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INCREMENTAL PROCESS AND SOLUTION FOR DETERMINATION OF CONTACT TENSIONS BETWEEN ORTHOTROPIC (COMPOSITE) MATERIALS BY USING THE BOUNDARY ELEMENT METHOD

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Abstract: *The boundary element formulation and the computer implementation of the 2-D contacvt problem with small displacements and strains between elastic anisotropic materials are presented in this paper.The contact program include isoparametric linear, quadratic,and quater-point-traction-singular elements. several contact zones with different friction coefficients berween the solids. Several exemples have been included, specially the computation od contact tractions in composite material plates with bolted joints or the influence on the stress intensity factor of the crack closure effects.*
Keywords:*contact, tension,composite,boundary,element*

1. INTRODUCTION

The increasing requirements in the design of machanical elements imply the necessity to include in the analysis differernt aspects that traditionally have only: appear inside them due to this effect.
It is true that, in most case, these stress are reduced to a very small region in the neighbourhood of the contact zon, and they do not effect the behavior of this structure. However, in order cases, the contact stresses are either the most important ort else they modify substantialy the reponse oThis is the case of joining elements, tribology or crack closure effects, among many others.
Over the last few years important advancements have been made in the inclusion of contact formulations into standard finite element, or boundary element programs. This last method seems to have proved advantageous in treating the linear contact problem, taht is the contact between lineart elastic solids with small displacements and strains, as occurs for instance along the crack lips of elastic bodies.
The formulation of the BEM is primarily incuded for completeness, so are the formulation and algoritma used to solve the contact problems between two solids. Finally several exemples are explained in detail, specially the study of contact traction in bolted joints in composite laminates.

2. INCREMENTAL PROCESS

The solution of a contact problem with friction implies the knowdlege of the complete reponse history, due to the irreversible nature.This implies the necessity of using an incremental solution approach. On the other hand, for a fritiomelles contact problem an interactive scheme can be followed to determinethe unknown zone and the distribution of the contact traction along it. Finally, for a contact problemwithout friction and a priori known contact zone, onlyone loading step is needed to know the final contact traction distribution, since it is eventually a linear problem. The only general approach is then the incremental one, and therefore, it is the one used here.
The next consideration refers to the convergence of the user's establishing of a load increment, with an additional internal iteractive process to determine the new contact zone or the changes between sliding and adhesive zones, or to use a node to node incremental process, with a known contact zone, obtaining then the load increment needed to contact two new nodes using the interative process only to detect the changes in the conditions of the different nodr pairs.This last approach is much simpler, the user need not look out for the best incremental step, but rather use congruent discretizations for the contact zones of both solids. This is not a very important drawback since we are assuming small displacements from the beginning so that the candidate zones have to be very similar in shape to have this type of contact problem.This last approach is the one that has been used here as in Refa.10 and Refs.16 due to its simplicity.

Therefore if a certain contact situation is given between both solids, divided into the different zones as described before, a new load increment is applied.assuming a linear process, so that we can write:

$$\Delta Q_n = f_n \cdot (Q - Q_{n-1}) \qquad (2.1)$$

which Q the total load to apply in the contact problem, Q_{n-1} the total load applied for the n-1 previous steps, and f_n the scaliung factor to determine, which corresponds to the minimum scaling factor which modifies the contact elements it may be shown[5] that when the centralnode is placed at a quarter of the element near the singular node and the usual geometric transformation for quadratic elements used (Fig.2.1), this is equivalent to using an approximation for the displacements inside these elements as:

$$u_i = u_i^1 + (4u_i^2 - u_i^3 - 3u_i^1)\sqrt{\frac{r}{l}} + (u_i^1 + 2u_i^3 - 4u_i^2)\frac{r}{l} \qquad (2.2)$$

which automatically includes the $\sqrt{r}$ fields which appears around the tip of the crack. In the same way, if the following approximation for the tractions

$$t_i = t_i^{-1}\phi^1\sqrt{\frac{l}{r}} + t_i^{-2}\phi^2\sqrt{\frac{l}{r}} + t_i^{-3}\phi^3\sqrt{\frac{l}{r}} \qquad (2.3)$$

with ϕ^i the standard aproximation functions for quadratic elements and t t_i^j the nodal value of t_i divided by the nodal value of $\phi^i \cdot \sqrt{\frac{l}{r}}$, it can be shown[13] that it is equivalent to considering the next type of approximation for the tractions

$$t^i = K\sqrt{\frac{l}{r}} + (2t_i^2 - 3K - 3t_i^3) + (2t_i^3 - 2t_i^2 + 2K)\sqrt{\frac{l}{r}} \qquad (2.4)$$

with $K = \lim r_i^1 \sqrt{\frac{l}{r}} \qquad r \to 0$

which corresponds to the $1/\sqrt{r}$ distribution of the analitycal solution for the tractions in the neighbourhood of the tip of crak (fig.2.1)

