# MATHEMATICAL MODEL TO ANALYZE THE INFLUENCE OF THE POSITION OF THE SUPPORT WHEEL AND THE VIRTUAL POINT ON THE STABILITY AND DYNAMICS OF THE TILLAGE MACHINERY

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***Abstract:*** *The tractor, together with the agricultural machinery back-mounted, forms one of the most important and used agricultural tillage for working the soil. Of these, the plows occupy the most important position, being some of the largest energy users. On the tillage machinery acts forces and torques having time varying sizes and directions. With the help of mathematical models different adjustments of the working and connecting elements can be studied, so that the forces and torques acting on them are minimized. If they have adequate values, the tillage machinery will work stable at optimum agro technical and energy parameters.*
***Keywords:*** *tillage machinery, plow, forces, virtual point*

## 1. GENERAL CONSIDERATIONS

Autumn is the period when in agriculture the soil is prepared for autumn crops or those that will follow in the spring. Although lately minimal tillage works are being used to reduce costs, plowing has remained the main autumnal tillage work. Therefore, consumption of energy, fuels, lubricants and spare parts at this time are the highest.
Tractor with agricultural machinery back-mounted form one of the most important and used agricultural soil working units. These aggregates are large consumers of energy and material resources and, therefore, both before and during operation must be made precise adjustments.
A complex of forces and torques with varying directions and sizes over time acts on the aggregates. The existence and position of the support wheel but also the position of the virtual point (CIR) greatly influence the stability of a ploughing machinery[1]. With the aid of mathematical models, can be studied for the various settings, the variation in the forces, torques that act, the position of virtual point of rotation (CIRv, CIRh), but also show the stability of the overall unit. As the adjusting elements allow the modification of the kinematic parameters of the agricultural machines, and the soils as working environment are not homogeneous, in order to analyze how they influence the dynamic characteristics of the agricultural aggregates, it is necessary to make mathematical models that allow the simulation of the operating conditions.

## 2. THEORETICAL CONSIDERATIONS

An tillage unit consisting of a wheeled tractor and a plow with support wheel are considered to operate at variable speed on a surface inclined at an angle α to the horizontal. The external forces acting in a vertical-longitudinal plane on the plowing tillage are shown in Figure 1, which represents the equivalent dynamic model of the tractor-plow assembly.
The plow is equipped with a support wheel and is coupled to the three-point suspension mechanism with floating mode operation. The suspension mechanism does not have automatic adjustment systems enabled. The plow has the working depth of a and the working width of a moldboard plow is *b* (the total working width of the plow is *B* = 3*b*). The support wheel can occupy a variable position of length $l_r$ and the virtual rotation point in vertical plane (CIRv) is located at the point of intersection of the axis of the central (compression) link with the plane of the draft links.
*External forces* of the dynamic model in Figure 1 have the following meanings:
$G_m = mg$ is the weight of the plow, placed in the weight center, which includes the weight of all the component elements (frame, troughs, disk knife, support wheel);
$F_{im}$ - the inertia force acting in the weight center of the plow;

$F_x$ , $F_z$ - the components of the plow strength;
$F_1$ - the force acting at the coupler head of the plow frame with the draft link;
$F_2$ - the force acting at the coupler head of the plow frame with the compression link;
$F_{cx}$ , $F_{cz}$ - the cutting force components of the cutting blade;
$F_{rx}$ - rolling resistance of the support wheel;
$F_{rz}$ - the normal reaction on the ground surface of the support wheel;
$M_{rr}$ - torque of rolling resistance of the support wheel;
$M_{cr}$ - torque of rolling resistance of the disk knife.

