



## DEFINING AND GEOMETRICAL MODELING OF A NEW FUNCTIONAL CHARACTERISTIC FOR BUCKET EXCAVATORS

Ioan Trif<sup>1</sup>, Ioan Szava<sup>2</sup>, Costel Bejan<sup>2</sup>

<sup>1</sup>SC 3F Impex Srl, Brasov, Romania, [ioan.trif@unitbv.ro](mailto:ioan.trif@unitbv.ro);

<sup>2</sup>Transilvania University of Brasov, Romania, [iet@unitbv.ro](mailto:iet@unitbv.ro); [cvbejan@yahoo.com](mailto:cvbejan@yahoo.com)

**Abstract:** The purpose of this paper-work is the definition and mathematical modelling of the surface described by the contact area between bucket and arm, during operation. The metric characteristics of the coupling area result from the kinematics of excavator's actuation mechanisms, however the surface described by the contact area becomes a functional feature by pairing the stress torsor of each point. The need to introduce this new functional feature is a direct consequence of the need to develop accessory coupling mechanisms on the machine arm. In the case of a coupling force generated through vacuum enclosures, the coupling area definition in the specified meaning, becomes essential for the adoption of this coupling mode, which is innovative in relation to the currently used mechanical or hydraulic couplings. Further, the authors deal with the geometric modelling of the contact surface between bucket and arm. This first step is necessary in the whole concept based on the kinematic study of the motion produced by actuating the three cylinders moving the excavator's arm. The shape and size of the contact zone are the result of modelling.

### 1. INTRODUCTION

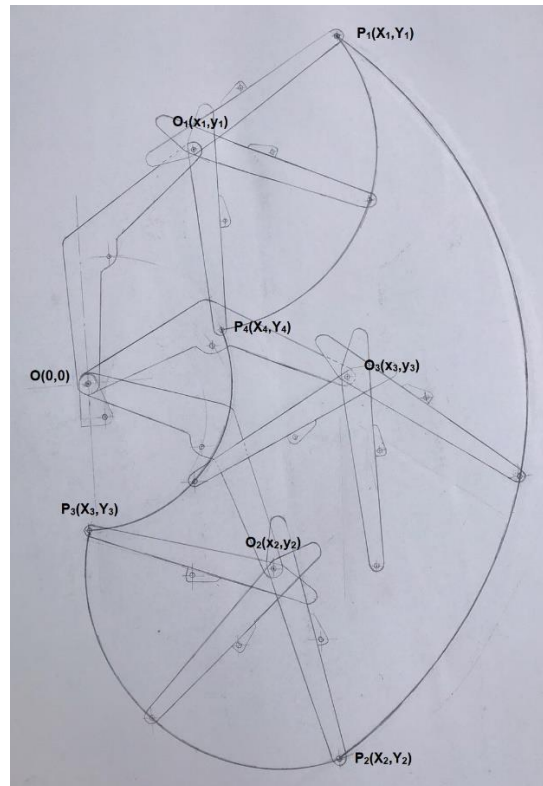
The components of excavator's arm mechanism are joints with cylindrical hinge type couplings, respectively bolt mountings. These elements are put in motion by hydraulic actuator cylinders. In order, the first one end articulated element on excavator's chassis in point O, is the boom. On the other end of the boom, in point O<sub>1</sub>, is articulated one of the arm extremities. On the other extremity, there is the joint point P between arm and excavator's bucket, joint point which is the subject of authors study (Figure 1). The excavators increase in performance is measured by the easiness, safety and rapidity of accessories changing and mounting process, like dipper buckets, grading buckets, grabs, rippers, hydraulic hammers, shearing buckets, crushers, etc. This process can be streamlined by using an accessory coupling system on the arm. The work will continue to refer to the arm-bucket joint because, according to the authors' experience, the excavation bucket stresses to the maximum this joint point.

### 2. CONSIDERATIONS ON THE SURFACE DESCRIBED BY THE ARM-BUCKET JOINT

The surface generated by point P is completely determined by two characteristics: the geometry and the stress torsor. The sizing and verification calculations under real operating conditions require deep knowledge on geometric elements and torsor components in each point of the surface in which an arm-bucket joint is located.

