotropic materials, can be reduced to (see ref. [24] or [31]).

$$\sigma_x=\frac{K_I}{\sqrt{2\pi r}}R_e\left[\frac{s_1 s_2}{(s_1-s_2)}\frac{s_2}{\mu_1^{1/2}}-\frac{s_1}{\mu_1^{1/2}}\right]+\frac{K_{II}}{\sqrt{2\pi r}}\mathrm{Re}\left[\frac{1}{(s_1-s_2)}\frac{s_2^2}{\mu_2^{1/2}}-\frac{s_1^2}{\mu_1^{1/2}}\right]$$

$$\sigma_y=\frac{K_I}{\sqrt{2\pi r}}R_e\left[\frac{1}{(s_1-s_2)}\frac{s_1}{\mu_2^{1/2}}-\frac{s_2}{\mu_1^{1/2}}\right]+\frac{K_{II}}{\sqrt{2\pi r}}R_e\left[\frac{1}{(s_1-s_2)}\frac{1}{\mu_2^{1/2}}-\frac{1}{\mu_1^{1/2}}\right]\tag{4}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} R_e\left[\frac{s_1 s_2}{(s_1 - s_2)}\frac{s_2}{\lambda_1^{1/2}} - \frac{1}{\lambda_2^{1/2}}\right] + \frac{K_{II}}{\sqrt{2\pi r}} R_e\left[\frac{1}{(s_1 - s_2)}\frac{s_1}{\lambda_2^{1/2}} - \frac{s_2}{\lambda_1^{1/2}}\right]$$

with $\lambda_i = \cos\theta + s_i \sin\theta$; r, θ defined in Fig.1 and s_i the roots of the characteristic equation of the material [1]

$$B_{11}s^4 + (2B_{12} + B_{33})s^2 + B_{22} = 0 \qquad (5)$$

It is remarkable that, as in the isotropic case, the singularity of stress is of order -1/2 and the parameters that determine the amplitude of singularity are again the SIFs' in mode I and II respectively. It must be also pointed out that in the expressions (4) it is assumed that the plane of crack is the principal plane of orthotropy x, z.

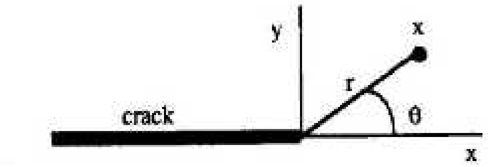

Figure 1. Polar coordinates at tip of a crack

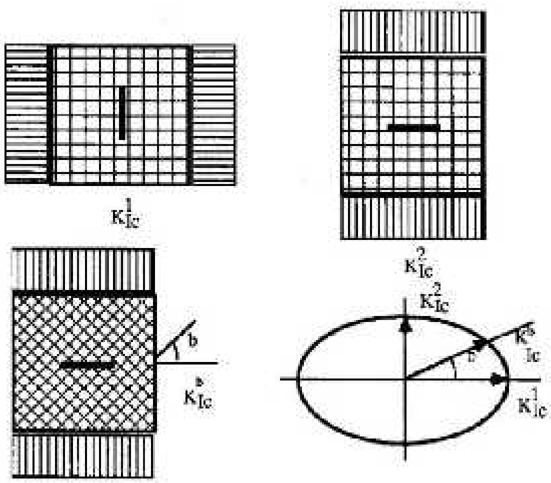

Figure.**2** Fracture toughness along any directions

For a crack forming a certain angle with the axis x, constants B_{ij} should be modified to the new crack axis, so that the elements B_{13} and B_{23} are not longer null and equation (5) would be also modified acquiring the general form of an anisotropic material . From this, it is interesting to point out that the order of the singularity is the same as the one appearing in an isotropic material, but the magnitude and the spatial distribution of the stresses around the crack tip are controlled not only by the values of the SIFs but also by the material proprieties and the angle of the crack with respect to the principal axis of orthotropy.