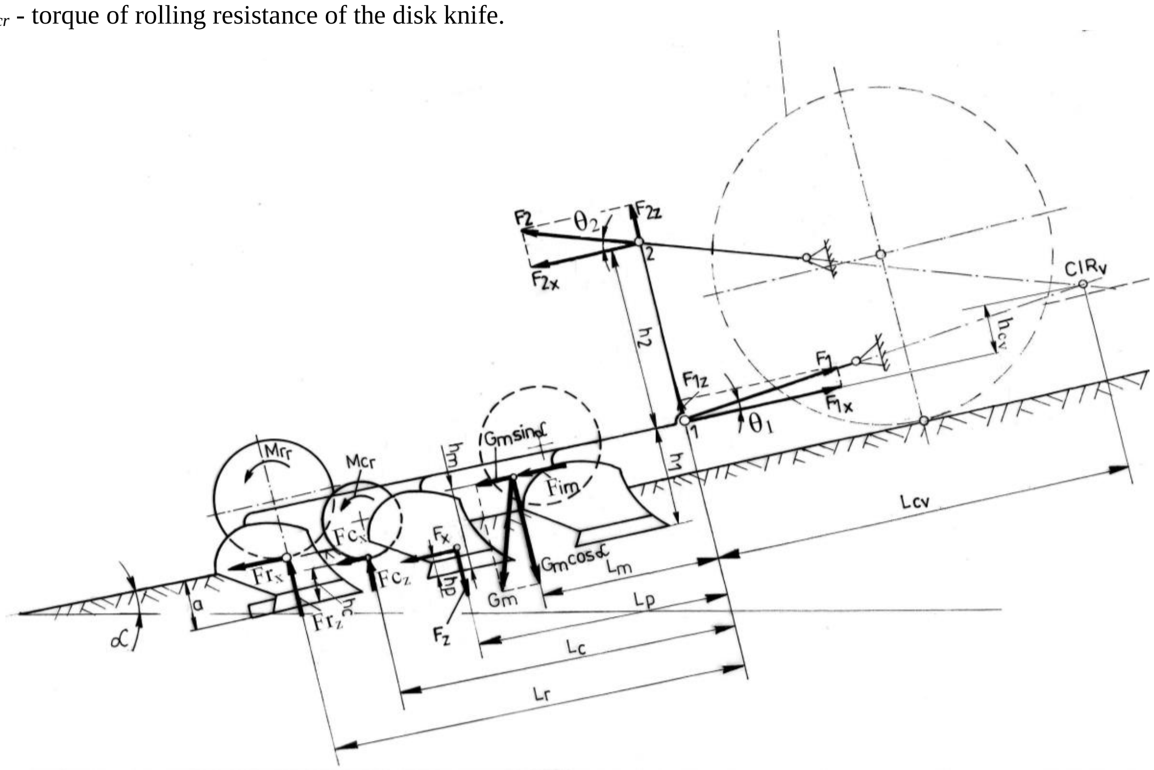

Figure. 1. The diagram of the forces that on the back-mounted plow with a support wheel in vertical-longitudinal plane on the plow when moving with variable speed on inclined surface [3].

*The geometrical and positioning elements* in the diagram in figure 1 are based on the reference point 1 of the plow frame coupling with the draft link and the coupling point 2 of the plow frame with the compression link;
For the component elements of the plow, the plane defined by the active edge of the coulters (fixed plane) was chosen as a reference surface, this being parallel to the surface of the soil against which it can have a variable position.
The determination of the forces acting at the coupling points 1 ($F_{1x}$ and $F_{1z}$) and 2 ($F_{2x}$ and $F_{2z}$) of the three point linkage, is made from the equilibrium condition of the system of figure 1. Thus, the following system of equations results [3]:

$$F_{1x} - F_{2x} - Fi_m - G_m \cdot sin\alpha - F_x - Fc_x - Fr_x = 0$$
$$F_{1z} + F_{2z} - G_m \cdot cos\alpha - F_z + Fc_z + Fr_z = 0 \quad (1)$$
$$Fr_z \cdot l_r + Fr_x \cdot (h_1 - a) + Fc_z \cdot l_c + Fc_x \cdot (h_1 - h_c) + F_x \cdot (h_1 - h_p) - F_z \cdot l_p + Fi_m \cdot (h_1 - h_m) - G_m \cdot l_m \cdot cos\alpha - F_{2x} \cdot h_2 - Mr_r - Mc_r = 0$$