The area generated by point P may be assimilated to the functional characteristics of bucket excavators under the following conditions;

- Its geometry is known, i.e. a function  $\phi_1(X_i, Y_i) = 0$ , in which  $X_i, Y_i$  are the coordinates of P point;
- It is known the stress torsor in each point of the surface, i.e. a function  $\phi_2(F_i, M_i) = 0$  where  $F_i$  and  $M_i$  respectively, represent the forces and moments acting in points  $P_i(X_i, Y_i)$ , determined by the variable force acting at the top of bucket teeth.



**Figure1** - Geometric generation of a surface described by arm-bucket joint

Further, for the coordinates, the following notations were adopted:

- in lowercase letters, the coordinates representing the joints of the boom on the chassis and of the arm on the boom, thus obtaining the points  $O(0,0)$  and  $O_1(x, y)$ , with  $x$  and  $y$  variables. The point  $O$  has two limit positions: the upper extreme position  $O_1(x_1, y_1)$  and the bottom extreme position  $O_1(x_2, y_2)$

-in capital letters, the coordinates of the mobile point  $P$ , mobile for its function in the frame of the mechanism, thus  $P_i(X_i, Y_i)$

With these considerations, it follows that the area determined by point  $P$  movement, is a functional feature of type:

$$\varphi(X_i, Y_i, F_i, M_i) = 0$$

The knowledge of this functional feature is necessary and sufficient to the correct adoption of any equalizer-bucket coupling mechanism, since the functionality of this mechanism is completely determined by the above function.

### 3. DELIMITATION OF THE SURFACE GENERATED BY THE ARM -BUCKET JOINT

For the delimitation of the surface generated by the arm-bucket joint, four phases of movement of the entire assembly shall be considered, the assembly consisting of the boom, the arm and the two motion cylinders.

The boom is fixed with one of its extremities on excavator's chassis, through the joint in point  $O$ . On the other extremity, respectively in point  $O_1$ , is the joint linking the boom to the arm. Considering that the cylinder that operates the boom is in a fully open position and the cylinder that operates the arm is in a completely closed position, it is obtained the highest point which the arm-bucket joint can reach, respectively the point  $P_1(X_1, Y_1)$ .

In a first phase, the arm's actuating cylinder is kept closed and the boom's actuating cylinder is operated. The arm extremity is moving from point  $P_1(X_1, Y_1)$  to point  $P_2(X_2, Y_2)$ , generating the quadrant  $P_1, P_2$  with the centre in point  $O(0,0)$ .

In the second phase, the boom's actuating cylinder is maintained completely closed, and the arm's actuating cylinder is operated. Through the action of this cylinder, the extremity at which the arm-bucket joint is located will move from point  $P_2(X_2, Y_2)$  to point  $P_3(X_3, Y_3)$  thus generating the quadrant  $P_2, P_3$  with the centre in point  $O_2(x_2, y_2)$ .

In the third phase, the arm's actuator cylinder is kept in a fully open position and is started the opening of the boom's actuator cylinder. In this way the equalizer-bucket joint will move from point  $P_3(X_3, Y_3)$  to point  $P_4(X_4, Y_4)$  on the  $P_3, P_4$  quadrant, with the centre in point  $O(0,0)$ .

Finally, in the fourth phase, the boom's actuator cylinder is maintained fully opened and the arm's actuator cylinder is opened. This way, the arm- bucket joint will move from point  $P_4(X_4, Y_4)$  to point  $P_1(X_1, Y_1)$  describing the  $P_4, P_1$  quadrant with the centre in point  $O_1(x_1, y_1)$  and thus, the surface generated by the arm-bucket joint is ended.

The aforementioned phases are illustrated in Figure 2. Based on the data resulting from this image, the following equations of curves (quadrants) wick delimitating the surface described by the arm-bucket joint, can be obtained, i.e.

