COMPENSATION OF TEMPERATURE EFFECTS FOR FREQUENCY ESTIMATION OF SLENDER STRUCTURES

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Abstract: *The paper presents a simple method to identify and eliminate the effects of slow temperature changes on the natural frequencies of slender beams. This temperature compensation is important if frequency evaluation is made for the purpose of damage detection, because these changes may overcome the changes due to the occurrence of the damage. The method is based on a mathematical relation found by the authors, which permits estimating the frequency at a temperature if it is measured at a different temperature. A compensation coefficient derived in a mode by mode manner is applied to the measured frequencies at a given temperature. So, the frequencies are found for the reference temperature at which the natural frequencies of the undamaged structure are known. The dissimilarity test is made by an original metric; in the case of exceeding a threshold previously set the occurrence of damage is signalized.*
Keywords: *frequency estimation, temperature changes, vibration, compression forces.*

1. INTRODUCTION

Damage detection in structures by vibration-based techniques implies the use of an advanced method of assessing natural frequencies [1]-[3]. However, due to the variation of environmental conditions the method can fail, because these changes can provoke even higher frequency changes as the damage itself [4]. Especially the temperature changes affect the static [5] and dynamic [6] behavior. The reason is the tensile or compression forces acting due to these changes.
In previous research, we have succeeded in implementing dissimilarity algorithms that compare the natural frequencies of a structure in different moments. In this way, we have been able to evaluate damages from the frequency changes that occur in the event of damage [7]-[9]. Also, we derived mathematical relations to calculate the frequency shift in case of temperature variation [10]. These relations can be used for perfectly clamped beams, but also for beams with elastic restraints [11]. We propose in this paper a new dissimilarity algorithm and involve it to find if the frequency changes occur due to temperature variation or are caused by damage.

2. TRANSVERSE VIBRATION OF BEAMS IN THE PRE-BUCKLING STAGE

A beam subjected to a temperature variation $-\Delta T$, hence suffering a temperature decrease, is subject of a tensile load. On the other hand, a beam subjected to a temperature increase ΔT is subject of a compression load. For the slender beam of length L, having the rectangular cross-section $b \cdot h$, the mass density ρ, the Young's modulus E and the thermal expansion coefficient α, which is in the initial state is fixed at both ends and has a temperature T_0, a temperature change ΔT generates an internal load P, which follows the relation:

$$P = \alpha \cdot E \cdot A \cdot \Delta T \tag{1}$$

If the temperature achieves the value T_{Cr-1}, i.e. the first critical temperature, the first critical buckling load P_{Cr-1} is accomplished and now the structure attain a transversal displacement (see Fig. 1).

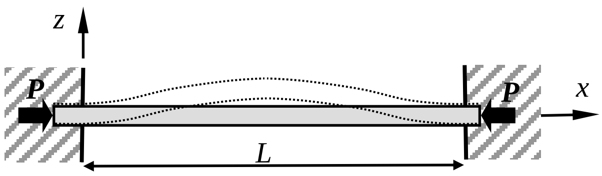

Figure 1: Beam fixed at both ends in the initial state (continuous line) and after exceeding the critical temperature for the first buckling mode (dotted line)

The transversal displacement of a slender beam is [12]:

$$EI\frac{d^4v}{dx^4}+P\frac{d^2v}{dx^2}=\frac{d^4v}{dx^4}+\varsigma^2\frac{d^2v}{dx^2}=0 \tag{2}$$

with the solution:

$$v(x)=c_1\sin(\varsigma x)+c_2\cos(\varsigma x)+c_3x+c_4 \tag{3}$$

For the beam fixed at it both ends, result the transcendental equation:

$$(\varsigma L)\sin(\varsigma L)+2\cos(\varsigma L)-2=0 \qquad \text{or} \qquad \zeta\sin(\zeta)+2\cos(\zeta)-2=0 \tag{4}$$

The roots $\zeta_i=\varsigma_i L$ of the transcendental equation are found by numerical or graphical methods. The first six values are indicated in Table 1.

