EQUIVALENT ELASTIC PROPERTIES OF THE COMPOSITE MATERIALS IN MACROMECANICS MODELING

Camelia Cerbu
Transilvania University of Brasov, ROMANIA, cerbu@unitbv.ro

Abstract: *In macro-mechanics of the composite materials, the symmetric special orthotropic laminated composite material plate can be equivalent to a fictitious orthotropic plate from mechanical behavior point of view. The paper presents the computing analytical method which gives the mathematical expressions of the equivalent properties that characterize the fictitious orthotropic plate. Two kinds of laminated composite materials are subjected to tensile test: a composite material whose layers are reinforced with flax fabric; a hybrid composite material whose core layers are reinforced with flax fabric while top and bottom shell layers are reinforced with glass fabric. It is shown the mechanical properties obtained in tensile test, corresponding to the both weft and warp directions of the reinforcing fabrics. The experimental results obtained for the equivalent moduli of elasticity are compared with the theoretical values obtained by using the analytical model. The errors were less than 2% showing a good accuracy of the experimental results.*
Keywords: *composite material, elastic properties, tensile test, flax, glass.*

1. INTRODUCTION

In micromechanics of the composite materials, it is shown the relations of the elastic properties of the composite materials related to the elastic properties of the fibers and matrix and their proportion. These relations are often used to compute the elastic properties of a thin composite layer that is called lamina.
The results obtained by using the relations from micromechanics or the results obtained in mechanical tests of a thin composite layer, are used as input data for macro-mechanical analysis of a laminated composite material that is made by stacking a number of composite layers having different orientations of the fibers. Once the properties of the lamina are known, the microstructural nature of the lamina is ignored in macro-mechanics [1-3]. Therefore, the lamina is considered a homogeneous and orthotropic material with respect with the local coordinate system 123. For example, in case of a composite layer unidirectionally reinforced with fibers, axis 1 is parallel to the fiber direction, axis 2 is perpendicular to the fiber direction and the third axis 3 is perpendicular to the reinforcement plane. In case of a composite layer bidirectionally reinforced with fibers (reinforcing with woven fabric), axes 1 and 2 are aligned with the two reinforcing directions.
Moreover, in macro-mechanics, there are some relations used in order to compute the equivalent elastic properties corresponding to the fictitious orthotropic plate whose mechanical behavior is similar with the one of the composite plate. This equivalence is valid in some particular cases concerning to both the loading of the plate and the structure of the plate [1, 4-6].
In practice, when a composite specimen is subjected to a mechanical test (tensile or bending test), the equivalent elastic properties are determined. This means that, the equivalent elastic properties of a laminated composite material can be experimentally determined or these can be computed by using the relations from analytical model.
The main objective of this paper is to compare the equivalent moduli of elasticity computed by using the analytical model with the experimental results obtained in case of two kinds of composites: (1) composite material having 10 layers made of epoxy resin reinforced with bidirectional flax woven fabric; (2) composite material having 6 core layers reinforced with flax woven fabrics and 2 bottom and 2 top shell layers reinforced with glass woven fabric. In case of each kind of composite material, the modulus of elasticity is computed by analytical method and experimentally determined in both warp and weft directions corresponding of the reinforcing woven fabric.

2. THEORETICAL ASPECTS

The first, some relations from the classical theory of the laminated composite plate are briefly described.

The composite materials analyzed in this paper are symmetric special orthotropic laminated composite materials because: the layers are stacked symmetrically with respect to the median surface; each layer is reinforced either with bidirectional flax or glass woven fabric; reinforcement fabrics have the same orientation in all layers.
Another assumed hypothesis is that the tensile force is applied either on warp direction or on weft direction of the reinforcing woven fabric.
In macro-mechanics of the composite materials, the symmetric special orthotropic laminated composite material plate can be equivalent to a fictitious orthotropic plate from mechanical behavior point of view. The fictitious orthotropic plate may be characterized by a set of elastic properties $\left(E_x, E_y, G_{xy}, \upsilon_{xy}\right)$ expressed in the equation (1):