$$\varphi_1(X_i, Y_i) = 0$$

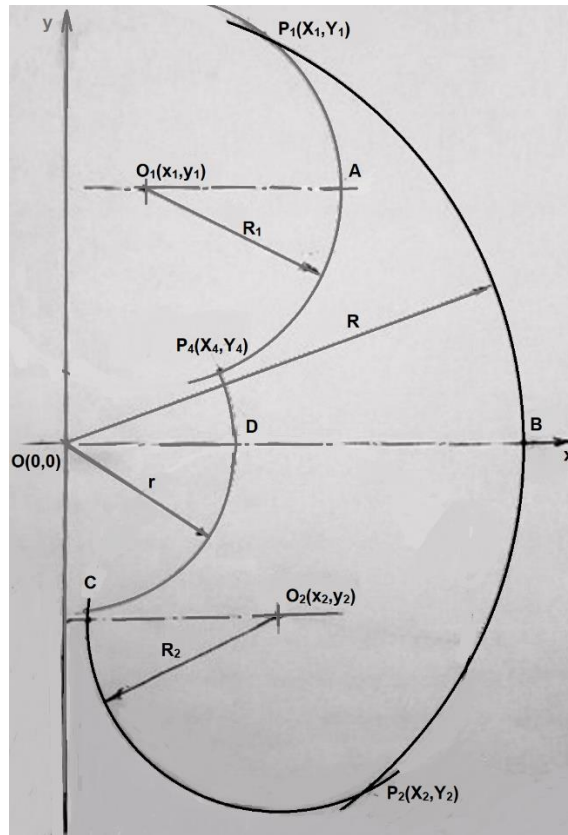


Figure 2-The area described by the arm - bucket joint

a.) For the circle centred in  $O_1$  and with radius  $R_1$ :

$$(x-x_1)^2 + (y-y_1)^2 = R_1^2 \quad \text{or} \quad y = y_1 \pm \sqrt{R_1^2 - (x-x_1)^2} \quad (1)$$

the equations for the quadrants  $P_1A$ , respectively  $AP_4$ , are as follows:

$$f_1(x) = y_1 + \sqrt{R_1^2 - (x-x_1)^2} \quad (2)$$

$$f_2(x) = y_1 - \sqrt{R_1^2 - (x-x_1)^2} \quad (3)$$

b.) For the circle having the centre in  $O(0,0)$  and the radius  $R$

$$x^2 + y^2 = R^2 \quad \text{or} \quad y = \pm \sqrt{R^2 - x^2}$$

the equations of quadrants  $P_1B$ , respectively  $BP_2$ , are as follows:

$$f_3(x) = \sqrt{R^2 - x^2} \quad (4)$$

$$f_4(x) = -\sqrt{R^2 - x^2} \quad (5)$$

c.) For the circle having the centre in  $O_2(x_2, y_2)$  and the radius  $R_2$ :

$$(x - x_2)^2 + (y - y_2)^2 = R_2^2 \text{ or } y = y_2 \pm \sqrt{R_2^2 - (x - x_1)^2}$$

the equations of quadrants P<sub>2</sub>C, respectively CP<sub>3</sub>, are the following:

$$f_5(x) = y_2 - \sqrt{R_2^2 - (x - x_2)^2} \quad (6)$$

$$f_6(x) = y_2 + \sqrt{R_2^2 - (x - x_2)^2} \quad (7)$$

d.) For the circle having the centre in O and the radius r:

$$x^2 + y^2 = r^2 \text{ or } y = \pm \sqrt{r^2 - x^2}$$

the equations of quadrants P<sub>2</sub>C, respectively CP<sub>3</sub>, are as follows:

$$f_7(x) = -\sqrt{r^2 - x^2} \quad (8)$$

$$f_8(x) = \sqrt{r^2 - x^2} \quad (9)$$

The functions f<sub>1</sub>(x), f<sub>2</sub>(x), ..., f<sub>8</sub>(x) represent the boundaries of the surface generated by the equalizer – bucket joint

The coordinates of points P<sub>1</sub>, P<sub>2</sub>, P<sub>3</sub>, P<sub>4</sub> are obtained by solving the equation systems below:

$$(S1) \quad P_1(X_1, Y_1) : P_1A : f_1(x) = y_1 + \sqrt{R_1^2 - (x - x_1)^2} \\ : P_1B : f_3(x) = \sqrt{R^2 - x^2}$$

$$(S2) \quad P_2(X_2, Y_2) : P_2C : f_5(x) = y_2 - \sqrt{R_2^2 - (x - x_2)^2} \\ : P_2B : f_4(x) = -\sqrt{R^2 - x^2}$$

$$(S3) \quad P_3(X_3, Y_3) : P_3C : f_6(x) = y_2 + \sqrt{R_2^2 - (x - x_2)^2} \\ : P_3D : f_7(x) = -\sqrt{r^2 - x^2}$$

$$(S4) \quad P_4(X_4, Y_4) : P_4A : f_2(x) = y_1 - \sqrt{R_1^2 - (x - x_1)^2} \\ : P_4D : f_8(x) = \sqrt{r^2 - x^2}$$