With respect to the fracture toughness, it is also well-known that more than one parameter is needed to characterization K_{Ic}^1 and K_{Ic}^2 if it is assumed that the fracture toughness has a tensorial character, in such a way that the toughness along any direction β (Fig.2) may be written as a function of two principal toughness as

$$K_{lc} = K_{lc}^{1} \cos^{2} \quad + K_{lc}^{2} \sin^{2} \tag{6}$$

Sometimes the approximated relation

$$K_{lc}^{1} = K_{lc}^{2} \frac{E_2}{E_1} \tag{7}$$

is used, allowing the characterization of the toughness of the material with only one parameter.

3. CRACK PROPAGATION IN ORTHOTROPIC MATERIALS

Among the many criteria established for the characterization of crack propagation in isotropic materials a review of these models may be found in ref [31], two of them appear as the most adequate to be extended to anisotropic bodies: the maximum circumferential stress criterium propose by Erdogan and Sih , and the minimum strain energy density, firstly propose by Sih . In this the first of these two is the one that has been use, because of its simplicity and its good performance in many kinds of anisotropic materials.

The maximum circumferential stress (MCS) criterium assumes that the crack propagates along the direction in which the maximum circumferential stress, defined in isotropic materials as:

$$\pm_{-}^{\max} = \max_{-} \left(\pm_{x} \sin^{2}{}^{-} + \pm_{y} \cos^{2}{}^{-} - 2²_{xy} \sin^{-} \cos^{-} \right) \tag{8}$$

is reached and tangential stress is zero.

Combining (4) this model may be extended to anisotropic bodies under a mixed fracture mode, defining then the MCS[31] as

$$\pm_{-}^{\max} = \left(\frac{K_I}{\sqrt{2®r}} \operatorname{Re}\left[A(s_1 B - s_2 C)\right] + \frac{K_{II}}{\sqrt{2®r}} \operatorname{Re}\left[A(B - C)\right] \right) \tag{9}$$

with

$$A = \frac{1}{s_1 - s_2} \; B = \left(s_2 \sin^{-} + \cos^{-}\right)^{3/2} \quad C = \left(s_2 \sin^{-} + \cos^{-}\right)^{3/2} \tag{10}$$

While for the isotropic case, the angle of crack propagation is obtained by just maximizing the expression (8), a different procedure must be followed when anisotropy is present. because the facture toughness may be a tensor function of θ, as was mentioned in (6). In this case, this MCS criterium is expressed in the form

$$\max_{-} \left(\frac{1}{K_{Ic}^{1} \cos^{2}{}^{-} + K_{Ic}^{2} \sin^{2}{}^{-}} \left\{ K_I \operatorname{Re}\left[A\left(s_1 B - s_2 C\right)\right] + K_{II} \operatorname{Re}\left[A\left(B - C\right)\right] \right\} \right) \tag{11}$$

or it equivalent

$$\max_{-} \frac{\operatorname{Re}\left[A\left(s_1 B - s_2 C\right)\right] + \frac{K_{II}}{K_I} \operatorname{Re}\left[A\left(B - C\right)\right]}{\cos^{2}{}^{-} + \frac{K_{Ic}^{1}}{K_{Ic}^{2}} \sin^{2}{}^{-}} \tag{12}$$

The angle along which reaches its maximum value is determined and if in this direction (26) is greater than I, then the crack propagates along this direction.

Finally, it must be pointed out that (12) become indeterminate in the isotropic, case being necessary to return to (11) or undo this indetermination by using the standard L'Hopital approach.

