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APPRAISING THE ACCURACY OF A NOVEL MODEL FOR L-SHAPED CRACKS

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Abstract: *In the current paper we aim to evaluate the accuracy of a previously developed method to determine the frequency drop of a cantilever beam having an L-shaped damage with a well-defined position and size. The method is based on the fact that the deformation energy lost by reducing a section of the beam is proportional to the deflection of the same beam under own mass. By setting the boundary and initial conditions for the cantilever, fixed at one end and free at the other, we applied equations of motion for the entire structure based on the Euler-Bernoulli beam model. By using the relation between the frequencies and the deformation energy we obtained a correction term that allows us to calculate the natural frequencies of the cantilever beam having a reduced section based on the frequencies of the undamaged beam. The method is applied through a program developed in the Python language by our research team. The obtained frequencies for the beam with a reduced section are compared with does deducted with the help of the ANSYS simulation software for the beam with an L-shaped crack. Tests performed to find the accuracy of the frequency estimates show that the results are very accurate and we concluded that this method can be used with confidence for modal analysis issues.*
Keywords: *natural frequency, stiffness reduction, modal curvature, energy distribution, modal analysis*

1. INTRODUCTION

Engineering structures can be affected during operation by various types of damages as material fatigue, corrosion or overloading. In order to evaluate the safe functioning of possibly damaged structures, in the last years, methods of early damage detection have been developed based on their dynamic response. Cracks, even at an early stage, can reduce the ability of a structure to store energy due to loss of rigidity, which results in a decrease of its dynamic parameters [1, 2], like natural frequencies, modal shapes and damping ratio.
Over time, various methods of detecting defects have been developed based on the measurement of modal parameters. Firstly, we distinguish the direct methods that consist in determining the frequency changes for a defect of known shape, position and size. Inverse methods rely on calculating the damage parameters, like crack length and location by using the frequency shift values. A comprehensive review of these methods is presented in [3]. Early attempts to detect damages by involving vibration-based methods are presented in [4-6]. More recent studies on detecting damages in beam-like structures were made in papers [7-9]. A successful detection of damages by measuring dynamic characteristics implies identifying small changes in modal parameters. In paper [10-12] a method to enhance the identification of natural frequencies is presented. In most literature the problem of detecting transversal cracks in trusses and beam-like structures is addressed. In more recent papers, complex damages have been taken into consideration too. These are delamination (longitudinal) cracks in composite structures [13,14]. In article [15], a method for determining the dimensions of L-shaped and inverted T-shaped cracks based on frequency measurements by using both direct and inverse methods is presented. Paper [16] describes a method of detecting the presence of cracks with different branch orientations in pipes by using the rotational spring model.
In papers [17-22] we present studies made to find the effects of complex shaped damages, meaning L and T shaped cracks by means of the finite element and analytical methods. L-shaped cracks may occur in composite materials and are described as transversal cracks followed by a delamination oriented at a straight angle. In the current paper we aim to determine the accuracy of predicting the effects of an L-shaped crack on the natural frequency shift of a cantilever beam, by comparing the results obtained using FEM analysis with values obtained with the help of a stiffness reduction algorithm embedded in the Python language.

2. THEORETICAL BACKGROUND

The structure studied is a cantilever beam with its main dimensions illustrated in figure 1.

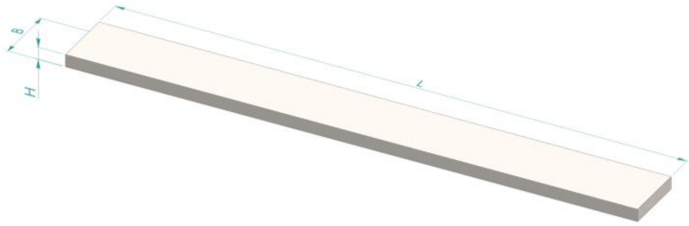


Figure 1: The undamaged cantilever beam

The cantilevers dimensions values as well as its physical properties are presented in table 1.

