THE MATHEMATICAL MODEL OF THE SIMULATION OF THE DOPPLER EFFECT

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Abstract: *As mathematically suggested, a kinematic system is technically required to reduce the emission power of the generator module and correlatively an increase in the emission bandwidth, where the kinematic system works. Thus, functional increments of the RADAR-type kinematic system can be achieved in relation to the acuity of the system itself without sacrificing the internal emission power of the emitter module by the use of specific compression protocols.*

Keywords: *doppler, kinematic, system, wave, electromagnetic*

1. INTRODUCTION

The Doppler effect is applied to determine the vector of momentary displacement velocity of a volume in space by calculating the known ratio between the incident and reflected frequencies of an electromagnetic wave (EM). The applicability in the present case is the case of an initial electromagnetic (EM) wave emitted by a transmitter and received by a receiver, both being coordinated by the same computing system (duplexer) which in turn is connected to a central computer system. This complex system analyzes the frequency difference of the incident electromagnetic (EM) wave with the reflected one, difference with which it will be possible to determine the speed of movement of the volume under investigation [4], figure 1.

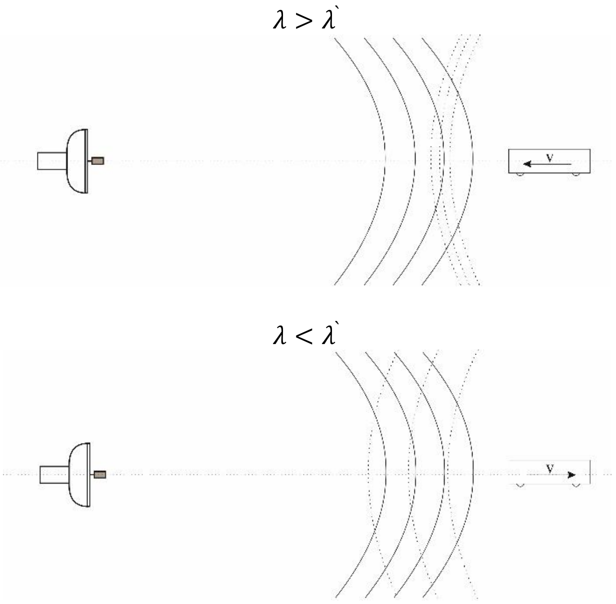


Figure 1: The speed of movement

The general scheme of operation of the kinematic system using the Doppler effect is shown below [1, 6], figure 2.

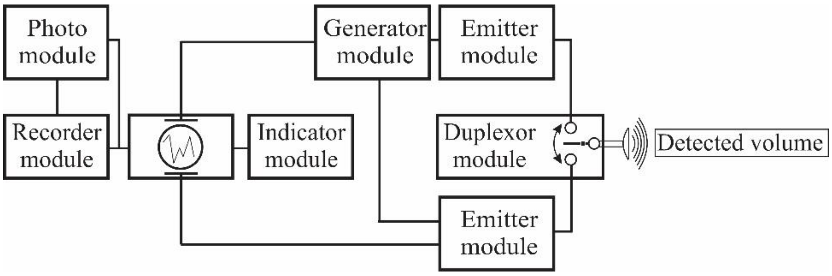


Figure 2: Operation mode of the kinematic system

We note that the duplexer ensures the synchronization of the transmitting frequency λ of the transmitter module with that of the receiver module reception.

Any volumetric object will have a geometrical characteristic of reflecting the electromagnetic (EM) waves with which they are bombarded. Therefore a radar system using the Doppler effect will process these frequency differences λ in order to extract information regarding the speed of movement of the volume subject to observation, which can be positive or negative, this principle can provide information of the type; speed, position, distance, angular position of the subject volume studied [2, 3].

We also iterate that modern radars can use pulsed or continuous electromagnetic (EM) waves to obtain measurements using the Doppler effect [1, 5, 7].