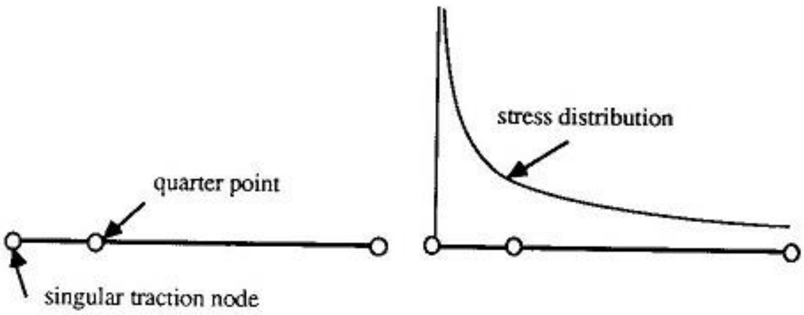

Fig.2.1 Singular element

With respect to the nodes considered in the program[8], and because some of them are placed on the edges of the elements, they can belong to two different contact zone. Because of that, the following kind of nodes are considered:

Nodes 11 – They are nodes that never will be in contact, and therefore they are treated in the same way as they are in a standard BE program;

Nodes IX (IX=2,3,4,5) – They are nodes with the left element in zone 1 and the right one in a contact zone (2,3,4 or 5);

Node XI (X=2,3,4,5) – Exactly the same as before but the left is now the one include in a contact zone while the right one is in zone 1;

Node 22 – Nodes in a candidate to contact zone;
Node 33 – Nodes in a currely sliding zone;
Node 44 – Node in a currely adhesive zone;
Nodes 5X 0r X5 (X=1,2,3,4,5) – Nodes with one of the elements rigidly joined to tje other body and the other element in any type of zone.
It is interesting to point out that the nodes 23 (32) , 24 (42) and 34 (43) have not been considered. This is due to the fact that, because of the approximation used in each element, it is imposible, in practice for a node to to appear in this situation, because the point wich separates the different zones will be always inside a certain element.
However, these types of nodes can be defined in the initial dataand the program changes them to an appropriate type. It is obvious that a node type ij of body A will be in contact with a node type ji of the solid B.
It is also interestimg topoint out that the nodes placed along asymmetry axis, when using an implicit symmetry treatement, have to be of type ii. If a different situation is detected, it is corrected. Finnaly, the singular nodes of the quater-point elements can only have conditions 52 (25), 53 (35) and 54 (45).

3. Structuring and solution of system of linear equations

Due to the nonlinear character of the contact problem and apart from the chosen nonlinear solving method: puerly incremental, puerly iterative or incremental-iterativ, it is necesary to assemble and solve system of linear equations an imoortant number of times throughout the process, that the greater part of the CPU time is spent in these operations. It is very important to chose the covert algoritm, in order to minimize this time.
With respect to the choice of the unknowns there are two possibilities. The first corresponds to the unknows strictly needed to apply to the integral equations which solve the problem. This is obtained by implicitly imposin g the contact conditions.[10] With this, thge number of equatyions is reduced of minimum, but the asembling process has to be done for each step since the unknowns and contact conditions are changing througouht the process (except for easy problem of known friclionless contacvt zone). The process implies the necessity of storing the integration constants corresponding to the contact zone, because a recomputation of them would be extremely inefficient.
The second possibility is the choice of a fixed unknown vector, in such a way that the contact conditions are imposed explicitly, changing only these equations from one step to the next.[8] The number of unknowns and equations is bigger (much bigger if the ratio length of the contact zone/total length of the boundary is large), but on the other hand, the assembling process is very easy due to simplicity of the contact conditions and due also to thge fact that the positions of the different unknows and equations are known from the beginning and unchanged throughout the process. Finally,[8] auxiliar discretized with 13 elements, with a refinement towards the singularity. In the secon case, a singular element with a length of 5mm was incluede in the singular edge.The material propertiesof the punch and fundations are almost the same as can be seen in (Fig.2), which gives an idea of the accuracy of the quadratic elements in problems with high traction gradients.