Table 1: The first six buckling eigenvalues ζ_i

Symmetric buckling modes	Eigenvalues	Asymmetric buckling modes	Eigenvalues
1	6.2831853071	2	8.9868189158
3	12.566370614	4	15.450503673
5	18.849555921	6	21.808243318

The critical loads are found from the well-known relation:

$$P_{cr-i}=\frac{\zeta^2 EI}{L^2} \tag{5}$$

For a vibrating beam subjected to axial loads, the equation of motion is:

$$EI\frac{d^4v(x,t)}{dx^4}\pm|P|\frac{d^2v(x,t)}{dx^2}-\rho A\frac{d^2v(x,t)}{dt^2}=0 \tag{6}$$

were $|P|$ is the absolute value of the axial force and the sign "+" is associated to a compressive force P, while the sign "–" is used for tensile loads.

If the load P is null, the natural frequency of the *i*-th mode is expressed by the known mathematical relation:

$$f_i=\frac{\lambda_i^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \tag{7}$$

where λ_i is the eigenvalue for that vibration mode.

It was shown in [11] that the stored energy distribution along the beam is quite similar for the buckling and the bending vibrations in the different modes. Compression diminishes the capacity of the beam to store energy, and in consequence the frequency (which is proportional to the stored energy) decreases. A relation to calculate the frequency of a simple supported beam for the *i*-th bending vibration mode if buckling manifests is given in [13]. We succeeded to generalize this relation [12]. Now, we can express the frequency of the *i*-th vibration mode of a compressed beam with the axial load P for any end support type with the relation:

$$f_i=\frac{\lambda_i^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}-\frac{|P|}{\rho A}\left(\frac{L^2}{\zeta_i^2}\right)} \tag{8}$$

that can be expressed in a concentrated form:

$$f_i(P)=f_i\sqrt{1-\frac{|P|}{|P_{cr-i}|}} \tag{9}$$

or, employing the if the temperature evolution are considered:

$$f_i(T)=f_i\sqrt{1-\frac{T-T_{ref}}{T_{cr-i}-T_{ref}}} \tag{10}$$

The term represented by the radical in eq. (9) and (10) is a correction coefficient, denoted in this paper by κ, which permits calculating in the beam's natural frequency for a known temperature T if the natural frequency f_i at a reference temperature T_{ref} are known. Hence:

$$\kappa_i = \sqrt{1 - \frac{|P|}{|P_{cr-i}|}} = \sqrt{1 - \frac{T - T_{ref}}{T_{cr-i} - T_{ref}}} \tag{11}$$

The coefficient is valid for the pre-buckling state. Therefore, for the lower vibration modes, it can be used for a narrower temperature interval while for higher modes the temperature range for which the coefficient can be employed is larger. However, in practice the temperatures lower than T_{cr-1} are of interest, because by exceeding this temperature the structure collapses. Numerical simulations were performed to confirm the validity of Eq. (10). The specimen is a double-clamped steel beam as those presented in figure 1, with the main dimensions and the physical and mechanical properties described in table 2.

Table 2: Main properties and dimensions of the specimen

Physical-mechanical properties					Main dimensions		
Mass density [kg/m^3]	Young modulus [N/m^2]	Poisson ratio [-]	Tensile strength [MPa]	Yield strength [MPa]	Length [mm]	Width [mm]	Thickness [mm]
7850	$2\cdot10^{11}$	0.3	470-630	355	1200	50	5

The frequency evolution with the temperature for was targeted. Eight weak-axis vibration modes were determined for the temperature range of 0-125°C. The reference temperature was chosen $T_{ref} = 24.85\,°C$. To be able to calculate the correction coefficient κ we first determined the critical temperatures. The results are presented in table 3. At these temperatures it is expected that beam frequencies become null.

