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STRESS INTENSITY FACTORS IN CRACK CLOSURE PROBLEMS

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Abstract: *The application of the Boundary Elemeny Method (BEM) to the computation of stress intensity factor (SIF) and the crack propagation angle in orthotropic materials is the aim of this paper*
Keywords *:crack,propagation,composite,boundary element*

1. INTRODUCTION

The rising expectations in the design of mechanical elements generate a need to incorporate, in more accurate ways, aspects that were previously solely approximated, or not even taken into consideration. Such is the case of the crack fatigue and propagation problems, both relevant when estimating the lifespan of a mechanical element that is subject to alternating loads, or that has initial cracks of certain extension.
In Linear Elastic Fracture Mechanics (LEFM), the most used parameter in terms of determining the cyclic fatigue life or the unstable nature of a process of monotonic loads is the Stress Intensity Factor (SIF). Many work studies are dedicated to the presentation of this parameter's values in different situations and to the specific programs developed in order to obtain it both in finite elements and in boundary elements.
However, the majority of such studies focus on cases in which the crack lips are almost completely open and smooth, respectively with a null crack friction coefficient. This case, that can result very relevant when it comes to a predominant one mode problems or in metals, becomes less relevant in mixed mode problems, especially in the anisotropic materials and composites. Due to the increasing use of this types of materials – like concrete, and especially fibre composites – this problem becomes one of unique importance and of great essence, if we take into account (1) the dramatic reduction that the consideration of such factors might lead to for the stress intensity factor and for the predicted cyclic fatigue life, and (2) the possible lack of crack propagation in situations in which a simple calculation of an open crack factor indicated a crack propagation. This is mainly the case of mode II cracks with increased friction between the crack lips.
First, it is analyzed the 2-D elastic problem for orthotropic materials via the Boundary Element Method, the formulation and algorithms used in order to solve the contact problem between two solids with or without friction against boundary loads, using a multi domain method that allows us to handle the crack closure problem as a mere contact one by solely including the corresponding distribution at the crack edges. Secondly, the method used in order to determine the stress intensity factor that allows us to easily identify the existing singular tips and fracture modes is defined. Finally, various examples that allow us to verify the accuracy of the present formulation are described.

2. FORMULATION OF THE BEM IN 2-D LINEAR ELASTICAL MULTI DOMAIN PROBLEMS

The first equation of the BEM, in its direct formulation, is the well-known Somigliana's identity, which expresses the displacement vector u_i *(Q)* of a point Q of a domain Ω as a function of the displacements u_i *(P)* and tractions t_i *(P)* of the boundary points of this dominium and the body forces X_i

$$C_{ik}u_i(Q)=\int_{\delta\Omega}U_{ik}(Q,P)t_i(P)d\delta\Omega-\int_{\delta\Omega}T_{ik}(Q,P)u_i(P)d\delta\Omega+\int_{\delta\Omega}U_{ik}(Q,P)X_i(P)d\Omega \quad (1)$$

where U_{ik} is the Kelvin fundamental solution of the Navier'equations, T_{ik} are tractions corresponding to those fundamental solutionof the Navier's equations, $T_{ik}$are the tractions corresponding to those fundamental solutions (the expresisions for the orthopic case are included in the Appendix), and C_{ik} can be expressed as:[9]

$$C_{ik} = \begin{cases} \delta_{ik} \rightarrow Q \in \Omega \\ C'_{ik} \rightarrow Q \in \delta\Omega \\ 0 \rightarrow Q \notin QU\delta\Omega \end{cases} \tag{2}$$

with

$$C'_{ik} = \frac{1}{4\pi(1-\nu} \times \begin{bmatrix} 2(1-\nu)(\pi+\alpha_1-\alpha_2) + \frac{1}{2(sen2\alpha_1 - sen2\alpha_2)} & sen^2\alpha_1 - sen^2\alpha_2 \\ sen^2\alpha_1 - sen^2\alpha_2 & 2(1-\nu)(\pi+\alpha_1-\alpha_2) - \frac{1}{2}(sen2\alpha_1 - sen\alpha_2) \end{bmatrix} \tag{3}$$