$$\begin{cases} E_x = \dfrac{1}{t\alpha_{11}} = \dfrac{A_{11}A_{22} - A_{12}^2}{tA_{22}}; \quad G_{xy} = \dfrac{1}{t\alpha_{66}} = \dfrac{A_{66}}{t}; \\ E_y = \dfrac{1}{t\alpha_{22}} = \dfrac{A_{11}A_{22} - A_{12}^2}{tA_{11}}; \quad \upsilon_{xy} = -t\alpha_{12}E_x = -\dfrac{\alpha_{12}}{\alpha_{11}} = \dfrac{A_{12}}{A_{22}}, \end{cases} \tag{1}$$

In the above equation (1), t is the total thickness of the composite material; A_{ij} are the terms of the stiffness matrix $[A]$ in plane expressed in the equation (2) in simplified form corresponding to the composite material with special orthotropic layers:

$$[A] = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}. \tag{2}$$

The terms of the stiffness matrix $[A]$ are computed by using the relation (3):

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k t_k, \tag{3}$$

where $\left(\overline{Q}_{ij}\right)_k$ is the term ij of the stiffness matrix $\left[\overline{Q}\right]_k$ corresponding to the composite layer number $k\left(k = \overline{1,n}\right)$ with respect to the global coordinate system; n represents the total number of layers; t_k is the thickness of the layer k.

The stiffness matrix $\left[\overline{Q}\right]_k$ links the stress vector $\{\sigma\}$ with the strain vector $\{\varepsilon\}$ expressed with respect to the global coordinate system $xOyz$. The stiffness matrix $[Q]_k$ links the stress vector $\{\sigma\}$ with the strain vector $\{\varepsilon\}$ expressed with respect to the local coordinate system (material coordinate system) defined by 1, 2, 3 axes.
In case of the composite materials tested in this paper, the tensile force is applied on one of the reinforcing directions (warp or weft) and the orientation of the reinforcing woven fabric is the same in all layers. Thus, the matrix $\left[\overline{Q}\right]_k$ is equal to the matrix $[Q]_k$ whose terms are given in the equation (4):

$$\left[\overline{Q}\right]_k = [Q]_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} = \begin{bmatrix} \dfrac{E_1}{\Delta} & \dfrac{\upsilon_{12}E_2}{\Delta} & 0 \\ \dfrac{\upsilon_{12}E_2}{\Delta} & \dfrac{E_2}{\Delta} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}, \tag{4}$$

where E_1, E_2 - modulus of elasticity in the reinforcing directions $1,2$ corresponding to the reinforcement fabric; G_{12} and ν_{12} are the shear modulus and Poisson's ratio in the reinforcement plane.
The quantity Δ is computed by using the relation (5):

$$\Delta = 1 - \nu_{12}\nu_{21} = 1 - \nu_{12}^2 \frac{E_2}{E_1}, \tag{5}$$

taking into account the following restriction between the elastic constants:

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}. \tag{6}$$

The first relation of the equation (1) is used in order to compute the equivalent modulus of elasticity in case of the both composite materials involved in this study.