The total plow resistance force is the result of the resistance forces of the three plow moldboards reduced at the level of the middle moldboard plow. It is divided into two components: the force $F_x$ parallel to the axis OX (parallel to the soil surface) and the force $F_z$ which is parallel to the axis OZ (perpendicular to the soil surface).
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The component $F_x$ parallel to the soil surface of the plowing force represents the tensile strength of the plow, determined by the relation (5.5) and the component $F_z$ perpendicular to the soil surface of the plowing force is determined by the relation, $F_z = F_x \cdot tg\beta$, where $\beta$ is the angle between the two components of the plowing force. For normal working conditions it is considered that $F_z = (0,2 \ldots 0,3) \cdot F_x$.
The force acting on the disk knife is decomposed into two components: parallel to the soil surface $F_{cx}$ and perpendicular to the ground surface $F_{cz}$, the dependence between the two components being given by the relation

$F_{cx} = F_{cz} \cdot f_c$ (where $f_c$ is the rolling resistance coefficient of the disk knife ). The moment of rolling resistance of the disk knife is determined by the relation: $M_{cr} = F_{cz} \cdot r \cdot cos\alpha$ where $r$, is the radius of the disk knife and $\alpha$, is the angle of the resultant force.
The component $F_{rx}$ is the rolling resistance of the support wheel and is given by the relation $Fr_x = Fr_z \cdot f_r$ (where $Fr_z$ is the load on the support wheel and $f_r$ is the rolling resistance coefficient of the support wheel. The support wheel torque is given by the relation: $M_{rr} = F_{rx} \cdot r = F_{rz} \cdot f_r \cdot r$
The components of the forces $F_1$ and $F_2$ acting at the head link points 1 and 2 of the three point linkage within plow frame are given depending to each other, by the connecting relations: $F_{1z} = F_{1x} \cdot tg\theta_1$ respectively, $F_{2z} = F_{2x} \cdot tg\theta_2$, where $\theta_1$ and $\theta_2$ are the angles formed by the draft link and the compression link against the soil surface.
Taking into account the above, the equations in system (1) become:

$$
\begin{aligned}
&F_{1x} - F_{2x} - Fi_m - G_m \cdot sin\alpha - F_x - Fc_z \cdot f_c - Fr_z \cdot f_r = 0 \\
&F_{1x} \cdot tg\theta_1 + F_{2x} \cdot tg\theta_2 - G_m \cdot cos\alpha - F_x \cdot tg\beta + Fc_z + Fr_z = 0 \qquad (2) \\
&Fr_z \cdot [l_r + f_r \cdot (h_1 - a)] + Z_c \cdot [l_c + f_c \cdot (h_1 - h_c)] + F_x \cdot (h_1 - h_p - l_p \cdot tg\beta) + G_m \cdot [(h_1 - h_m) sin\alpha - \\
&- l_m cos\alpha] + Fi_m \cdot (h_1 - h_m) - F_{2x} \cdot h_2 - Mr_r - Mc_r = 0
\end{aligned}
$$

Since the number of constants that make up the free term of each equation of the system (2) is large, they are grouped and noted as follows [3]:

$$
\begin{aligned}
&\Phi = Fi_m + G_m \cdot sin\alpha + F_x + Fc_z \cdot f_c ; \\
&\Gamma = G_m \cdot cos\alpha + F_x \cdot tg\beta - Fc_z ; \qquad (3) \\
&\Psi = Fc_z \cdot [l_c + f_c \cdot (h_1 - h_c)] + F_x \cdot (h_1 - h_p - l_p \cdot tg\beta) + G_m \cdot [(h_1 - h_m) sin\alpha - l_m cos\alpha] + Fi_m \cdot (h_1 - h_m) - \\
&- (Mr_r + Mc_r)
\end{aligned}
$$

By replacing the constants with the above notations, the system (2) becomes:

$$
\begin{aligned}
&F_{1x} - F_{2x} - Fr_z \cdot f_r - \boldsymbol{\Phi} = 0 \\
&F_{1x} \cdot tg\theta_1 + F_{2x} \cdot tg\theta_2 + Fr_z - \boldsymbol{\Gamma} = 0 \qquad (4) \\
&-F_{2x} \cdot h_2 + Fr_z \cdot [l_r + f_r \cdot (h_1 - a)] + \boldsymbol{\Psi} = 0
\end{aligned}
$$

