# NUMERICAL MODEL FOR THE ANALYSIS OF ACOUSTIC WAVE PROPAGATION IN AN EXHAUST DRUM

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***Abstract:*** *Nowadays, noise level is a very important factor of pollution of the environment, of the comfort of life, especially in cities. One of the most important sources of noise is the noise produced by motor vehicles. In this context, an element to reduce the noise level is the exhaust drum, which over time has experienced various constructional and dimensional variants, which will make it as effective as possible. In addition to an experimental study, today we can add a numerical study of the propagation of acoustic waves in the exhaust drum. This paper presents a hypothetical numerical study of an exhaust drum, which can be developed and adapted for any engine, for any constructive and dimensional solution. The numerical analysis presented is performed by the finite elements method, using the ANSYS program, which offers a number of advantages, among which the finite elements dedicated and the consideration of the frequency of the sound waves are the most important offered facilities. The paper also presents a model for analyzing the results through post-processing graphics, which allows to substantiate the conclusions.*
***Keywords:*** *acoustic wave, exhaust drum, acoustic pressure, finite element method*

## 1. INTRODUCTION

The noise level of motor vehicles is fundamentally determined by the exhaust drum. This must produce an attenuation of the characteristics of the sound waves as strong as possible. Achieving this goal is done both experimentally and through theoretical studies. The study with finite elements is one of the most efficient ones, which, besides the results offered, also offers a very useful graphic post-processing for establishing the conclusions. In this paper we present a numerical study of a hypothetical silencer, made with the Ansys program using the finite element method.

## 2. THEORETICAL FUNDAMENTALS

For a fluid, under the hypothesis of mass conservation, linearized momentum equation, adiabatic process and homogeneous medium, using Hooke and Newton laws, the second order equation of acoustic waves [2] is :

$$\frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 0 \qquad (1)$$

In the case of an stady state armonic variation of the acoustic waves, the pressure can be written:

$$P = \overline{P} \cdot e^{j\omega \cdot t} \qquad (2)$$

where $\overline{P}$ is the pressure amplitude, $j = \sqrt{-1}$, $\omega = 2\pi \cdot f$ and $f$ is the frequency of acoustic wave. Under these conditions, equation (1) becomes:

$$\nabla^2 P + k^2 P = 0 \qquad (3)$$

This relation (3) is known as Helmholtz equation, where $k$ is the wave number, defined by relation,

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \qquad (4)$$

where $\lambda$ is the wave length, $\omega$ is angular velocity and $c$ is the sound speed. Considering the propagation of the acooustic wave without lossless energy and entering the notations:

$$\{L\}^T = \nabla \cdot (\ ) \qquad (5)$$

$$\{L\} = \nabla(\ ) \qquad (6)$$

The equation (1) is written:

$$\frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} - \nabla \cdot \nabla P = 0 \tag{7}$$

or,

$$\frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} - \{L\}^T \{L\} P = 0 \tag{8}$$

The finite element matrices can be obtained starting from relation (8) by Galerkin procedures. Multiplying equation (8) by a virtual change in pressure and integrating over the volume of the domain [11] with some manipulation yields:

$$P = \{N\}^T \{P_e\} \tag{9}$$

$$u = \{N'\}^T \{u_e\} \tag{10}$$

where, $\{N\}$ is the element shape function for pressure, $\{N'\}$ is the element shape function for displacements, $\{P_e\}$ is the nodal pressure vector, and $\{u_e\} = \{u_{ex}\}, \{u_{ey}\}, \{u_{ez}\}$ is the nodal displacement component vectors.

Applying the matrix operator $\{L\}$ to the element shape functions, we can write:

$$\{B\} = \{L\}\{N\}^T \tag{11}$$

Using relations (1)...(11) in the integral form of the lossless wave equation [12], the following equation is obtained:

$$\int_V \frac{1}{c^2}\{dP_e\}^T \{N\}\{N\}^T dV \{\ddot{P}_e\} + \int_V \{dP_e\}^T [B]^T [B] dV \{P_e\} + \\ + \int_S \rho_0 \{dP_e\}^T \{N\}\{n\}^T \{N'\}^T dS \{\ddot{u}_e\} = \{0\} \tag{12}$$

where $\{n\}$ is the normal at the fluid boundary. Then the equation (12) is written:

$$\frac{1}{c^2}\int_V \{N\}\{N\}^T dV \{\ddot{P}_e\} + \int_V [B]^T [B] dV \{P_e\} + \rho_0 \int_S \{N\}\{n\}^T \{N'\}^T dS \{\ddot{u}_e\} = \{0\} \tag{13}$$