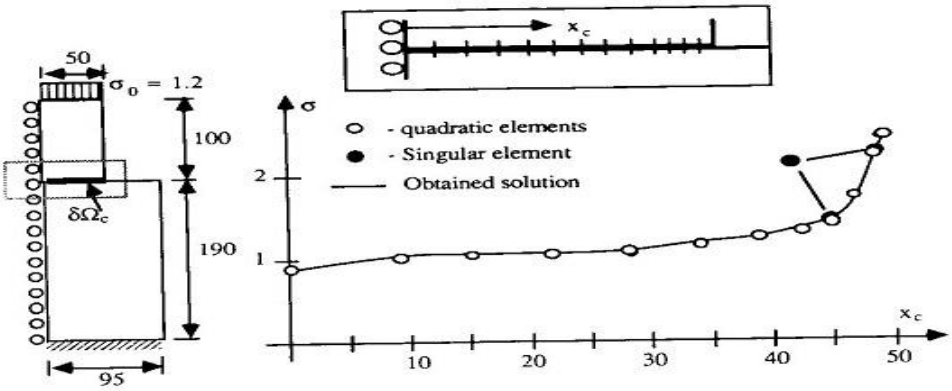

Fig.2. Rectangular punch over an elastic foundation

The second exempleis the Hertz problem, also included in Refs 4 and 10, that is is thw contact between a circular cylinder and a prismatic foundation with different friction coefficients and with a vertical load acting on the cylinder aloing the axis of symmetry. (Fig.3)[7] swows this model for which implicit symmetry has again been considered. The load is applied until achieving the initially established contact zone. This contact zone has been discretized with 11 linear elements in one case and 6 quadratic elements for another. The load needed to obtain that contact zone in the withH=190mm, μ=0.005 and α=15^0, for exemple, is P=568.38 N , which has been modelled wirh a linear distribution along only one element

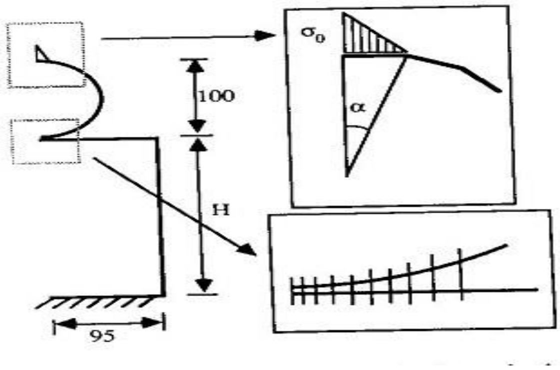

Fig.3. Cylinder over an elastic foundation.

Table 1 shows the results obtained for the maximum normal and tangential tractions, and also the ratio between the adhesive zone B_i and the contact zone B for different lenghts of the fundations, and load angles. The influence of these variables is very important on the values of the tangential tractions.A smaller lenghth of the foundationand a bigger load angle increase the tangential tractions, and with this the sliding zone which is absolutely necessary when completely defining the model if a good comparison of result is to be made.Two values included[7] in the table for the ratio B_i/B correspond to the limits of element inside which the change of corresponds to the results when using quadratic elements, which, in this case , do not differ very much from thoseobtained by the linear ones. Finally, the last result corresponds, to the same problem without friction and with an orthotropic cylinder with material proprieties E_x=1000MP_a; E_y=8000MP_a; E_z=1000MP_a;ν_{xy}=0.3 ;G_{xy}=8.33 MP_a,. The load needed to obtain the contact zone is, in this case, P=1063 · N_w.
Once the program[8] has been ratified with standard testas, a very important example is studied. It is the problem of a bolted joint, similar to the one included in Refs 4 but withan orthotropic plate. It is a rivet under a tangential load, in which the contact tractions along the central section are analyzed(Fig.12)
The traction distribution arround the boundary of a hole in a composite material matrix is an interesting problemthat has been studied experimantally by Gowin et all[11] and Uering et all[21] . However. theconditions of the node pair, that is

$$f_n = \min_\alpha f_n^\alpha \qquad (2.20)$$

with f_n^α the scaling factor needed to modify the contact conditions α. Once f_n has been computed, the new displacement and traction increment distributions are given by

