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MODERN DIMENSIONAL ANALYSIS INVOLVED IN STATIC DISPLACEMENT'S EVALUATION

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Abstract: *The Modern Dimensional Analysis (MDA) method, introduced by Professor Thomas Szirtes, can be applied in many engineering problems. The authors illustrate the main advantages of this method in a static case: in the displacement's evaluation of a straight bar, loaded by a concentrate force, perpendicular-disposed to its longitudinal axis. Several variants based on different suppositions of the Dimensional Set were analyzed, mainly how one can toggle the main variables from the whole potentially involved variables, which have some influence on the analyzed phenomena. It is proved, that one can find such Dimensional Sets, in which the corresponding sets of the independent variables can assure dimensional similarity between the prototype and model without geometric similarity. It can represent one of the main advantages of the MDA method.*
Keywords: *Modern Dimensional Analysis (MDA), static case, displacement's evaluation, straight bar*

1. INTRODUCTION

In the engineering investigations the experimental approach is very important. In the case of the relatively large (or very small) structures, named *prototypes*, instead of the direct measurements (on the prototypes) are widely applied the results obtained on scaled structures (usually on the reduced ones), defined as *models*.
During the last century, several analytical approaches were developed, like Geometric Analogy, Theory of Similitude, as well as the Classical Dimensional Analysis, analyzed in detail in several representative contributions [1-6; 10; 11; 13; 14].
Unfortunately, all of them carry up several difficulties, especially on the finding the involved dimensionless variables in this process of the establishing correlations between the prototype's and the model's behaviours.
The author of the contributions [7; 8], professor Thomas Szirtes, developed a modern version of the Dimensional Analysis, named in the following as *Modern Dimensional Analysis* (*MDA*).
This new approach solves all of these difficulties and is able offering both the complete set of the dimensionless variables and an easy-, and unitary procedure.
In the following, some results of the authors' investigations, detailed in [12], will be briefly presented.

2. THE ANALYSED CASE

As the *prototype* (indexed with "1") one is considered a straight bar fixed on the end and free in the other, having length L, a rectangular cross-section (axb), loaded by a vertical force F, which is related to a tri-rectangular system of axis $xGyz$, presented in Figure 1. It is supposed that this beam has relatively large dimensions.
Also, one other smallest beam will be considered as the *model* (indexed with "2"); all measurements will be performed on this model.
The authors propose to establish, by *MDA* [7; 8], the most suitable Model Law, in order to predict the prototype's free end's vertical displacement „ v_1 " , based on the similar measurements performed on model (i.e.: v_2).

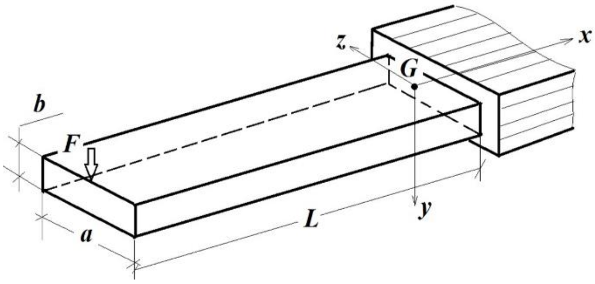

Figure 1: The schema of the cantilever beam

3. BRIEF EVALUATION OF DIFFERENT DIMENSIONAL SETS OF THE AUTHORS

How was mentioned before, the authors performed a detailed study of this case, described in [12]. As an illustration of the *MDA*'s efficiency, in the following will be presented some of them.

Taking into consideration the theoretical layouts of the method [7; 8], instead of a single variable for length, were adapted the lengths' separation along the main rectangular directions (z,y,x), i.e. m_x, m_y, m_z. Consequently, the number of the involved dimensions will be increased together with the diminishing of the number of the dimensionless variables π_j in the Model Law.

The version V2 from [12] will offer the following Dimensional Set, presented below:

Table 1: Version 2 (V2)

	B		A			
	V	E	L	F	a	b
m_x	0	1	1	0	0	0
m_y	1	-2	0	0	0	1
m_z	0	-1	0	0	1	0
N	0	1	0	1	0	0
π_1	1	0	0	0	0	-1
π_2	0	1	-1	-1	1	2
	D		C			

One has to mention that matrix *A* is constituted by the chosen independent variables' exponents (L, F, a, b) and matrix *B* contains the remained variables' exponents from their dimensional definitions.

The matrixes (*B*+*A*) are completed with matrix $C = -\left(A^{-1} \cdot B\right)^T$, as well as one adequate, *n*-ranked unit matrix $D \equiv I_{nxn}$; here n=2.