Relation (13) can be written in a matriceal form,

$$\left[M_e^P\right]\{\ddot{P}_e\} + \left[K_e^P\right]\{P_e\} + \rho_0 [R_e]^T \{\ddot{u}_e\} = \{0\} \tag{14}$$

where the following notations were used, these representing the finite element matrices for acoustic analysis:

$$\left[M_e^P\right] = \frac{1}{c^2}\int_V \{N\}\{N\}^T dV \quad \text{= fluid mass matrix (fluid);} \tag{15}$$

$$\left[K_e^P\right] = \int_V [B]^T [B] \cdot dV \quad \text{= fluid stiffness matrix (fluid);} \tag{16}$$

$$\rho_0 [R_e] = \rho_0 \int_S \{N\}\{n\}^T \{N'\}^T dS \quad \text{= coupling mass matrix (fluid-structure interface).} \tag{17}$$

## 3. PROBLEM FORMULATION

The considered exhaust drum (silencer) is one made of steel sheet, cylindrical in shape, with a diameter of 200 mm and a length of 0.50 m; at the ends, at the level of the horizontal diameter, the inlet/outlet pipes with a diameter of 40 mm, the length of 0.10 m, with an eccentricity of 0.05 m are placed, as it can be seen in the Figure 1. The inside of the drum is free, without additional walls, without extensions of the inlet/outlet pipes.
The propagation medium of the acoustic waves through the silencer is the air, having a density of 1.21 kg/m$^3$, the acoustic impedance of 415.03 Ns/m$^3$ and the speed of sound propagation 343 m/s. An acoustic pressure at the input of 100 dB and a frequency range of 0 ... 3500 Hz were considered. Considering an internal combustion engine with 4-stroke operation, we adopted for the numerical analysis the harmonic analysis in the considered frequency range.

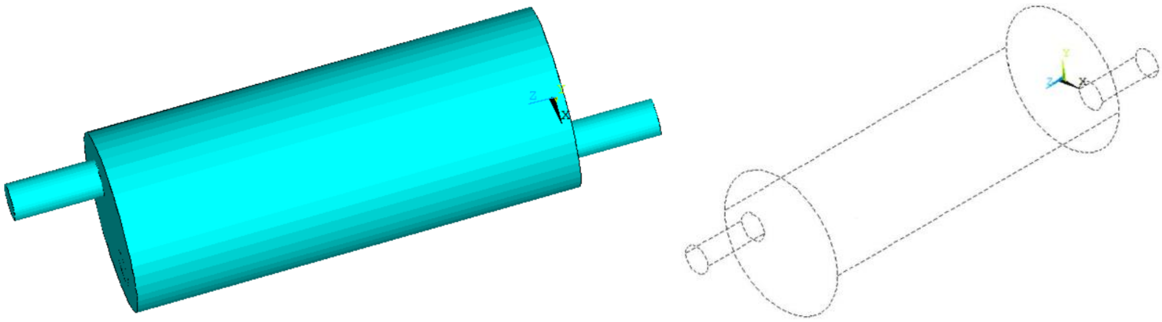


**Figure 1:** Geometrical model of the exhaust drum

## 4. FINITE ELEMENT MODEL

The finite element model is shown in Figure 2. The model contains a number of 139791 finite elements and 195889 nodes. The finite elements are tetrahedral type with 10 nodes/element, an quadratic one, as it can be seen in the Figure 3. The finite element dimension is 0.010 m for all elements. This finite element is named 10 node tet 221 belonging to Ansys finite element library.