Table 1: Main dimensions and physical properties of the studied beam geometry

Length L [mm]	Width B [mm]	Thick. H [mm]	Mass density [kg/m3]	Young modulus [N/m2]	Poisson ratio [-]	Tensile strength [MPa]	Yield strength [MPa]	Min. elongation [%]
1000	50	5	7850	$2 \cdot 10^{11}$	0.3	470-630	355	20

The damages considered in the current paper are described as two L-shaped cracks, one with the delamination component oriented to the right indicated as L_R (figure 2) and the second oriented to the left indicated as L_L (figure 3).

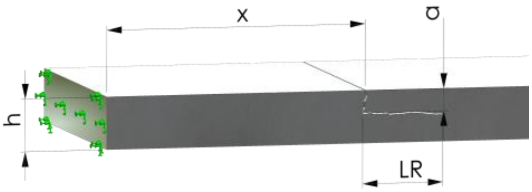


Figure 2: L-shaped crack oriented to the right

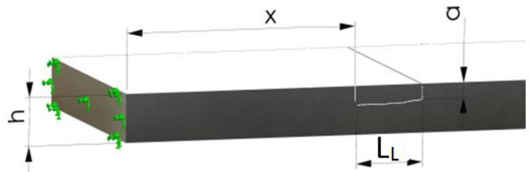


Figure 3: L-shaped crack oriented to the left

The positions and dimensions of the damages are presented in table 2.

Table 2: L-shaped crack main dimension and orientation

Damage location x [mm]	Damage interval a-b [mm]	Delamination length L_R or L_L	Damage depth a [mm]
300	300-350	50	1
350	350-400		
400	400-450		
280	280-230		
330	330-380		

In order to test the reliability of the stiffness reduction method we compare the results obtained in the developed software with the natural frequencies obtained via ANSYS simulation software for the cantilever beam affected by the previously described damages.
The Python embedded algorithm works by reducing the stiffness for the segment *a-b* of the cantilever, where the damage is presumed to be, but maintaining in the same time a constant mass of the beam, as shown in figure 4.

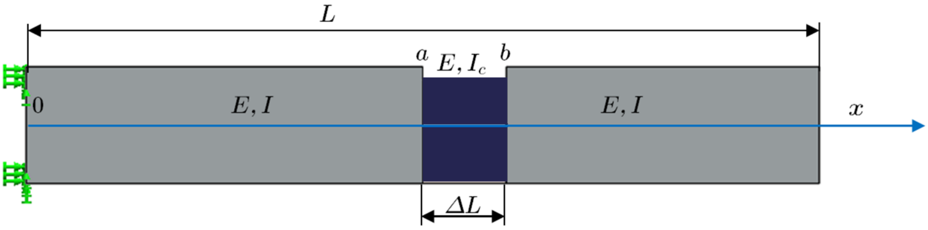


Figure 4: The cantilever beam with reduced section *a-b*

For the analyzed cantilever beam the boundary conditions have been set and the dimensionless wave numbers for the first six modes of transversal vibration are given by:

$$\cos(\alpha L)\cosh(\alpha L) + 1 = 0 \tag{1}$$

where α is the dimensionless wave number.

By determining the dimensionless wave numbers $\alpha L = \lambda$, for the cantilever beam, we can obtain the natural frequencies of the undamaged beam by applying relation:

$$f_i = \frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI_y}{mL^3}} \tag{2}$$

The relation for calculating the normalized modal forms is:

$$\bar{\phi}_i(x) = 0.5\left\{\frac{\cos\alpha_i L + \cosh\alpha_i L}{\sin\alpha_i L + \sinh\alpha_i L}\left[\sin(\alpha_i x) - \sinh(\alpha_i x)\right] - \cos(\alpha_i x) + \cosh(\alpha_i x)\right\} \tag{3}$$

The normalized modal curvature is determined with the help of relation:

$$\bar{\phi}_i''(x) = 0.5\left\{-\frac{\cos\alpha_i L + \cosh\alpha_i L}{\sin\alpha_i L + \sinh\alpha_i L}\left[\sin(\alpha_i x) + \sinh(\alpha_i x)\right] + \cos(\alpha_i x) + \cosh(\alpha_i x)\right\} \tag{4}$$