Since the number of equations equals the number of unknowns, the system (2) is compatible with a unique solution, if the kinematic elements are considered constant. The three forces considered unknown which have variable values are given by the relations [3]:

$$
F_{1x} = \frac{\Phi \cdot \{h_2 + [l_r + f_r \cdot (h_1 - a)] \cdot tg\theta_2\} + \Gamma \cdot [l_r + f_r \cdot (h_1 - a + h_2)] + \Psi \cdot (1 - f_r \cdot tg\theta_2)}{h_2 \cdot (1 + f_r \cdot tg\theta_1) + (tg\theta_1 + tg\theta_2) \cdot [l_r + f_r \cdot (h_1 - a)]}
$$

$$
F_{2x} = \frac{\Psi \cdot (1 + f_r \cdot tg\theta_1) + (\Gamma - \Phi \cdot tg\theta_1) \cdot [l_r + f_r \cdot (h_1 - a)]}{h_2 \cdot (1 + f_r \cdot tg\theta_1) + (tg\theta_1 + tg\theta_2) \cdot [l_r + f_r \cdot (h_1 - a)]} \qquad (5)
$$

$$
Fr_z = \frac{h_2 \cdot (\Gamma - \Phi \cdot tg\theta_1) - \Psi \cdot (tg\theta_1 + tg\theta_2)}{h_2 \cdot (1 + f_r \cdot tg\theta_1) + (tg\theta_1 + tg\theta_2) \cdot [l_r + f_r \cdot (h_1 - a)]}
$$

The depth of work - $a$, the specific resistance of soil to the plow - $k$, the angle of the soil surface - $\alpha$ and the angle between the support of the forces $F_x$ and $F_z$ have variable values. Therefore, to simplify the system, some simplifying assumptions are used:
• soil work is homogeneous, having specific resistance to plowing and constant slope;
• the tractor speed is constant;
• the position of the support wheel is fixed during plowing;
• the positions of the head point of the three point linkage are fixed, only changing the inclination angle from the vertical plane with the working depth;
The only variable parameter is the working depth, which determines the change of the angle of the compression and draft links and of the virtual rotation point (CIRv) position.
Variable parameters that influence the forces acting on the plow are presented in table 1[3].

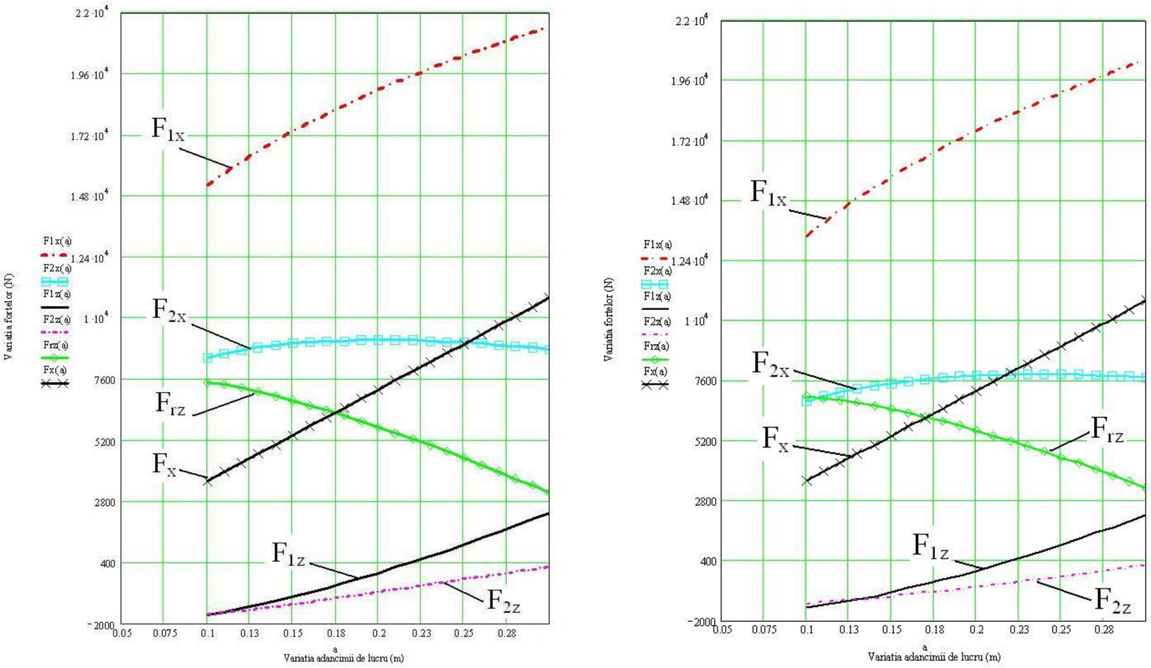
**Table 1: Values of variable influence parameters**