Table 3: First eight critical temperatures T_{cr-i}

T_{cr-1} [°C]	T_{cr-2} [°C]	T_{cr-3} [°C]	T_{cr-4} [°C]	T_{cr-5} [°C]	T_{cr-6} [°C]	T_{cr-7} [°C]	T_{cr-8} [°C]
29.816	34.783	44.716	54.649	69.549	84.349	104.051	123.686

The evolution of the first four frequencies is shown in figure 2. One can observe that indeed, the beam frequencies become zero if the system's temperature achieves a critical one. Also, it is observed the frequency decreases with the temperature increase. The closer the temperature to the critical value, the faster the decrease is. In addition, for all four modes, the temperature at which the frequency becomes null coincide with the critical temperature. This qualifies the simulations to be set as references for the results obtained by calculus.

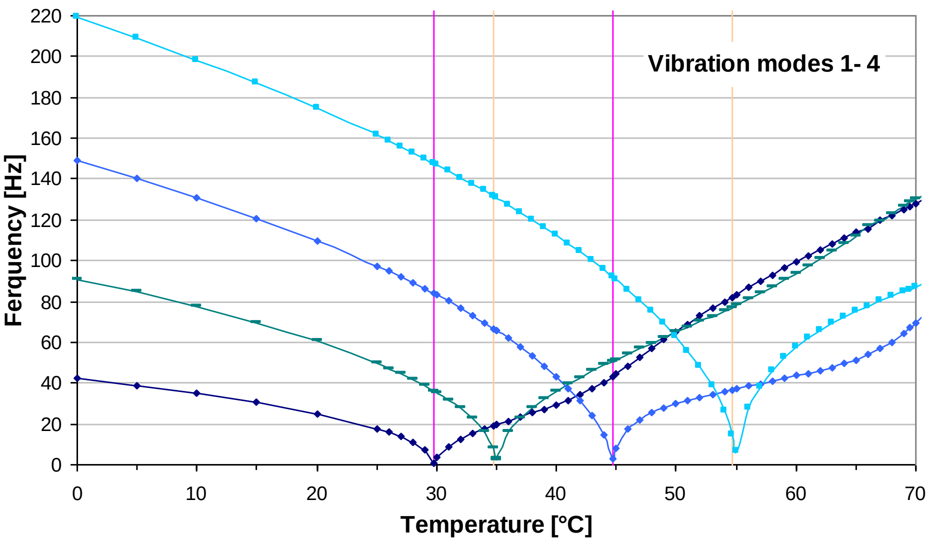

Figure 2: Frequency evolution with the temperature

After applying Eq. (10) to the frequency values found for the reference temperature by simulation we succeed to plot curves indicating the frequency evolution with the temperature. An example of such a curve is presented in figure 3 (for vibration mode three) in comparison with the curve plotted based on simulation results. A good concordance was found, the errors being less that 2.5% for a wide range.

The best fit was found in the vicinity of the reference temperature, while for temperatures close to the critical temperature the errors can achieve 8%. This error expressed in percents is significant because the frequencies close to the critical temperature, to which we refer here, are low. In absolute values the errors can be neglected.