As an example, of the study of the propagation of a crack in a rectangular plate as the one shown in Fig.3.a The crack forms initially an angle of -45^0 with the axis I of orthotropy andhas an initial length of a_{ini}=0.1. The material properties that have been considered are

E_1=25000 E_2=1750 G_{12}=770 ν_{12}=0.27

and the fracture toughness ratio between the two principal toughnesses is $K_{Ic}^1 / K_{Ic}^2 = 0.5$, with the tensorial variation presented in (6) (Fig.3.8a). The variation of the propagating angle has been studied with two different increments of the crack length Δa=0.1 and Δa=0.05. The results corresponding to the first case are:

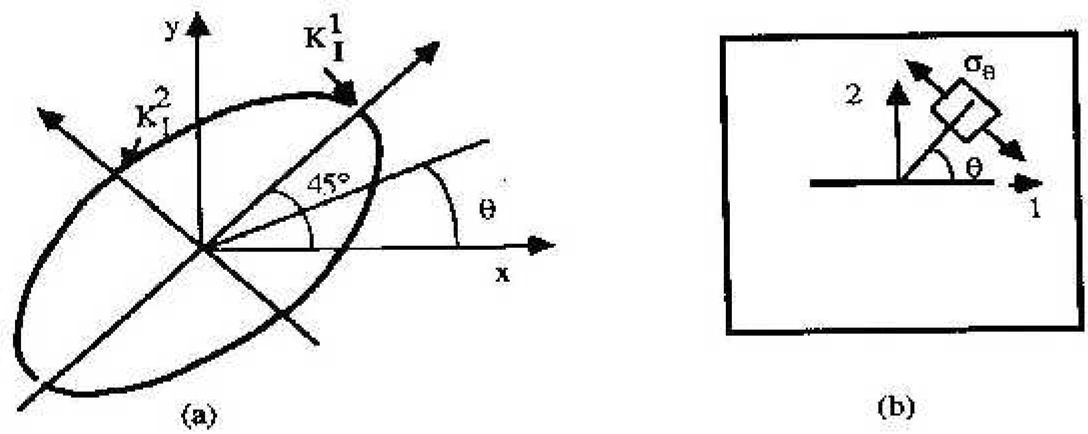

Figure 3. Variation of the fracture toughness with the direction.