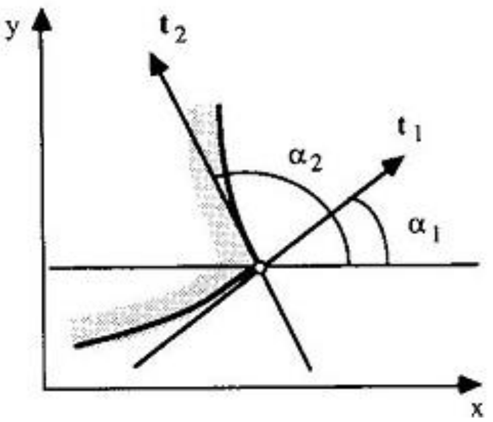

Figure 1 Geometrical mean of α_1 and α_2

where U_{ik} is the fundamental solution of the Navier equation, T_{ik} are the tractions corresponding to the mentioned fundamental solution, included in the Appendix on the orthotropic case, and C_{ik} can be expressed as☹formula(2.for isotropic materials, where $\alpha_1$and α_2 have the geometrical meaning shown in Figure 1, δ_{ik} is the Kronecker tensor, **r** the radiovector joining the points P and Q, *n* the outward normal to the boundary at point P and *v* the Poisson coefficient [for plane stress, this value must be modified by the well-known expression $\nu^* = \frac{\nu}{1+\nu}$

Under some circumstances, the domain integral in (1) can be rewritten as the sum of two boundary integrals, in such a way that is possible to express the displacement of any point of the domain Ω in terms of only boundary integrals. In this work, however, no body forces have been considered, hence such integral disappears, and the equation (1) is directly expressed based on the boundary integral function.(fig1)

If a boundary discretization with *Ne* elements is used, and the displacements and tractions are approximated inside each element in terms of nodal values, in the standard form of BEM, as (formula (4)

$$u_i^j = \sum_{k=1}^{Nnj} (u_i^j)_k \varphi_k \qquad t_i^j = \sum_{k=1}^{Nnj} (t_i^j)_k \varphi_k \tag{4}$$

where Nnj is the number of nodes of the element j, and φ_k the shape function for 2-D continuous elements, then the eq (1) can be approximated by

$$C_{ik} u_i(Q) = \sum_{j=1}^{Ne} \int_{\delta Q_j} U_{ik}(Q,P) \left[\sum_{m=1}^{Nnj} (t_i^j)_m \varphi_m \right] d\delta\Omega_j - \sum_{j=1}^{Ne} \int_{\delta\Omega_j} T_{ik}(Q.P) \left[\sum_{m=1}^{Nnj} (u_{ij})_m \varphi_m \right] d\delta\Omega_j \qquad \textbf{(5)}$$

For example, in the case of linear elements (two nodes per element), equation (4) can be rewritten as (formula 6)

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} + \sum_{j=1}^{Ne} \begin{bmatrix} A_{111}^{kj} & A_{211}^{kj} & A_{112}^{kj} & A_{212}^{kj} \\ A_{121}^{kj} & A_{221}^{kj} & A_{122}^{kj} & A_{222}^{kj} \end{bmatrix} \times \begin{bmatrix} (u_i^j)_1 \\ (u_i^j)_2 \\ (u_i^j)_3 \\ (u_i^j)_4 \end{bmatrix} = \sum_{j=1}^{Ne} \begin{bmatrix} B_{111}^{kj} & B_{211}^{kj} & B_{112}^{kj} & B_{212}^{kj} \\ B_{121}^{kj} & B_{221}^{kj} & B_{122}^{kj} & B_{222}^{kj} \end{bmatrix} \begin{bmatrix} (t_i^j)_1 \\ (t_i^j)_2 \\ (t_i^j)_3 \\ (t_i^j)_4 \end{bmatrix} \qquad \textbf{(6)}$$

If this expression is applied to each of the nodes and the corresponding boundary conditions are also included, it is possible to compute an algebraic linear system with [$2\sum_j (Nnj - 1)$] equations and unknowns, corresponding to the displacements and tractions of the boundary nodes.

If the collocation point is not one of the nodes of the element along which the integrals in (5) are computed, a standard Gauss-Legendre quadrature is used. On the other hand, when it is placed from a node inside the adjacent element, singular integrands appear in the integrals of (5). In this case, the constants *B* are computed by using a quadrature with logarithmic weight function, while the constants *A* are computed, together with the free term C_{ik}, by imposing a rigid body condition to the studied body.

At each node two equations and six unknowns (two displacements, and two tractions for each of the elements to which the node belongs) can then be established. Most of the times, these tractions are expressed in local coordinates being necessary to transform the traction vector based on these coordinates.