By solving the S1 system, it follows:

$$X_1 = \frac{x_1}{2} \left( \frac{R^2 - R_1^2}{x_1^2 + y_1^2} + 1 \right) + \frac{y_1}{2} \sqrt{\left( \frac{2R}{\sqrt{x_1^2 + y_1^2}} \right)^2 - \left( \frac{R^2 - R_1^2}{x_1^2 + y_1^2} + 1 \right)^2}$$

But  $x_{12} + y_{12} = OO_{12}$ , thus:

$$X_1 = \frac{x_1}{2} \left( \frac{R^2 - R_1^2}{OO_1} + 1 \right) + \frac{y_1}{2} \sqrt{\left( \frac{2R}{OO_1} \right)^2 - \left( \frac{R^2 - R_1^2}{OO_1} + 1 \right)^2}$$

R, R<sub>1</sub>, OO<sub>1</sub> are dimensional characteristics (lengths) of the drive mechanism elements.

If we note:  $k_1 = \frac{1}{2} \left( \frac{R^2 - R_1^2}{OO_1} + 1 \right)$ ;  $k_2 = \frac{1}{2} \sqrt{\left( \frac{2R}{OO_1} \right)^2 - \left( \frac{R^2 - R_1^2}{OO_1} + 1 \right)^2}$ , with  $k_2 = \sqrt{\left( \frac{2R}{OO_1} \right)^2 - k_1^2}$ , the final result is:

$$X_1 = k_1 x_1 + k_2 y_1 \quad (10)$$

$$Y_1 = \sqrt{R^2 - X_1^2} \quad (11)$$

By solving the S2 system, the result is:

$$X_2 = \frac{x_2}{2} \left( \frac{R^2 - R_2^2}{x_2^2 + y_2^2} + 1 \right) + \frac{y_2}{2} \sqrt{\left( \frac{2R}{\sqrt{x_2^2 + y_2^2}} \right)^2 - \left( \frac{R^2 - R_2^2}{x_2^2 + y_2^2} + 1 \right)^2}$$

But  $x_{22} + y_{22} = OO_{22}$ , therefore:

$$X_2 = \frac{x_2}{2} \left( \frac{R^2 - R_2^2}{OO_2} + 1 \right) + \frac{y_2}{2} \sqrt{\left( \frac{2R}{OO_2} \right)^2 - \left( \frac{R^2 - R_2^2}{OO_2} + 1 \right)^2}$$

If we note  $k_3 = \frac{1}{2} \left( \frac{R^2 - R_2^2}{OO_2} + 1 \right)$ ;  $k_4 = \frac{1}{2} \sqrt{\left( \frac{2R}{OO_2} \right)^2 - \left( \frac{R^2 - R_2^2}{OO_2} + 1 \right)^2}$  with  $k_4 = \sqrt{\left( \frac{2R}{OO_2} \right)^2 - k_3^2}$ , the final result is:

$$X_2 = k_3 x_2 + k_4 y_2 \quad (12)$$

$$Y_2 = \sqrt{R^2 - X_2^2} \quad (13)$$

By solving the S3 system, the result is:

$$X_3 = \frac{x_2}{2} \left( \frac{r^2 - R_2^2}{x_2^2 + y_2^2} + 1 \right) - \frac{y_2}{2} \sqrt{\left( \frac{2r}{\sqrt{x_2^2 + y_2^2}} \right)^2 - \left( \frac{r^2 - R_2^2}{x_2^2 + y_2^2} + 1 \right)^2}$$

But  $x_{22} + y_{22} = OO_{22}$ , thus:

$$X_3 = \frac{x_2}{2} \left( \frac{r^2 - R_2^2}{OO_2} + 1 \right) - \frac{y_2}{2} \sqrt{\left( \frac{2r}{OO_2} \right)^2 - \left( \frac{r^2 - R_2^2}{OO_2} + 1 \right)^2}$$