| Nr. crt | **Parameter** | **Value ranges or fixed values** | | |
|---|---|---|---|---|
| 1 | Depth work, $a$ [$cm$] | 0…30 | | |
| 2 | Specific resistance of soil to the plow, $k$ [$kN/m^2$] | 20…90 | | |
| 3 | Angle of the soil surface, α [°] | 0…12 | | |
| 4 | Angle between the support of the forces $F_x$ and $F_z$, $\beta$ [°] | -12…+12 | | |
| 5 | Support wheel position, $l_r$ [$mm$] | 410…1100 | | |
| 6 | Three point linkage distance, $l_{12}$ [$mm$] | 460…610 | | |
| 7 | Compression link position, $z_3$ [$mm$] | 395 | 435 | 475 |

In order to simulate and determine the variation of the forces acting on the plow in Figure 1, a Mathcad Professional Edition program was used with parameters whose values are known. The simulation was performed in order to analyze the forces acting on the plowing unit according to all the variable parameters.

## 3. DETERMINATION OF FORCE VARIATION OF THE CONNECTING ELEMENTS AND THE FORCES ACTING ON THE SUPPORT WHEEL

Figure 2 shows the variations of the forces acting on the back-mounted plow with a support wheel, depending on the working depth, when the distance between the head link points and thus the CIRv position is changed as follows: $h_2$ = 520*mm* (fig.1a) and $h_2$ = 590 *mm* (fig.2b)




a b

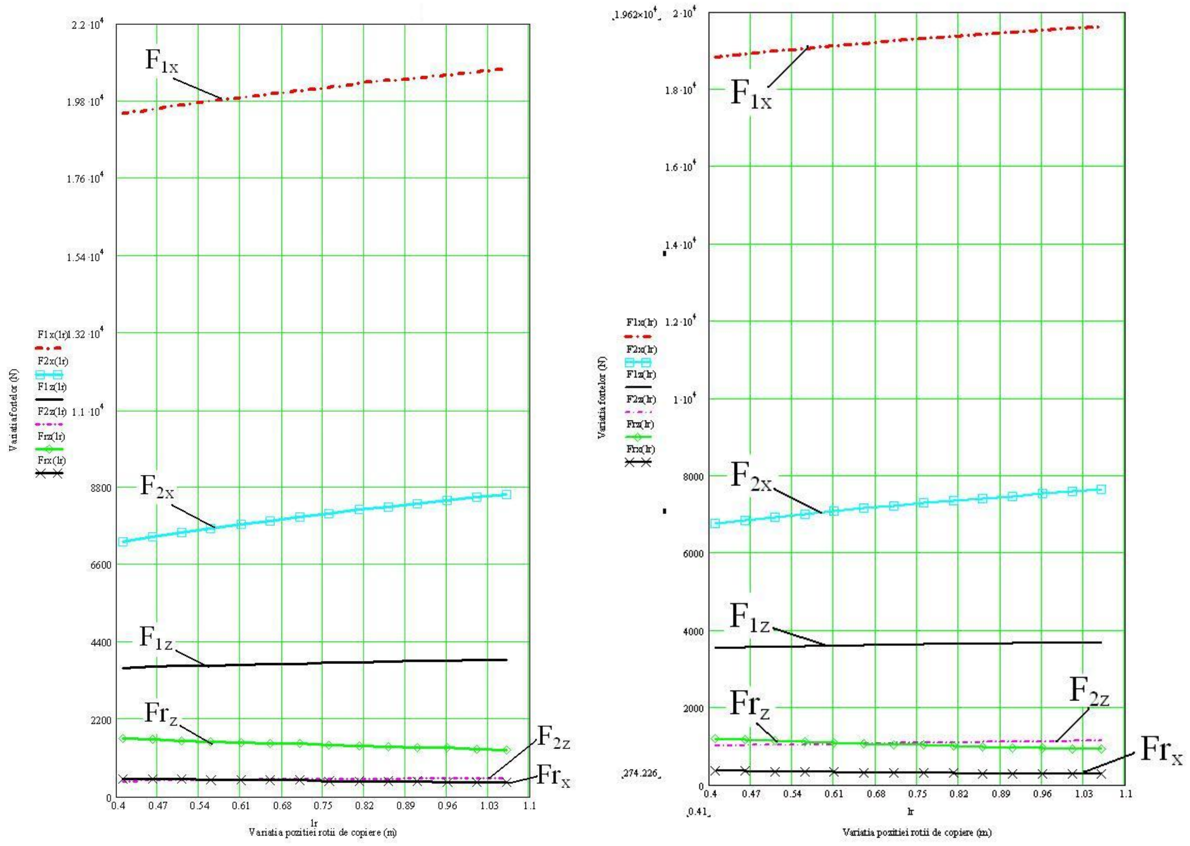
**Figure 2**. Variation of the forces acting on the back-mounted plow according to the working depth, a = (10…30) cm and for the two values of the distance between head links , $l_{12}$ = 510mm (a) and $l_{12}$ = 590mm (b). Constant values of related parameter: k = 40000N/m$^2$; β = 12º; α= 0º; $z_3$ = 435mm [3]