2. THE RANGE OF ACTION OF THE RADAR SYSTEM

From the schematic representation of the operation of a radar-type kinematic system using the Doppler effect [6], figure 2, its operating mode follows:

a. The generator module has the task of generating an EM (pulsed or continuous) synchronized electromagnetic wave necessary for the operation of the entire system.

b. This electromagnetic wave (EM) will be transmitted to the transmitter module which in turn transmits it to the radar system antenna through the duplexer module.

c. The duplexer module performs the synchronization of the emitted electromagnetic waves (EM) as well as of the received ones, thus realizing the dual role of the unique antenna of the radar system.

d. The receiver module is the module that, when receiving the transmitted electromagnetic wave (EM) analyzes it, and analyzes the possible deviations of the frequency of the received electromagnetic wave (EM) thus doing the analysis and transforming this information into the required value of the system, respectively the speed of displacement of the volume studied.

e. The indicator module is the module that has the role of indicating the results of these synchronized measurements and implicitly to graphically or numerically reproduce the displacement speed of the studied volume.

To determine the range of a kinematic system using the Doppler effect we will apply the general relation (1):

$$R = \frac{c\Delta t}{2} \qquad (1)$$

Where R represents the radius of action of the system, Δt represents the effective time in which a single OPO pulse (one ping only) travels the distance from the transmitter module to the receiver module, the EM electromagnetic pulse having travel speeds $c = 2{,}998 \times 10^8$ m/s.

Graphically representing the initial EM electromagnetic wave emitted by the emitter module as well as the received EM electromagnetic wave received by the receiver module we will observe with the help of an oscilloscope the graphical differences as shown in figure 3.

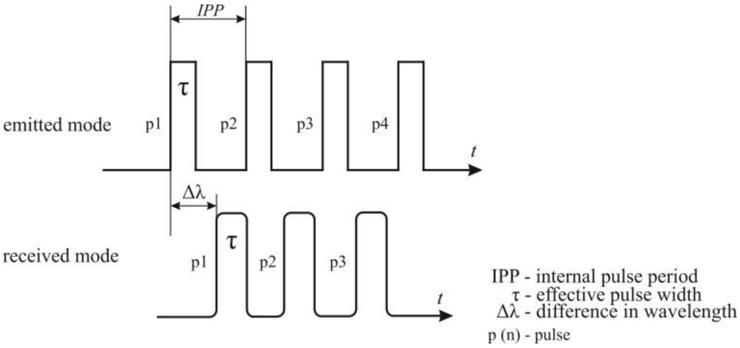


Figure 3: Wave emitted by the emitter module and received by the receive

Thereby, one can graphically observe the difference Δλ of the received electromagnetic wave (EM) frequency with respect to the emitted electromagnetic wave (EM) frequency. It is precisely this difference Δλ interpreted mathematically represents the speed of movement of the studied volume.

In general, the *IPP* (internal generated pulse of the system) is a constant and its value is known both by the transmitter module and the receiver module, so the difference is determinable.

As far as we are concerned, in the experiment carried out, the optimum range of action of the radar system must be related to the ideal angle of the main lobe of the electromagnetic wave (EM) within the band *Ka*, which has an angle identified by 8^0, figure 4.

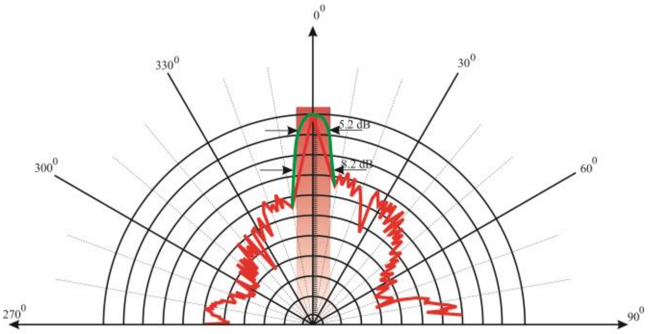


Figure 4: The main lobe of the electromagnetic wave *Ka*

Thus, having the topographic elements of the proposed route, we will identify trigonometrically the ideal distance at which the actual measurements will be made. We reproduce these elements in figure 5, from which results the distance "*d*" at which the measurements will actually be made. In order to preserve the specificity of the determinations made practically by the operator of the cinemometer, we will keep the human element of the system of determination, so the errors arising from the measurements will have the specific characteristic encountered in practice.