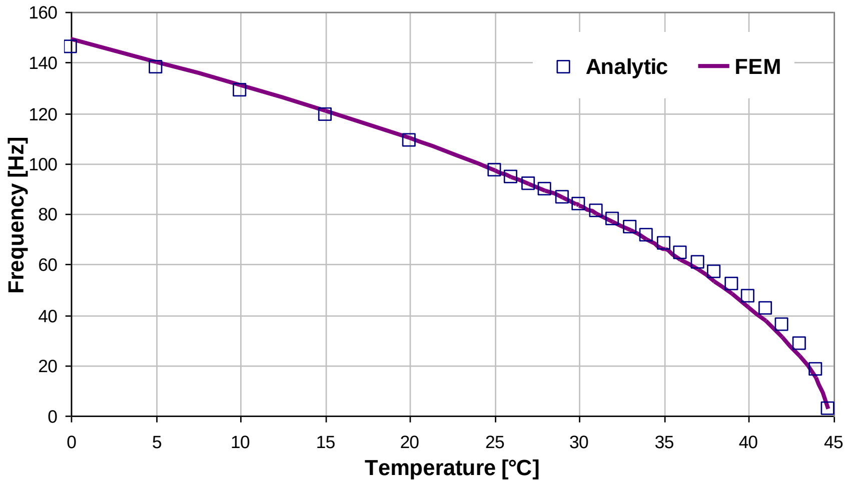

Figure 3: Comparison of curves plotted based on numerical simulations respectively calculus

The conclusions previously formulated permit us to consider the correction coefficient κ proper to suppress the effect of temperature changes if modal analysis is performed.

3. EMPLOYING THE DISSIMILARITY ARGORITHM TO FIND IF TEMPERATURE CHANGE ACTS

Structural changes change the dynamical behavior of structures; hence the modal parameter changes become a reliable indicator of damage. Natural frequencies are the mostly used modal parameters in structural health monitoring. However, the sensitivity to damage of the natural frequencies is small. In addition, the environmental factors can also influence the frequencies, and so damage could be masked or false damage alerts are possible. To avoid this, the modal parameter evaluation must be performed at reference values of the perturbing environmental factors.

In this section, we demonstrate how the effect of the temperature on the natural frequencies can be identified and suppressed if it affects the measured natural frequencies of beam-like structures. The first several measured frequencies are taken as a matrix and compared with the original frequencies and those to which the correction coefficient κ is applied for a series of temperatures differing from the reference. If for one temperature similarity with the original frequencies is found we can assume a temperature change. If not, a structural change occurred.

Let us consider the beam described in the previous section, for which the weak-axis vibration modes are measured at the reference temperature T_{ref}. The sequence can be written:

$$F(T_{ref}) : \{ f_1, f_2, f_3, f_4, f_5, f_6 \}^T \tag{12}$$

If a new measurement is later performed, it is possible to get other results, for instance:

$$F(T_X) : \{ f_{1X}, f_{2X}, f_{3X}, f_{4X}, f_{5X}, f_{6X} \}^T \tag{13}$$

The question is whether the change of frequencies is due to temperature changes or because of damage. To find out this, we calculate the structure frequencies at different temperatures using Eq. (10), having as reference the frequencies indicated by Eq. (13). The more numerous the temperatures for which the natural frequencies are calculated, the more accurate the assessment will be. In this example the frequencies are calculated for successively altered temperatures with a step equal to 1°C. Obviously, the pre-buckling state is of interest. For the beam considered in the previous section, we consider the temperature range 0-29°C. Thus, 29 sequences of natural frequencies are obtained by calculus, resulting as the matrices presented in the relation below:

$$
\begin{aligned}
&F(T_0):\{f_{1-0}, f_{2-0}, f_{3-0}, f_{4-0}, f_{5-0}, f_{6-0}\}^T \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&F(T_j):\{f_{1-j}, f_{2-j}, f_{3-j}, f_{4-j}, f_{5-j}, f_{6-j}\}^T \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&F(T_{29}):\{f_{1-29}, f_{2-29}, f_{3-29}, f_{4-29}, f_{5-29}, f_{6-29}\}^T
\end{aligned} \tag{14}
$$

We have now 30 matrices (including the original measurement), which are compared one-by-one with the second measurement results. The way how the comparison is made is presented in figure 4.