$$\Delta u^k = f_n \bar{u}_k \qquad \Delta t^k = f_n \cdot \bar{t}^k \qquad (2.21)$$

with u^k and t^k the displacements and tractions for the node k, computed by using the boundary conditions of the step n-1 and loaded with $Q - Q_{n-1}$.
The diierent changes that are considered by the program[8] are:
a) The formation of a new contact pair. Change from an adhesive situation to free node (12, 21 or 22) to a contact zone (sliding 13,31 or 33); or adhesive (14,41 or 44). This situation appears when the non-penetration conditions is violated. After this the sliding conditions is checked.
b) Separation of the nodes in contact. Change from a sliding node (33, 13, or 33); to free (12, 21 or 22). This situation is considered when tension tractions are detected in the node.
c) Violation of the adhesion condition. Change from an adhesive node (14, 41 or 44) to a sliding zone (13, 31 or 33). The adhesion conditions is violated when the tangential tractions are greater than the friction (eq.9).

It is interesting to point out that the direct change from node 5 to node 1 has been included, in order to take into account the crack propagation phenomenum. However, it is usually implies the necessity to modify the mesh, wich also means new integration constants and new matrix. The criterium considered for this type of change is that one of the maximum circumferential stress.17

Besides these situations that allow the compulation of the scaling load factors, checking of the incompatilibilities is also needed in order to modify the assumed contact,condition which are not satisfied inside a certain load step. This is the typical case of the initial contactconditions established by the user, althrough they can also appear throughout the loading process. The incompatibity conditions that have been taken into account are:

a) Appering of tension tractions along adhesive or sliding zones. The node is changed to adhesion.

b) Incompatibility of the sliuding condition (the tangential traction and the tangential displacement have the same direction). The node is changed to adhesion.

c) The tangential traction is greater than the friction. The node is changed to side.

The program also iuncludes the possibility of multiple load stages, thus a new load stage is automaticaly started with the contact conditions of the final step of the previous load stage. Two different situations corresponding to loading and unloading are shown in Fig.4 with the following meanings:

F^n – load factor corresponding to the end of the increment n;

FI – load limit of the load for which the contact conditions corresponding to F^nremain hanged;

FS_1– upper limit of the load for which the contact conditions corresponding to F^n remain unchanged,when F^n is lower than the next load peak;

FS_2 – upper limit of the load for which the contact conditions corresponding to F^n remain unchanged,when F^n upper than the next load peak:

F_1^{n+1} - load factor corresponding to the increment n+1 for the case FS_1

F_2^{n+1} - load factor corresponding to the increment n+1 fot the case FS_2

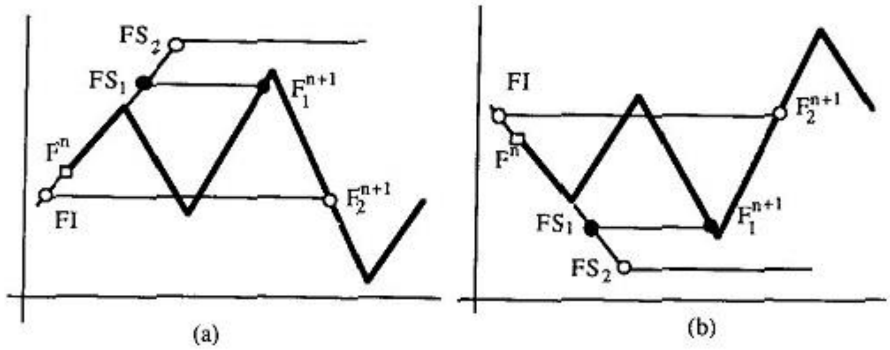

Fig.4. Loading and unloading process

4. CONCLUSIONS

It has been sshown that the B.E.M. may be used yo study the problem of propagating craks in orthotropic bodies in a similar form yo the previouis works on isotropic materials. Also, the singular boundary elements give very good results in the computationof stress intensity factors every with very coars meshes, specially using a direct traction being only necessary the modification of the fundamental solution of a standard isotropic boundary element method.

In the most of cases, the method which gives rise to the best results in the SIF is the one the singular traction approximation,using the nodal value of the singular node as ther parameter wkich allows the obtantion of the SIF, although it is very impostant the choice of the length of siongular element.

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