The equivalent mass of the beam m_{ech-i} positioned at the free end is given by the relation:

$$m_{ech-i} = \bar{m}L\int_0^L [\bar{\phi}_i(x)]^2\,dx = \varphi_i^{0-L}\bar{m}L \tag{5}$$

$$\int_0^L [\bar{\phi}_i(x)]^2\,dx = \varphi_i^{0-L} = 0.25 \tag{6}$$

From relations 2 and 5, the natural frequency of the undamaged beam is given by relation:

$$f_i = \frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI_y}{\bar{m}LL^3}} = \frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI_y}{4m_{ech-i}L^3}} \tag{7}$$

The coefficient of the strain energy for any vibration mode *i*, for the segment *a-b*, is given by the relation:

$$\kappa_i^{a-b} = \int_a^b \left[\bar{\phi}_i''(x)\right]^2 dx \tag{8}$$

From relation 8, we can derive for any vibration mode:

$$\kappa_i^{0-L} = \int_0^L \left[\bar{\phi}_i''(x)\right]^2 dx = 0,25 \tag{9}$$

Knowing the relation between the natural frequencies and strain energy is $f_i \approx \sqrt{U_i}$, the frequency for the beam with reduced cross-section becomes:

$$f_{Ci} = f_i\sqrt{1 - 4\kappa_i^{a-b}\frac{I - I_C}{I}} \tag{10}$$

Relation (10) can be expressed as:

$$f_{Ci} = f_i\sqrt{\frac{\kappa_i^{0-a} + \frac{I_C}{I}\kappa_i^{a-b} + \kappa_i^{b-L}}{\kappa_i^{0-L}}} \tag{11}$$

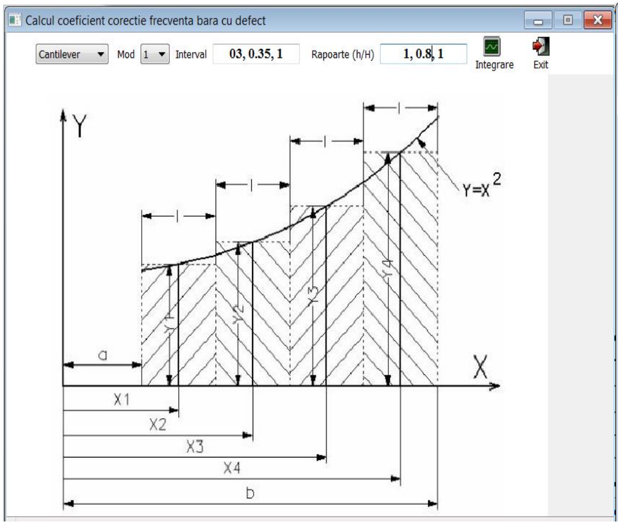
3. THE DEVELOPED SOFTWARE

The software developed for frequency estimation, nominated INTEGRATION and presented in figure 5, because it involves an integration of the strain energy of the part of the healthy beam, as well as the segment with stiffness reduction, which substitutes the damage positioned at a specific location. For conducting of calculations, the program requires the following steps, by entering the necessary parameters, as illustrated:

a) Selection of the beam type, options are: cantilever, double-clamped, free-free and simply supported;
b) Selection of the mode number. The program is capable of calculating the coefficients for the first ten modes of transversal vibrations;
c) Setting the position and length of the reduced segment extremities;
d) Setting the ratio between the height of the healthy beam and the height for the segment with reduced stiffness.

As we can observe in figure 5, the program calculates the frequency correction coefficient κ_i^{a-b} for mode number 1 of vibration for a cantilever beam having a presumed damage of 1 mm depth starting from location *a*=300 mm extending to *b*=350 mm.