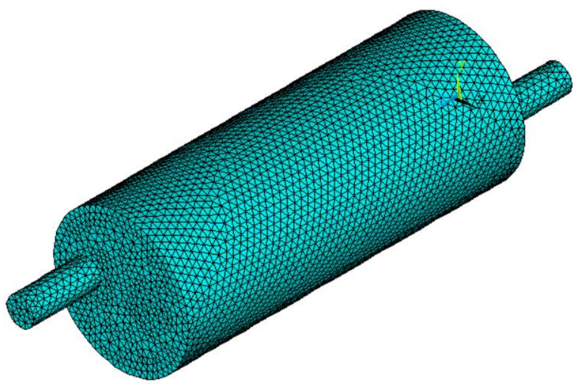


**Figure 2:** Finite element model of the exhaust drum

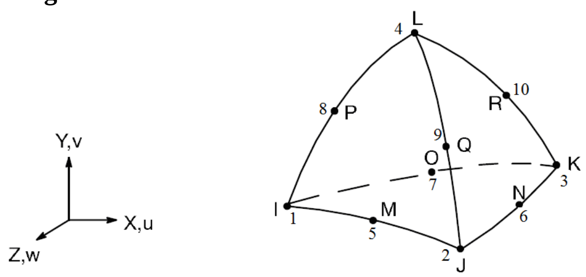



**Figure 3:** The 3-D 10-nodes tetrahedral finite element

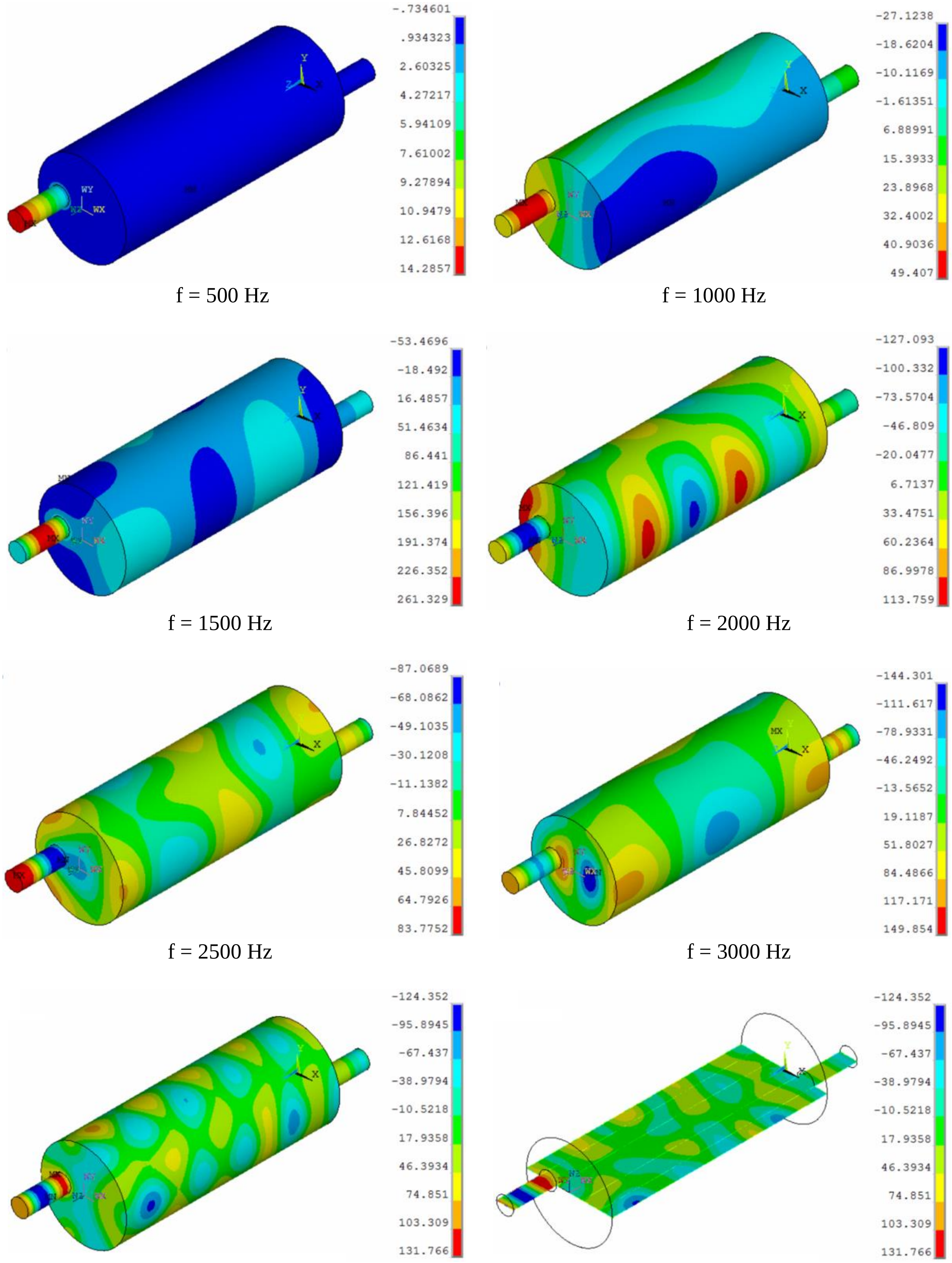
Given the recommendation [2] regarding finite element size, to be smaller than $\lambda/6$ for linear finite elements and smaller than $\lambda/2$ for parabolic elements, the adopted dimension fulfills these requirements for the full frequency range, as shown in Table 1.