Consequently, the set of the dimensionless variables (π_1, π_2) will be obtained. In the next step, they will be equalized with "1", i.e. π_j=1, j=1,2. After than, will be expressed the corresponding dependent variable from them. As a final step, all of the involved variables ω will be substituted with their Scale Factors S_ω, obtaining finally the constitutive elements of the Model Law:

$$\pi 1 = \mathrm{v} \cdot b^{-1} = \frac{\mathrm{v}}{b} \rightarrow S_{\mathrm{v}} = S_b \tag{1}$$

$$\pi 2 = \frac{E \cdot a \cdot b^2}{L \cdot F} \rightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2 \rightarrow S_E = \frac{S_F \cdot S_L}{S_a \cdot (S_b)^2}$$

In this case, for the model can be chosen a priori: its length L_2, its loading force F_2, as well as its cross-sectional dimensions a_2xb_2. Of course, for the prototype all dimensions and its loading conditions are known, except its end's vertical displacement "v_1".

Due to the fact that the Young modulus *E* is in matrix *B*, which contents the dependent variables, the E_2 has to be determined by calculus using the second expression of the Model Law.

Performing measurements on the model, one will obtain the magnitude of "v_2", and after than, by applying the first expression of the Model Law, one will obtain by calculus the predictable magnitude of "v_1".

How was proved in [12], by changing the involved independent variables, similar calculi can be performed and the authors obtained several useful versions (V3, V4, V5, V6 and V7) with practically the same (equivalent) Model Laws.

In this sense, in version V7 the Young modulus *E* was considered as independent variable instead of "a":

Table 2: Version7 (V7)

	B		A			
	V	a	F	b	L	E
m_x	0	0	0	0	1	1
m_y	1	0	0	1	0	-2
m_z	0	1	0	0	0	-1
N	0	0	1	0	0	1
π_1	1	0	0	-1	0	0
π_2	0	1	-1	2	-1	1
	D		C			

The obtained Model Law presents practically the same correlations:

$$\pi 1 = \mathrm{v} \cdot b^{-1} = \frac{\mathrm{v}}{b} \rightarrow S_{\mathrm{v}} = S_b$$

$$\pi 2 = \mathrm{a} \cdot \frac{E \cdot b^2}{L \cdot F} \rightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2 \rightarrow S_a = \frac{S_F \cdot S_L}{S_E \cdot (S_b)^2}$$

Consequently, E_2 can be chosen a priori, but a_2 has to be obtained only by calculus (using the second expression of the Model Law).

Some other useful variants were elaborated by the authors in reference [12], where, by *MDA* can be eliminated the restriction of the geometric similarity. Consequently, it is not necessary to use a geometric similar model (with rectangular cross-section), only to respect the corresponding Scale Factors S_ω of the moment of inertia I_z.

Version V8, presented below, satisfies this requirement with the adequate Model Law:

Table 3: Version 8 (V8)

	B	A			
	V	F	I_{zG}	L	E
m_x	0	0	0	1	1
m_y	1	0	3	0	-2
m_z	0	0	1	0	-1
N	0	1	0	0	1
π_1	1	1	-1	1	-1
	D	C			

$$\pi 1 = \frac{V \cdot F \cdot L}{E \cdot I_z} \rightarrow S_V \cdot S_F \cdot S_L = S_E \cdot S_{I_z} \rightarrow S_V = \frac{S_E \cdot S_{I_z}}{S_F \cdot S_L}$$

In this case, all variables (not only for prototype but also for the model, too), excepting the pursued vertical displacement „ v”, can be chosen a priori and the model’s cross-section can has one other shape.

3. CONCLUSION

- The analyzed variants allow us to observe a large number of options concerning on the variables, which can be included in the Dimensional Set, as well as in the Model Law;
- Depending on the selected variables (both the independent and the dependent ones), one can obtain a diminishing of the constitutive elements of the Model Law, as well as a simplicity of the analysis;
- Even there are several expressions of the Model Law, finally they will offer the same requested „v_1” displacement for the prototype;
- In the authors’ opinion, the *MDA* method [7; 8], elaborated by Professor Thomas Szirtes, represents the most simple, advanced and unitary way in the prototype’s behaviour analysis via the model’s ones;
- Applying the *MDA* method, the number of the experimental investigations will be drastically diminished and simplified;
- All experimental measurements will be performed exclusively on the model (which represents an easier manner, in comparison with the prototype’s ones), offering also a significant diminishing of the involved costs;
- The version V8 offers the great facility of a good dimensional similarity between the prototype and model without a geometric similarity, because I_z was an independent variable, that one can choose freely;
- Consequently, in this case (V8) the cross-section can be of any type (circular, tubular, rectangular or any other) with the single constrain of the equality of their I_z magnitudes (numerical values).

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