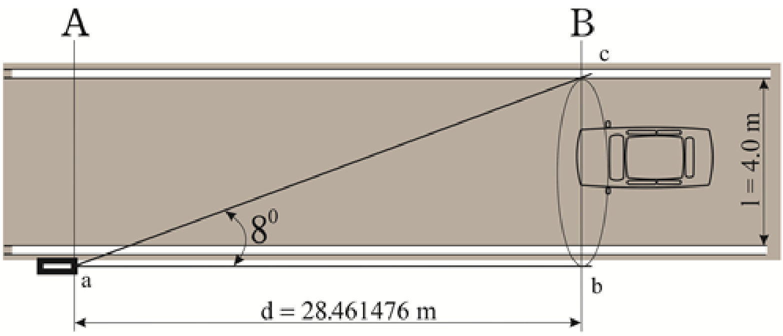


Figure 5: Optimum action range of the radar system

The determination of the optimum distance of detection of the kinematic system "*d*" was done by applying the trigonometric formula ctg related to the topographic data imposed by the experiment, thus the distance resulting by applying the following equation (2):

$$\text{ctg}\, 8^0 = \frac{d}{l} = \frac{d}{4m} \tag{2}$$

Resulting (3):

$$d = 4m\ x\ ctg\ 8^0 = 4m\ x\ 7.115396 = 28.641476\ \text{m} \tag{3}$$

During each PRI (pulse repetition interval), the transmitter module of the radar system emits an electromagnetic wawe (EM) wave of an internally generated frequency known only for a period of τ seconds, representing the effective pulse width, following for the receiver module to receive the frequency of the echo of the signal emitted by the studied volume by referring it to the PRI initially generated and known by the system, and the duplexer system analyzing the detected differences will process mathematically, resulting in the vector of the displacement speed of the studied volume.

The corresponding distance between two successive echoes reflected represents the ambiguity of the radar system *Ru*. This ambiguity can be represented by figure 6:

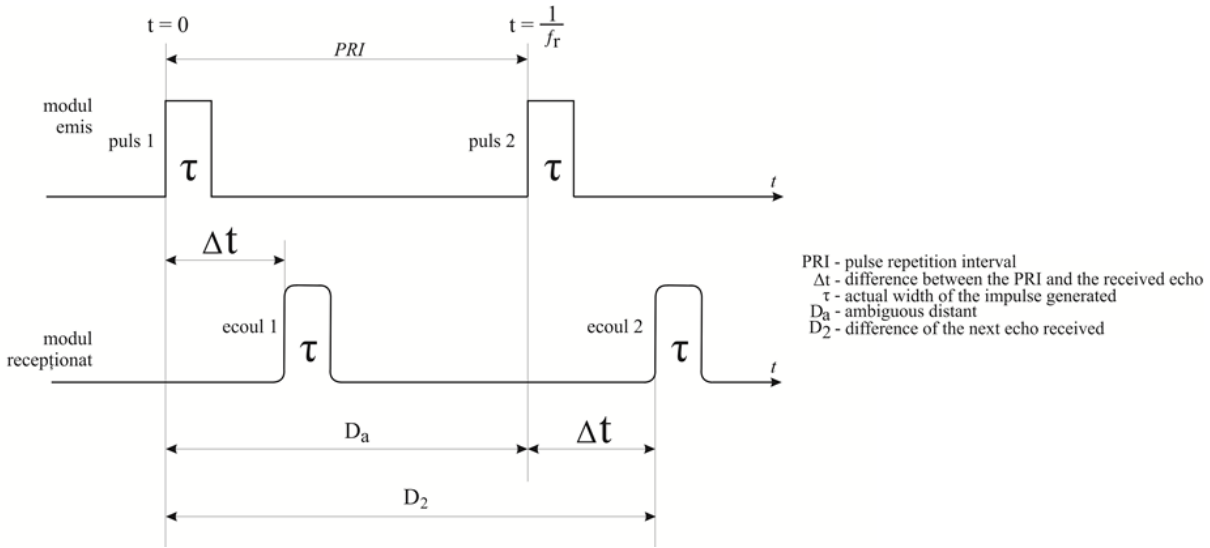
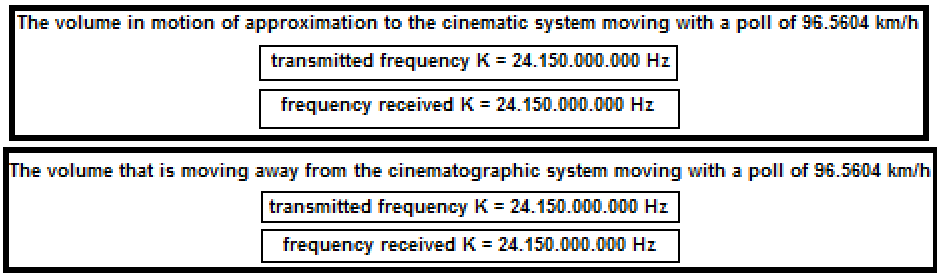