3.MATERIALS AND WORK METHOD

3.1. Materials tested

Two kinds of composite materials were tested: (1) composite material having 10 layers made of Epolam epoxy resin reinforced with bidirectional flax woven fabric; (2) composite material based on Epolam epoxy resin whose six core layers are reinforced with flax woven fabrics while two bottom shell layers and another two top shell layers are reinforced with glass woven fabric. It is said that the second composite material is a hybrid one because it contains layers that are reinforced with different kinds of materials. Mass fiber ratio was equal to 24.16% in case of the first composite material reinforced only with flax fabric while the mass fiber ratio was equal to 33.09% in case of the hybrid composite material.
Firstly, one plate of each composite material previously described is manufactured by using hand-layup technology. The dimensions of each plate are $450x500\,mm^2$. The average thickness of the plate made of Flax/ epoxy composite is equal to 6.07mm while the thickness of the plate made of Glass/ flax/ epoxy is equal to 4.47mm.
It is assumed that the reinforcing materials have the same orientation in all layers that means that the weft direction in all layers is parallel to the same edge of the composite plate.
The glass fabric used is a bidirectional one for which the glass yarn on the weft direction has the same properties with the yarn corresponding to the warp direction. The density of the glass fabric is $200\,g/m^2$.
The flax woven fabric used for reinforcing has different kinds of yarns in warp and weft directions as it is shown in the photo acquired by using a digital microscope (Fig. 1). Therefore, in case of the tested composite materials, the mechanical properties corresponding to the weft direction of the flax fabric are not the same with the properties corresponding to the warp direction. This is the reason for which two sets of tensile specimens are cut from each kind of composite plate: one set containing tensile specimens whose length is parallel to the weft direction corresponding to the flax fabric; another set whose length is parallel to the warp direction.
The shape and dimensions of the tensile specimens were in accordance with standard [7]. Five tensile specimens were tested in case of each composite material involved in this paper.

Figure 1: Flax woven fabric used as reinforcing material

3.2. Work method

Tensile tests were conducted on the universal testing machine type LFV 50-HM, 980 (Walter & Bai, Switzerland) having digital control. The universal machine whose maximum force is 200 kN is equipped with devices for tensile, compressive and bending tests. The software of the machine permits recording with a high acquisition frequency, of the following data: tensile force F, elongation Δl and the time t. The speed of loading was 1 mm/min. in tensile tests. The data of the elastic portion of the tensile stress-strain $(\sigma-\varepsilon)$ curve were approximated by using a linear function whose slope represents the modulus of elasticity E (Young's modulus).
The first relation of (1) is used to compute the equivalent elastic properties of the composite materials involved by knowing the elastic properties of each layer obtained in tensile tests. Finally, the results will be compared with the results obtained in tensile test.

4. **RESULTS**

4.1. Experimental results

Stress-strain $(\sigma-\varepsilon)$ curves recorded in tensile test in case of the composites tested are shown in the figure 2.

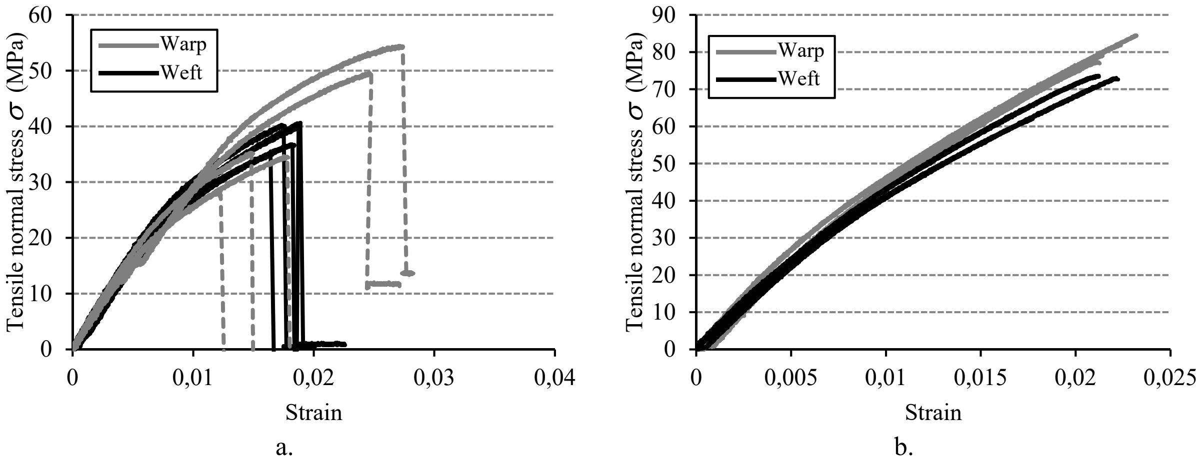

Figure 2: Stress-strain $(\sigma - \varepsilon)$ curves: a. Flax /epoxy composite; b. Glass / flax / epoxy composite.