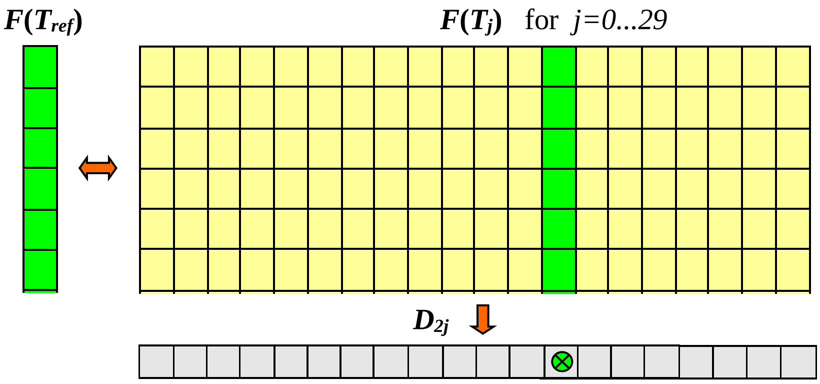

Figure 4: Similarity test algorithm

The similitude test is made using the second-order Minkowski Distance:

$$D_{2j}(F(T_j), F(T_X)) = \sqrt{\sum_{i=1}^{6}\left(f_{i-j} - f_{iX}\right)^2} \qquad j = 0 \ldots 29 \tag{15}$$

and is applied for the 30 cases. The index j, for which a minimum is found, should indicate the reference temperature at which the original frequencies were acquired. In this example, the original frequencies were measured at 24.85°C and the second measurement was made by 27°C, presumed as unknown. The measurement results are:

$F(T_{ref})$: 17.8 Hz, 49.5 Hz, 97.4 Hz, 161.3 Hz, 241.1 Hz, 337.1 Hz
$F(T_X)$: 9.6 Hz, 39.0 Hz, 86.9 Hz, 149.9Hz, 229.7 Hz, 325.2 Hz

The 30 results obtained with Eq. (15) after correcting the values of $F(T_X)$ are graphically represented in figure 5.

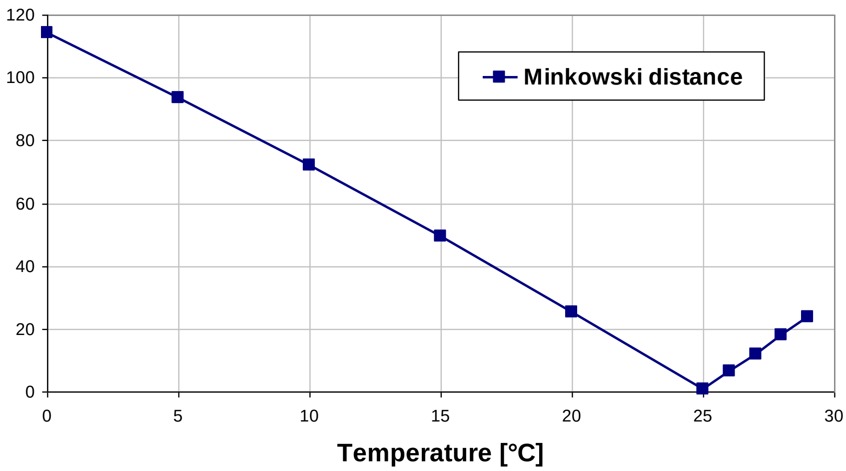

Figure 5: Minkowski distance applied to find the reference temperature

In figure 5 can be seen the reference temperature is correctly indicated, and the Minkowski distance calculated for this temperature is D_{2j} = 0.818, thus very low. This means no structural change occurred in the structure. If the reference temperature is correctly found but the Minkowski distance is relatively high means a damage in incipient phase is present. Finding false reference temperatures indicate the presence of a serious damage.

4. CONCLUSION

This paper introduces a correction coefficient that permits evaluating the frequency of beams at a reference temperature if the actual frequency and temperature are known. This coefficient applies for all beam boundary conditions if the critical temperatures are correctly calculated. Based on this coefficient we developed an algorithm to envisage temperature changes during measurements. We successfully employed the algorithm for detecting temperature changes in several simulations, demonstrating the reliability of the proposed method.

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