Figure 6: The ambiguity of the Doppler system

Thus, we can identify a case of ambiguity of the Doppler kinemometric system studied as being the possibility of the system to receive and record a value other than the one desired by identifying the second echo which of course does not reflect the actual measurement of the proposed velocity vector.

From the measurements made available by the manufacturer of the PYTHON SERIES II kinemometer by MPH INDUSTRIES INC. with which we have made the measurements, the values with which the echo of the initial electromagnetic wave (EM) emitted are given, depending on the direction of the velocity vector of the volume under study, figure 7.



The volume in motion of approximation to the cinematic system moving with a poll of 96.5604 km/h

transmitted frequency K = 24.150.000.000 Hz

frequency received K = 24.150.000.000 Hz

The volume that is moving away from the cinematographic system moving with a poll of 96.5604 km/h

transmitted frequency K = 24.150.000.000 Hz

frequency received K = 24.150.000.000 Hz

Figure 7: The received frequency values

Thus, we can observe what is the frequency difference (λ) received by the kinematic system depending on the direction of the velocity vector of the volume under study.

3. ACCURACY OF THE CINEMATOGRAPHIC SYSTEM

The accuracy of the kinematic system used can be defined as the ability of the kinematic system to determine the speed of movement of the volume studied without the results being distorted by other random elements that may occur during a measurement [2, 3, 4].

The accuracy of the cinematographic system, defined as ΔR, is the ability of the system to detect the speed of movement of the target volume, within a certain tolerance that is legally acceptable, compared to other volumes close to the target volume.

The cinematographic system used has the ability to operate within the parameters described above between certain optimal distances represented by the difference between the maximum distance ($Rmax$) and the minimum distance (R_{min}). This optimal distance is in turn divided into submultiple distances N each having a value ΔR as can be seen from the relation (4):

$$N = \frac{(R_{max} - R_{min})}{\Delta R} \qquad (4)$$

Thus, the volumes subjected to the cinematographic measurements that, in their displacement are at a distance greater than or equal to ΔR will be able to be identified individually by the used cinematographic system, correlatively all the volumes that will not be included in this value will not be able to be identified individually , this representing an ambiguity of the II^{nd} degree of the used cinematographic system. However, using specialized mathematical protocols of the Doppler-based kinematic system, and these velocities of displacement of these volumes within ambiguity of rank II can be identified individually [1, 5, 6].

Considering two volumes (the target volume - subject to the cinematographic measurements - and a volume in its immediate vicinity), R_1 and R_2 respectively having the time delay correspondent determined by the difference between (t_1) and (t_2), it follows that, the accuracy ΔR, of Ist rank of the kinematic system, it can be defined as the ability of the kinemometric system based on the Doppler effect, to detect the echo of the pulse width generated (τ) by the generator module of the R_2 volume different from the volume intended for the proposed kinemometric determination, respectively R_1 thus obtaining- there is an ambiguity of the proposed kinemometric determination, relation (5).

$$\Delta R = R_2 - R_1 = c\,\frac{(t_2 - t_1)}{2} = c\,\frac{\Delta t}{2} \qquad (5)$$

The relation (5) indicates the difference between the distances of two successive volumes moving successively in the same direction and in the same sense subjected to the cinematographic measurements.