Table 1: Mechanical properties of Flax /epoxy composite material obtained in tensile test

Direction	Modulus of elasticity E (MPa)	Max. tensile stress σ_{max} (MPa)	Poisson's ratio ν_{12}
Weft	2927.3 (107.3)	39.1 (2.47)	0.2514 (0.0796)
Warp	3394.9 (192.7)	40.5 (11.0)	0.2803 (0.0255)

**** a (b): a represents the average value; b is the Stdev.*

Table 2: Mechanical properties of Glass/ flax /epoxy composite material obtained in tensile test

Direction	Modulus of elasticity E (MPa)	Max. tensile stress σ_{max} (MPa)
Weft	5056.2 (288.5)	68.6 (5.7)
Warp	5450.9 (355.6)	78.0 (6.9)

**** a (b): a represents the average value; b is the Stdev.*

The experimental results (tensile modulus of elasticity E, maximum normal stress σ_{max}) obtained for all specimens tested are statistically processed and the average and standard deviation (Stdev) values are synthesized in Table 1 and Table 2, respectively. The Poisson's ratio ν_{12} of Flax/ epoxy composite (last column of Table 1) was experimentally determined by author in another work that was partially published in [8, 9]. Analyzing the values of Stdev shown in Tables 1 and 2, it may remark that the degree of scattering of the results is lower in case of the Flax/ epoxy composite material than in the case of the Glass/ flax/ epoxy composite.

4.2. Elastic properties computed by using the analytical model

Firstly, the terms of the matrix $[Q]$ which respect to the local coordinate system are computed by using their relations shown in the equation (4). The results are shown in the Table 3 in case of each composite material tested in the both weft and warp directions.

Table 3: Terms of the matrix $[Q]$ corresponding to each layer of the composite materials tested

Layer type	Thickness (mm)	Direction	E_1 (MPa)	E_2 (MPa)	ν_{12}	Δ	Q_{11} (MPa)	Q_{12} (MPa)	Q_{22} (MPa)
Flax / epoxy	0.607	Weft	2927.3	3394.9	0.25	0.9275	3156.119	915.067	3660.270
		Warp	3394.9	2927.3	0.28	0.9325	3640.643	878.975	3139.196
Glass / epoxy	0.206	Weft or Warp direction	14082.6	14082.6	0.15	0.9775	14406.752	2161.013	14406.752

In case of the Flax/ epoxy composite layer, the values of the moduli of elasticity E_1, E_2 and of the Poisson's ratio ν_{12} , in weft or warp direction, are those experimentally determined (Table 1). The elastic properties of Glass/ epoxy composite layer were published by author in another work [10]
The thickness of each kind of composite layer is computed by knowing the thickness of two kinds of composite plates and by assuming that the layers reinforced with the same kind of fabric have the same thickness.
It may observe that the experimental results obtained for the moduli of elasticity (E_1, E_2) and for the Poisson's ratios obey in a good approximation, the restriction between the elastic constants expressed by relationship (6).

Table 4: Terms of the stiffness matrix $[A]$ and equivalent modulus of elasticity E_x computed by using analytical model

Type of laminated composite	Direction	A_{11}	A_{12}	A_{22}	Equivalent modulus of elasticity E_x (MPa)
Flax / epoxy (10 layers)	Weft	19157.64	5554.46	22217.84	2927.35
	Warp	22098.70	5335.38	19054.92	3394.53
Glass / flax / epoxy (10 layers)	Weft	23365.75	5113.35	25201.87	4999.61
	Warp	25130.39	4981.90	23304.12	5388.57

Taking into account that the matrix $[\overline{Q}]$ and matrix $[Q]$ are the same in case of each kind of layer, the terms of the stiffness matrix $[A]$ are computed by using the equation (3). Finally, the equivalent modulus of elasticity E_x is computed by using the first relation of equation (1). The results obtained for the terms of the matrix $[A]$ and for the equivalent modulus of elasticity E_x are synthesized in Table 4 in case of the both composite materials involved subjected to tensile force in weft or warp direction.