By deducing the area *a-b* for the reduced stiffness segment and by integration of the function of the normalized modal curvature given in relation (4), the program plots the modal curvature of the beam for the selected mode of vibration and highlights the area where the modal curvature is disrupted by the stiffness reduction. In the same time using relations (8) and (9), the frequency correction coefficients are calculated. In figure 6 we illustrate the third mode of transversal vibration.



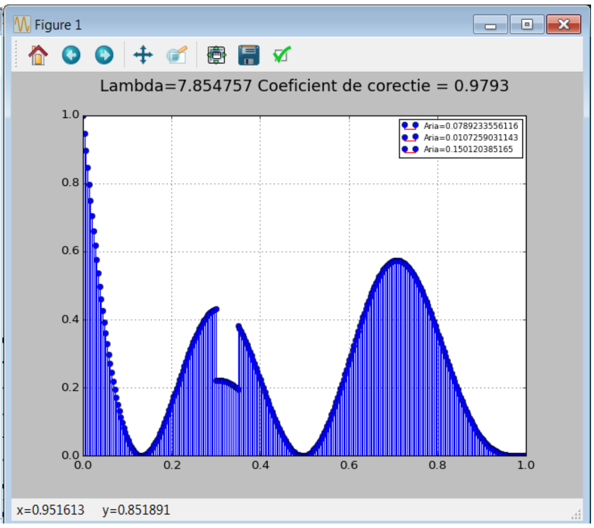


Figure 5: Interface of the developed program

Figure 6: The interface of the curvature obtained

The plotted normalized modal curvature defines the stiffness reduction section in the selected location, and by integration of the area the frequency correction coefficient is given.

By knowing the undamaged beams natural frequencies and applying relation 11 with the help of the frequency correction coefficient, the beam's natural frequencies values can be calculated.

4. RESULTS AND DISCUSSION

In order to prove the model is reliable and in consequence the INTEGRATION algorithm applicable, after calculating the natural frequencies for all damage cases presented in table 2, we compared the obtained frequency values with those given by the ANSYS software for the cantilever having the L-shaped cracks presented in figures 2 and 3. The errors obtained are presented in table 3 for the L_R crack, and in table 4 for the L_L crack.

Table 3: Errors obtained for the cantilever beam having an L-shaped crack oriented to the right

Mode no. [-]	Damage interval 300-350 [%]	Damage interval 350-400 [%]	Damage interval 400-450 [%]	Damage interval 280-330 [%]	Damage interval 330-380 [%]
1	1.37	1.07	0.82	1.49	1.19
2	0.62	1.13	1.60	0.43	0.92
3	-1.59	1.20	0.58	1.64	1.40
4	0.30	0.13	0.97	0.61	0.07
5	1.14	2.58	2.08	0.46	2.19
6	10.95	8.77	7.33	11.10	9.56

Table 4: Errors obtained for the cantilever beam having an L-shaped crack oriented to the left

Mode no. [-]	Damage interval 300-350	Damage interval 350-400	Damage interval 400-450	Damage interval 280-330	Damage interval 330-380
1	1.37	1.07	0.82	1.49	1.19
2	0.62	1.13	1.59	0.43	0.92
3	1.59	1.20	0.58	1.64	1.40
4	0.30	0.13	0.97	0.61	0.07
5	1.14	2.57	2.08	0.45	2.19
6	10.95	8.53	7.35	11.08	9.53

As we can observe from tables 3 and 4, except for mode of vibration 6, the maximum error obtained is 2.58% for the damaged interval 350-400 mm.

The large error occurs for mode six because the cantilever reaches at this mode a shape as presented in figure 7. This reflects the vibration of the small element of the L-shaped crack, and that natural frequency is improper to predict the cantilever beam's natural frequency for the sixth transverse mode.