Analyzing mathematically this type of ambiguity of the 2nd degree we will identify the elements taken into account taking into account the hypothesis described above., figure 6.

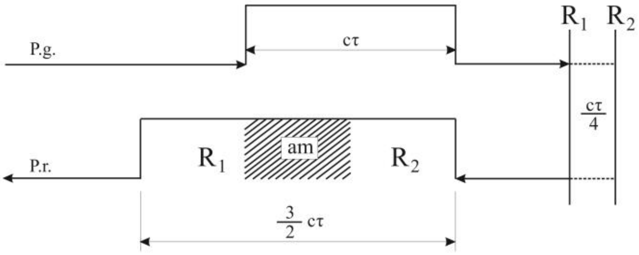


Figure 8: 2nd degree ambiguity

We thus identify the following distinct elements; P.g. and P.r. being the pulse generated and the pulse received by the cinematographic system respectively, R_1 and R_2 being the volume subject to the kinemometric detection and the volume in its immediate vicinity respectively.

Considering that, the width of the generated pulse has a value of cτ, in the present hypothesis figure 8, the distance between the two subject volumes of the R_2 - R_1 kinematic determination will have a value of $\frac{c\tau}{4}$, where τ represents the width of the pulse generated by the internally generated module.

In this case, when the pulse generated by the generator module is reflected by volume R_2 (echo wave) it will travel an additional distance that will have a value of $\frac{c\tau}{4}$ value which, together with the value of the initiated pulse generated cτ, will generate a deviation of $\frac{3}{2}c\tau$, value that represents the 2nd rank ambiguity of the kinematic system. There is also a limiting evidence of 2nd rank ambiguity.

Considering the case where the distance between the two volumes R_2 - R_1 will have a value of at least $\frac{c\tau}{2}$ In this case, the pulse emitted by the transmitter module (τ) will be reflected by the subject volume of the R_1 kinematic measurements, and at the moment when the pulse emitted by the emitter mode will be reflected by the R_2 volume, two distinct determinable kinematic measurements will be observed, as it results from figure 9.

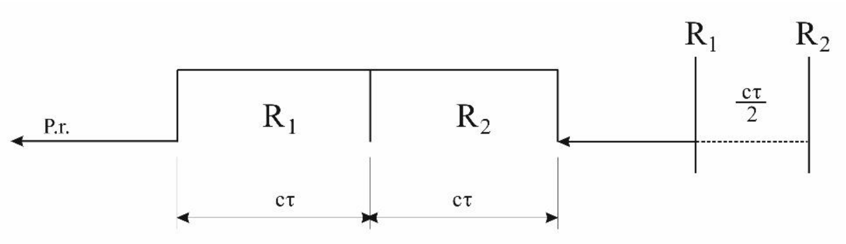


Figure 9: 2nd rank ambiguity

Therefore, in this probative case, the distance between the two volumes subjected to the kinematic determination will require a reflection of two distinct kinematic determinations according to the equation (6):

$$\Delta R = \frac{\Delta\tau}{2} = \frac{c}{2B} \quad (6)$$

Where the notion of the emitted bandwidth of the RADAR type (B) kinematic system appears and has a value of $B = \frac{1}{\tau}$.

This is the reason why, in general, manufacturers of RADAR type cinematographic systems try to reduce the distance of ambiguity ΔR in order to obtain determinations of a higher accuracy of the produced cinematographic systems.

4. CONCLUSION

As can be suggested by equation (6), in order to obtain an increased acuity of RADAR (B) type kinematic systems, the width of the pulse generated by the generator module of the system must be reduced. In this situation, mathematically suggested, the kinematic system is technically required to reduce the emission power of the generator module and correlatively an increase in the emission bandwidth, in our case *Ka*, where the kinematic system works. Thus, functional increments of the RADAR-type kinematic system can be achieved in relation to the acuity of the system itself without sacrificing the internal emission power of the emitter module by the use of specific compression protocols.

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