**Table 1:** Maximum finite element sizes

| FREQUENCIES | | Frecvența [Hz] | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | **500** | **1000** | **1500** | **2000** | **2500** | **3000** | **3500** |
| *Wave length $\lambda$ [m]* | | *0.686* | *0.343* | *0.2286* | *0.1715* | *0.1372* | *0.1143* | *0.098* |
| F. E. Size [m] | Linear | 0.1143 | 0.0572 | 0.0381 | 0.0285 | 0.0228 | 0.0190 | 0.0163 |
| | Parabolic | **0.3430** | **0.1715** | **0.1143** | **0.0857** | **0.0686** | **0.0571** | **0.0490** |

## 5. RESULTS

In the Figure 4, the acoustic pressure field [dB] is presented, inside along the exhaust drum, for an input of 100 dB pressure at the frequencies of 500, 1000, 1500, 2000, 2500, 3000 and 3500 Hz respectively.




f = 500 Hz
f = 1000 Hz
f = 1500 Hz
f = 2000 Hz
f = 2500 Hz
f = 3000 Hz
f = 3500 Hz
f = 3500 Hz; Horizontal longitudinal slide

**Figure 4:** Acoustic pressure field inside the exhaust drum

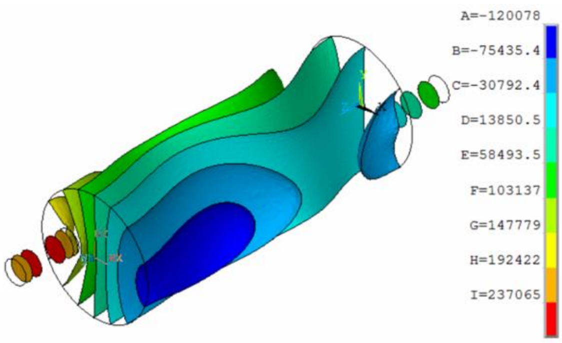



**Figure 5:** f = 1000 Hz; Vertical longitudinal slide

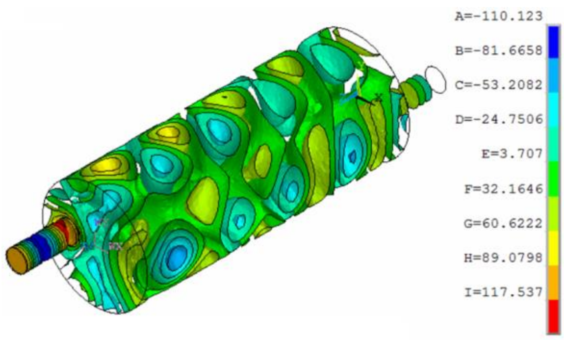



**Figure 6:** f = 3500 Hz; Isosurface representation of the inside acoustic pressure field

As we can see looking at the Figures 4…6, great turbulence occure inside the exhaust drum. These turbulences are cause of a strong variation in the values of the accoustic pressure field. As their effects are concerned, such turbulences are stronger when inside the exhaust drum some walls, with or without holes, exist.

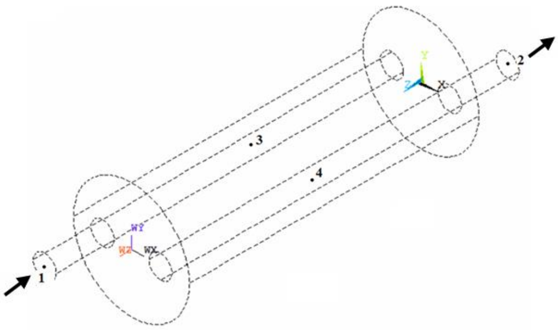



**Figure 6:** The points of pressure determination

In the Figure 7, the curves representing the acoustic pressure versus frequencies, determined in those four nodes presented in the Figure 6 are shown. As we can see just the input acoustic pressure depend on the frequency; his maximum value occur at the frequency of 3545 Hz and its variation is nearly linear. For frequencies lower than 2000 Hz, the most attenuation of the acoustic pressure occur. At the frequency of 3000 Hz, the acoustic pressure at the output point overcome the input acoustic pressure.