### 3.1 Consequences:

- $F_x$ has a linear variation in both cases in the range (3600… 10800) *N* depending on the working depth, according to the definition relation;
- $F_{1x}$ and $F_{1z}$ have an increasing variation, but increasing the distance between the head links and towards displacement of the CIRv, causes the decrease of the value of $F_{1x}$.
- $F_{2x}$ has an increasing variation for depths up to 20 cm, after which, its value decreases so that it has approximately equal values for depths in the range 13 ... 30 cm and $F_{2z}$ increases linearly with working depth;
- The $Fr_z$ reaction on the support wheel, has a different evolution from the other forces. It decreases with increasing working depth due to the increase of components $F_{1z}$ and $F_{2z}$ which tend to lift the plow from the furrow, which reduces the load on the support wheel.
- *The increase of the distance between the head-links (the displacement towards the CIRv) leads to the decrease of the load on the support wheel, especially for working depths less than 18 cm.*

In figure 3. the variations of the forces acting on the back-mounted plow with a support wheel are presented in comparison with the position of the support wheel on the plow frame, if the position of the compression link on the body of the tractor is changed as follows: $z_3$ = 395*mm* (fig.3a) and $z_3$ = 475 *mm* (fig.3b)

In case of changing the position of the support wheel, the variation of the forces acting on the plow, looks as follows:




a b

**Figure 3**. Variation of the forces acting on the back-mounted plow according to the position of the support wheel for $l_r$ = (410… 1100) mm and for $z_3$= 395mm (a) respectively $z_3$ = 475 mm (b). Fixed values of the connection parameters: a = 30 cm; k = 40000N/m$^2$; β = 12°; α= 0°; $l_{12}$ = 510 mm[3]

## CONCLUSION

- The forces $F_{1x}$ and $F_{2x}$ which act in parallel with the direction of travel, have an increasing variation as the position of the support wheel moves to the rear of the plow, even if the working depth remains constant. This is an advantage in case of the use of force sensors mounted on the clamping bolts of the draft links on the body of the tractor, integrated in an automatic SRA adjustment system of force;
- The vertical components $F_{1z}$ and $F_{2z}$ of the forces acting at the coupled head have a constant variation but the change of the position of the coupling point of the compression link on the body of the tractor $z_3$ from 395 to 475 mm (from the lower position to the upper position), causes the movement to the front of the virtual point (CIRv) and the increase of $F_{2z}$ value from 459 *N* to 1100 *N* (average values).
- The two components of the force acting on the support wheel $Fr_x$ and $Fr_z$ have a slightly decreasing variation which shows that, by moving the support wheel towards the back of the plow, the rolling resistance force $Fr_x$ decreases with positive consequences for the total resistance to plowing draught.

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