If we note  $k_5 = \frac{1}{2} \left( \frac{r^2 - R_2^2}{OO_2} + 1 \right)$ ;  $k_6 = \frac{1}{2} \sqrt{\left( \frac{2r}{OO_2} \right)^2 - \left( \frac{r^2 - R_2^2}{OO_2} + 1 \right)^2}$  and we have  $k_6 = \sqrt{\left( \frac{2r}{OO_2} \right)^2 - k_5^2}$ , the final result will be:

$$X_3 = k_5 x_2 + k_6 y_2 \quad (14)$$

$$Y_3 = \sqrt{r^2 - X_3^2} \quad (15)$$

By solving the S4 system, it will result:

$$X_4 = \frac{x_1}{2} \left( \frac{r^2 - R_1^2}{x_1^2 + y_1^2} + 1 \right) + \frac{y_1}{2} \sqrt{\left( \frac{2r}{\sqrt{x_1^2 + y_1^2}} \right)^2 - \left( \frac{r^2 - R_1^2}{x_1^2 + y_1^2} + 1 \right)^2}$$

But  $x_{12} + y_{12} = OO_{12}$ , thus :

$$X_4 = \frac{x_1}{2} \left( \frac{r^2 - R_1^2}{OO_1} + 1 \right) + \frac{y_1}{2} \sqrt{\left( \frac{2r}{OO_1} \right)^2 - \left( \frac{r^2 - R_1^2}{OO_1} + 1 \right)^2}$$

If we note:  $k_7 = \frac{1}{2} \left( \frac{r^2 - R_1^2}{OO_1} + 1 \right)$ ;  $k_8 = \frac{1}{2} \sqrt{\left( \frac{2r}{OO_1} \right)^2 - \left( \frac{r^2 - R_1^2}{OO_1} + 1 \right)^2}$  with  $k_8 = \sqrt{\left( \frac{2r}{OO_1} \right)^2 - k_7^2}$ , the final result will be :

$$X_4 = k_7 x_1 + k_8 y_1 \quad (16)$$

$$Y_4 = \sqrt{r^2 - X_4^2} \quad (17)$$

3.) Remarks on Pi limit points (  $i = 1 \div 4$  )

If the position of Pi points is imposed, the positions of  $O_1$  and  $O_2$  points are resulting

It is exemplified the calculating method of  $O_2$  points coordinates, using the system below:

$$\begin{cases} k_1 x_1 + k_2 y_1 = X_1 \\ k_7 x_1 + k_8 y_1 = X_4 \end{cases}$$

The system has a solution if

$$\Delta = \begin{vmatrix} k_1 & k_2 \\ k_7 & k_8 \end{vmatrix} = k_1 k_8 - k_2 k_7 \neq 0$$

For the system have technical solutions, the following condition must be fulfilled:

$$\frac{k_1}{k_7} \neq \frac{k_2}{k_8}$$

System determinants are:

$$\Delta = \begin{vmatrix} k_1 & k_2 \\ k_7 & k_8 \end{vmatrix}; \Delta_{x_1} = \begin{vmatrix} X_1 & k_2 \\ X_4 & k_8 \end{vmatrix}; \Delta_{y_1} = \begin{vmatrix} k_1 & X_1 \\ k_7 & X_4 \end{vmatrix},$$

with the solutions:

$$x_1 = \frac{\Delta_{x_1}}{\Delta}; y_1 = \frac{\Delta_{y_1}}{\Delta}$$

Analogue for point  $O_2$ .

## 4. CONCLUSIONS

The paper-work develops the geometric modelling of the surface described by the coupling mechanism between equalizer and bucket

This area is proposed to be included in all the functional characteristics of excavators. To substantiate this point of view, the paper-work defines a function corresponding to all coordinates, with the form  $(X, Y, F, M) = 0$ . It shall be demonstrated that all points have the coordinates expressed by linear combinations of geometric elements defining the excavator's working mechanism.

## **REFERENCES**

- [1] Mihailescu, St. Mașini de construcții și pentru prelucrarea agregatelor. Editura Didactică și Pedagogică, București, 1983.
- [2] Vlase, S. Mecanică. Statică. Editura Infomarket, Brașov, 2008.
- [3] Udriste, Constantin. Algebra liniară, geometrie analitică. Geometry Balkan Press, București, 1996.