Ultimately, once the coefficient and independent term vector matrix is assembled, and the boundary conditions are applied, an algebraic system is obtained in the form (7)

$$Kx=f \qquad \textbf{(7)}$$

in which the unknowns, **x,** correspond to boundary displacements and/or tractions. The solution of this system is performed by any standard method, depending on its size.

Once the unknown displacements and tractions have been obtained, the displacements of any internal point are also obtained by (1), while the stresses may be computed by applying the stress operator to it.

Focusing solely on the contact problem formulation between to elastic solids, with their interface initially in a full contact, and normal for both solids. This is the only case of interest for this context. The non-traction condition for the mentioned point and with the data (tipology of zone) described in Figure 2 is expressed as

$$u_N \leq 0 \qquad \textbf{(8)}$$

where u_N is the projection of relative displacement between equivalent points (equal to the post-contact position) above normal.

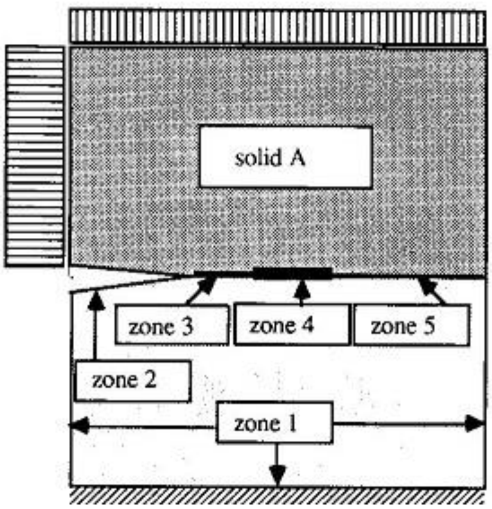

Figure 2 Tipology of zone

The static boundary conditions, in the unilateral case proposed in this work and based on the Coulomb's Law of Friction like the one used here, can be expressed as formula 8

Besides that, the compatibility and equilibrium conditions are to be met between the two solids, in the points in which contact has been established. For this, the following different areas are defined in terms of the global boundary of each solid (Figure 2.)
- No contact area (area no. 1) – the area that shall never establish contact
- Candidate to contact area (area no. 2) – the area that still has not established a contact, that might establish one at a specific load level.
- Slip area (area no. 3) – $\left| r = \mu\sigma_N \right|$
- Adherent area (area no. 4) – $\left| r \prec \mu\sigma_N \right|$
- Welding area (area no. 5) – the contact area in which both solids are considered welded, thus recognizing the traction stresses.
The contact problem between two solids, or better said between two domains of one body, as in this case, consists therefore in approaching the BEM equations to each contact solid, including implicitly or explicitly (in this case the second option was chosen) the boundary conditions (compatibility and equilibrium) in the contact area for each load level, as well as the boundary conditions in the other areas for each one of the aforementioned solids.
The program that has been implemented includes linear, quadratic and quarter-point-singular-traction elements (–1/2 singularity), all of them with stresses and displacement continuity, as long as area no. 1 is checked for special nodes (nodes with excess or no unknowns), treated in an analogue mode in [2]. In case of friction, the friction coefficient is defined independently for each element, as it is possible to have independent contact areas between two solids with different friction coefficients.

3. CONCLUSIONS

It has been shown that the B.E.M. may be used to study the problem of propagating cracks in orthotropic bodies in a similar form to the previous works on isotropic materials. Also,the singular boundary elements firstly proposed by Blandford et.all. give very good results in the computation of stress intensity factors every coarse meshes, specially using a direct traction approach like the one presented by Martinez and Dominguez, being only necessary the modification of the fundamental solution of standard isotropic boundary element program.
In most of cases, the method which gives rise to the best results in the computation of the SIF is the one that uses the singular traction approximation, using the nodal value of the singular traction approximation, using the nodal value of the singular node as the parameter which allows the obtention of the SIF, although it is very important the choice of the length of this singular element.
Also the maximum circumferential stress criterium may be very easily included as a postprocessor in a standard B.E. code. In this case, was expected, the choice of shorter increments of the crack propagation effort and with only a few redefinitions of the mesh.
In comparison with domain methods, the mesh needed to produce similar results are much simpler, which is always needed in a crack propagation problem.

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