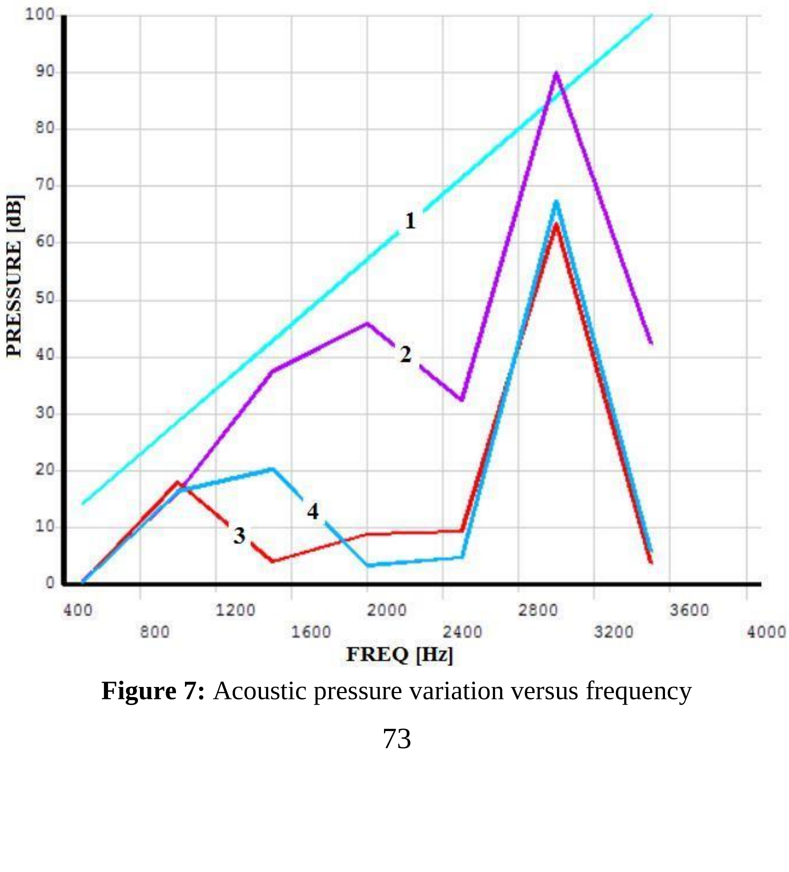



**Figure 7:** Acoustic pressure variation versus frequency

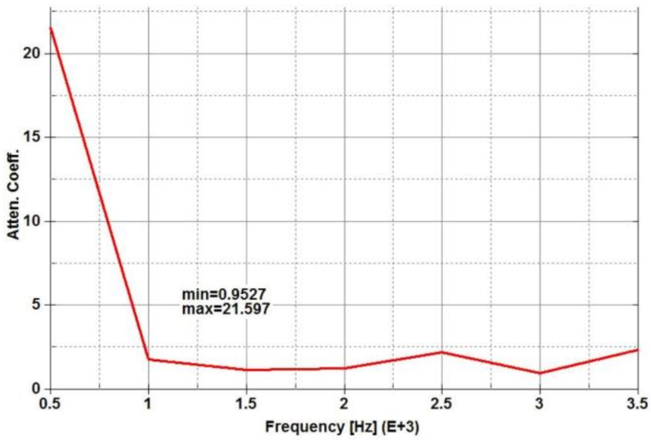



**Figure 8:** Attenuation coefficient versus frequency

By introducing the attenuation coefficient defined by ratio of input/output acoustic pressure (curves 1 and 2 from Figure 7), we see that the attenuation strongly depend on the frequency, as the Figure 8 shows. Beyond the frequency of 1000Hz the attenuation coefficient strongly goes down.

## 6. CONCLUSION

The numarical analysis by finite element method of the acoustic wave propagation in an exhaust drum is a very efficient way for the analysis of the its performance regarding the noise attenuation. Next to it, this way allow us to know what happens inside the drum. So, a large constructive solutions can be studied both for sound level attenuation and for a flow with acceptable hydrodynamic resistance.

The studied case presented in this paper is a hypothetical one without inside walls, so with minimal hydrodynamic resistance.

The numerical model presented in this paper is a model which can be improved by taking into account different constructive solutions beginig with thw geometrical shapes and dimensions and finishing with the present of the walls inside the exhaust drum. Of course, the numerical analysis has to be accompanied with experimental studies.

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