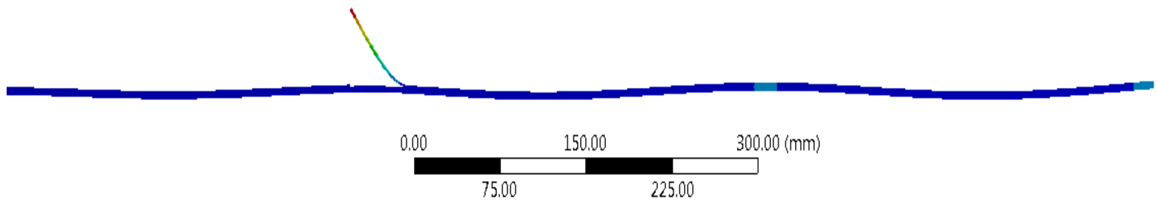


Figure 7: Mode shape for the transverse vibration mode six of the cantilever beam

Furthermore, in order to decrease the errors presented in tables 3 and 4 between the calculated frequencies using the described algorithm and the frequencies obtained by simulation, an empirically developed method can be applied. By including the supplementary occurred bending moments at the damage extremities which are proportional with the squared normalized modal curvature, we can calculate the correction coefficients k_i^a and k_i^b which we multiply with the obtained frequency correction coefficient k_i^{a-b} to give more accurate frequency estimation. These are:

$$k_i^a = 1 - s\left[\bar{\phi}_i''(a)\right]^2 \tag{12}$$

$$k_i^b = 1 - s\left[\bar{\phi}_i''(b)\right]^2 \tag{13}$$

where s is the severity of the crack. For a crack of depth a=1 mm we found the severity s=0.02 empirically, by applying a procedure described in [23]. In further study we aim to determine an algorithm to find these severities for different damage depths.

The proposed coefficient is:

$$k_S = k_i^a \cdot k_i^{a-b} \cdot k_i^b \tag{14}$$

which considers the stiffness loss due to cross-section reduction between points a and b respectively the supplementary bending moments at the damage extremities. Now, the frequencies can be predicted using a model introduced herein that involves the coefficient k_S instead of the root square in relation (12). Tests made involving this model predicted the frequencies with a maximum error of 0.83% for the3 first five modes, as shown in table 5, which is a considerable improvement of the model that only involves the stiffness reduction.

Table 5: Errors obtained for the cantilever beam having an L-shaped crack

Mode no. [-]	Damage interval 300-350 [%]		Damage interval 350-400 [%]		Damage interval 400-450 [%]		Damage interval 280-330 [%]		Damage interval 330-380 [%]	
	L_R	L_L	L_R	L_L	L_R	L_L	L_R	L_L	L_R	L_L
1	0.11	0.11	0.09	0.09	0.07	0.07	0.11	0.11	0.10	0.11
2	0.03	0.03	0.02	0.02	0.02	0.02	0.04	0.04	0.03	0.03
3	0.05	0.05	0.02	0.02	0.01	0.01	0.04	0.04	0.04	0.04
4	0.01	0.01	0.09	0.09	0.01	0.01	0.02	0.02	0.08	0.08
5	0.28	0.28	0.83	0.81	0.66	0.66	0.02	0.02	0.70	0.6
6	9.14	9.14	7.96	7.72	6.86	6.87	9.33	9.30	8.47	8.24

Therefore, in order to obtain a more precise prediction of the natural frequencies of beams with complex cracks we propose a model which takes into account the supplemental strain energy at the damage extremities.

5. CONCLUSION

The paper proposes a model which makes a link between the effect of the stiffness loss and the natural frequency changes for beams with complex-shaped cracks that introduces correction coefficient k_S.. First, an actual model that involves the stiffness loss coefficient k_i^{a-b} developed previously by us for corrosion damage was tested for complex shaped-cracks. We found that for complex shaped-cracks, meaning L_R and L_L cracks, the error is up to 2.58%. Because we observed important slope at the damage ends, we concluded that in order to have a more precise prediction of the natural frequencies of beams with complex cracks we need to take into account this reality. So, we introduced two coefficients which consider the supplemental equivalent bending moments at the crack extremities (proportional with the local curvature) and a damage severity *s* found empirically in respect with the cross-section change. This model shows a maximum error of 0.83% which can be further decreased by analytically finding the damage severity for the specified cracks.

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