4.3. Comparing the results

In Table 5, the results obtained for the equivalent moduli of elasticity E_x by using the analytical model are compared with the corresponding values obtained by tensile test.

Table 5: Comparison between the equivalent modulus of elasticity E_x computed and experimental result in case of the composite materials tested

Composite material	Direction	Analytical model	Experimental	Error (%)
Flax /epoxy	Weft	2927.35	2927.3	0.00
	Warp	3394.53	3394.9	0.01
Glass / flax /epoxy	Weft	4999.61	5056.2	1.13
	Warp	5388.57	5450.9	1.16

The errors of the experimental results with respect to the theoretical results are also shown in the last column of Table 5. It may remark that all errors are less than 2% and these results indicate the accuracy of the experimental results.

5. CONCLUSION

In this paper, it is shown the methodology used to compute the equivalent modulus of elasticity E_x in case of the symmetric special orthotropic laminated composite materials, by using the analytical model shown in macro-mechanics of the laminated composite materials. The small values of the errors between the experimental results and the theoretical results show us that the analytical model from macro-mechanics can be used to predict the equivalent modulus of elasticity E_x corresponding to the symmetric special orthotropic laminated composite materials. Very good results are obtained even in the case of the hybrid composite material (Glass/ flax/ epoxy) containing layers with different reinforcement materials.
The small results of the errors that were less than 2%, show also a good accuracy of the experimental results.

REFERENCES

[1] Barbero E.J., Introduction to composite materials design, CRC Publisher, USA, ISBN 978-1560327011, 1998.

[2] Berthelot, J.M., Chapter: Classical laminate theory, Book: Mechanics of Composite Materials and Structures, Vallouise, France, 2015.

[3] Rebhi L., Dinulović M., Dunjić M., Grbović A., Krstić B, Calculation of the Effective Shear Modulus of Composite Sandwich Panels, FME Transactions, 45, 2017, pp. 537-542.

[4] Chawla K.K., Macromechanics of Composites, Book: Composite Materials. Science and Engineering, The 3rd edition, ISBN 978-0-387-74364-6, Springer, 2013.

[5] Cerbu C., Curtu I., Mecanica şi rezistenţa materialelor compozite, Editura Universităţii Transilvania din Braşov, ISBN 978-973-598-614-8, 2009.

[6] Cerbu C., Botiș M., Numerical Modeling of the Flax / Glass / Epoxy Hybrid Composite Materials in Bending. The 10th International Conference Interdisciplinarity in Engineering, INTER-ENG 2016, 6-7 October 2016, Tîrgu-Mureș, România. Procedia Engineering, ISSN 1877-7058, Elsevier, Vol. 181, 2017, pp. 308–315.

[7] EN ISO 527-4:1997. Plastics – Determination of tensile properties. Part 4: Test conditions for isotropic and orthotropic fiber-reinforced plastic composites.

[8] Cerbu C., Xu D., Wang H., Roșca I.C., The use of Digital Image Correlation in determining the mechanical properties of materials, The 3rd China-Romania Science and Technology Seminar (CRSTS 2018), IOP Conf. Series: Materials Science and Engineering, 399, 012007, 2018.

[9] Lazar V., Cerbu C., Diploma project: Aspects concerning to the properties of the composite materials reinforced with vegetable textile fibers, Faculty of Mechanical Engineering, Transilvania University of Brasov, July 2018.

[10] Cerbu C., Curtu I., Constantinescu D.M.; Miron M.C., Aspects concerning to the transverse contraction in the case of some composite materials reinforced with glass fabric, Materiale Plastice, ISSN 0025-5289, Vol. 48, Nr. 4, 2011, p.341-345.