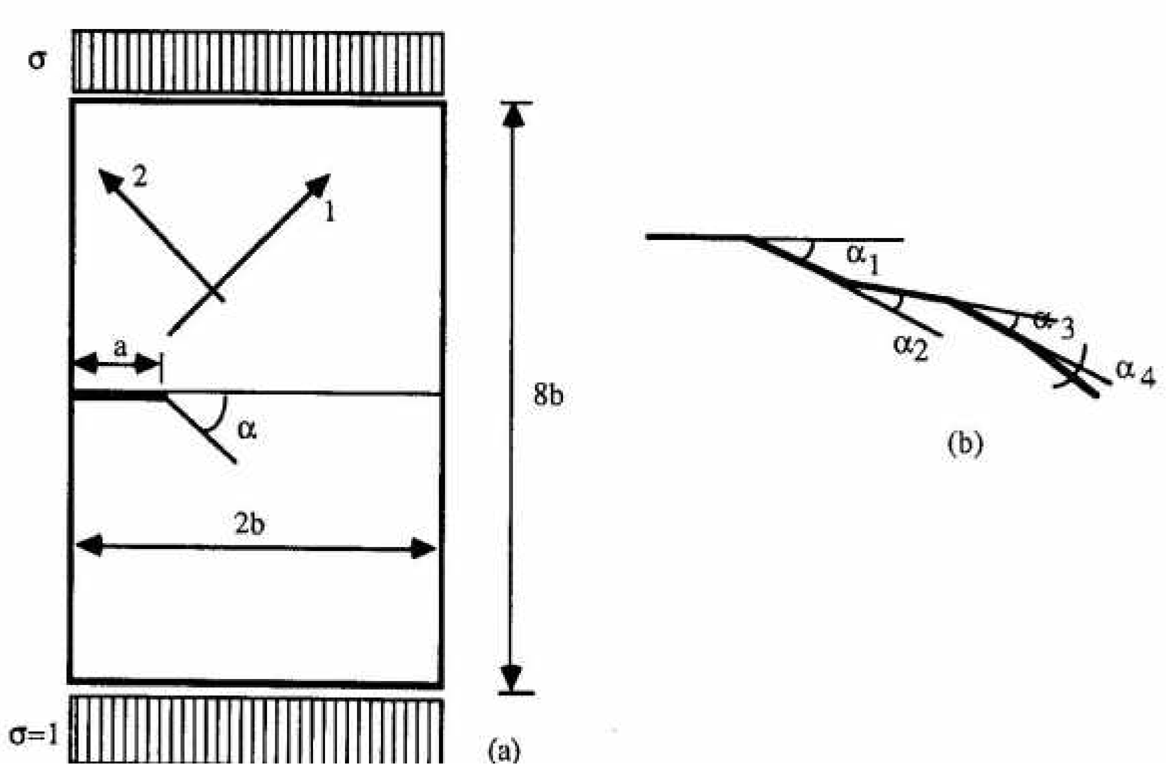

Figure 4. Propagating crack in on orthotropic plate

Step 1. Crack length a=0.1. Length of singular element t=0.025. Initial angle of the crack=-45^0.
Results: t_I=1.8654; t_{II}=-0.38088; K_{II}/K_I=-0204191. Angle of crack propagation with respect to the initial crack profile=-32.

Step2.Crack length a=0.2 Length of singular element l=0.08. Initial angle of the crack=-77^0.
Results: t_I=0.65169; t_{II}=-0.49311; K_{II}/K_I=0.7638. Angle of crack propagation with respect to initial crack profile =11^0.

Step 3. Crack length a=0.3. Length of singular element l=0.0767. Initial angle of the crack=-66^0.
Results: t_I=2.1242; t_{II}=-0.045989; K_{II}/K_I=-0.02165. Angle of crack propagation with respect to initial crack profile=-14^0.

Step 4: Crack length a=0.4. Length of singular element l=0.07934. Initial angle of the crack=-80^0
Results: t_I=2.3971; t_{II}=0.4762; K_{II}/K_I=0.199497. Angle of crack propagation with respect to initial crack profile=-1^0.

The change of the crack direction along the process is shown in Fig.11b, getting finally a propagating angle of -81^0 with respect to the axis 1 of orthotropy. If the same procedure is used with increments of the length of the crack Δa=0.05 the next results are now obtained:

Step 1. Crack length a=0.1. Length of singular element l=0.025. Initial angle of the crack=-45^0
Results: t_I=1.8654; t_{II}=-0.38088; K_{II}/K_I=-0.204181. Angle of crack propagation with respect to initial crack profile=-32^0.

Step 2. Crack length a=0.15. Length of singular element l=0.025 Initial angle of the crack=-77^0
Results: t_I=6.8633; t_{II}=0.1.133; K_{II}/K_I=0.16508. Angle of crack propagation with respect to initial crack profile=-3^0.

Step 3. Crack length a=0.2. Length of singular element l=0.03 Initial angle of the crack=-80^0
Results: t_I=6.9622; t_{II}=1.3767; K_{II}/K_I=0.1977. Angle of crack propagation with respect to initial crack profile =-1^0 getting the same result of -81^0 with only 3 analyses

6. CONCLUSIONS

It has been shown that the B.E.M. may be used to study the problem of propagating cracks in orthotropic bodies in a similar form to the previous works on isotropic materials. Also,the singular boundary elements firstly proposed by Blandford et.all. give very good results in the computation of stress intensity factors every coarse meshes, specially using a direct traction approach like the one presented by Martinez and Dominguez, being only necessary the modification of the fundamental solution of standard isotropic boundary element program.

In most of cases, the method which gives rise to the best results in the computation of the SIF is the one that uses the singular traction approximation, using the nodal value of the singular traction approximation, using the nodal value of the singular node as the parameter which allows the obtention of the SIF, although it is very important the choice of the length of this singular element.

Also the maximum circumferential stress criterium may be very easily included as a postprocessor in a standard B.E. code. In this case, was expected, the choice of shorter increments of the crack propagation effort and with only a few redefinitions of the mesh.

In comparison with domain methods, the mesh needed to produce similar results are much simpler, which is always needed in a crack propagation problem.

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