A MOTION STUDY OF THE QUADRILATERAL PLANE MECHANISM WITH SOLIDWORKS

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Abstract: The objective of the paper is to calculate the kinematic behavior of the quadrilateral plane mechanism and compare the theoretical with the following SolidWorks Motion results: the positional analysis, kinematic analysis of velocities and kinematic analysis of accelerations.
Keywords: mechanism, kinematic, SolidWorks, motion

1. INTRODUCTION

The widely used articulated quadrilateral mechanism is consisting of four hinged beams, three of which are movable and one fixed. The elements 1 and 3 (beams OA and BC), hinged at the base, are called cranks in the case of full rotation or swing in the case of partial rotation (oscillatory movement). The element 2 (AB beam), non-articulated at the base and having a plane movement, is named connecting rod. Two important applications of this type of mechanism are: opening the boot of a car and riding a bicycle [1].

2. THE QUADRILATERAL MECHANISM GEOMETRY AND MAIN PARAMETERS

The quadrilateral mechanism geometry presented in (Fig.1), in accordance to [1], is composed from three beams identified as ①, ② and ③ with 6 mm thickness and the following lengths: L_1=100 mm, L_2=280 mm, L_3 = 180 mm. The point O has the coordinates (0,0) and the point C has the coordinates (-200,-200). The initial position of the mechanism is defined by the angle φ_1=0°, (Fig.2), angle variable during mechanism movement between 0° ÷360°. The mechanism is driven by beam ① with constant rotational velocity n_1=6 rot/min equivalent with angular velocity ω_1=π/5 rad/sec. The maximum time of the study is imposed at 10 seconds. The successive positions of the quadrilateral mechanism are presented in (Fig.3).
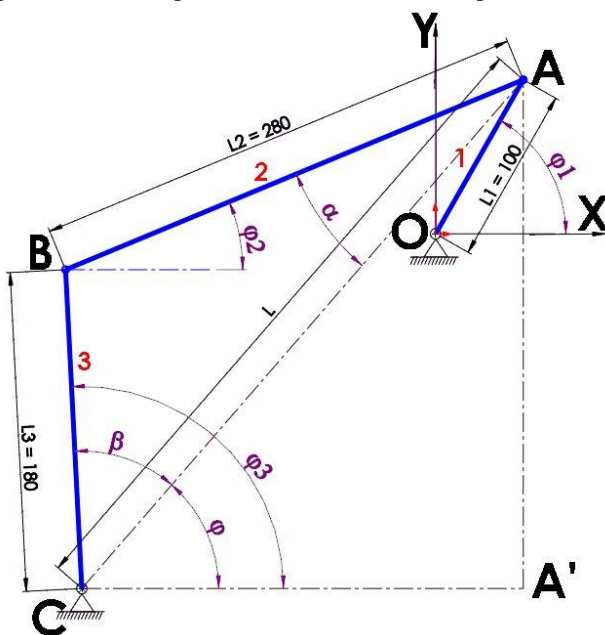

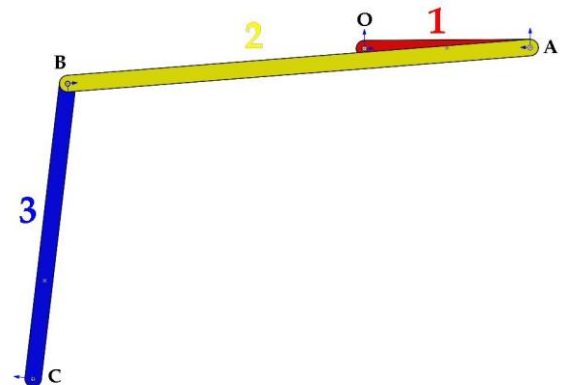**Figure 1:** The quadrilateral mechanism geometry

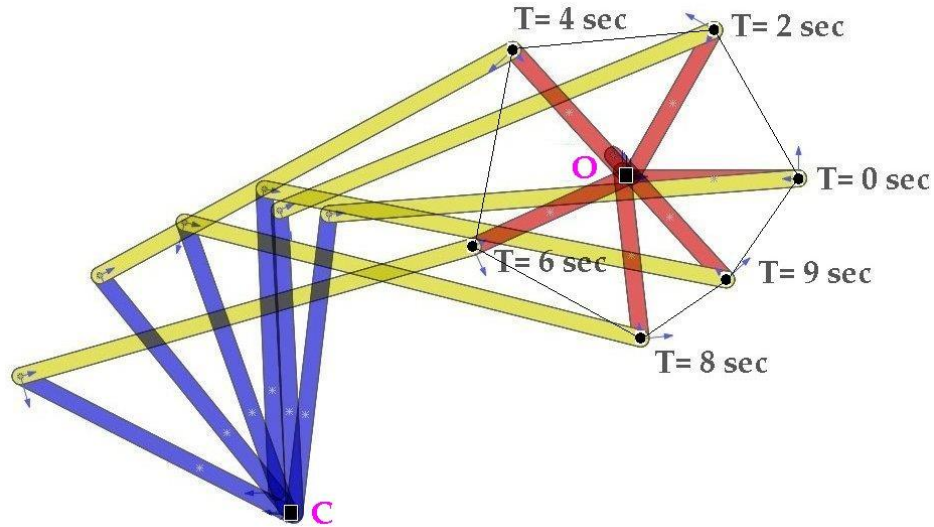**Figure 2:** The initial position of the mechanism

Figure 3: The successive positions of the quadrilateral mechanism

3. THEORETICAL BACKGROUND AND NUMERICAL RESULTS

The positional parameters of the mechanism, depicted in (Table 1), represent the linear displacements of the component's specific points A and B, which can be calculated, in accordance to [1], with the analytical relations (1) to (4):

$$X_A = L_1 \cdot \cos(\varphi_1) \tag{1}$$

$$Y_A = L_1 \cdot \sin(\varphi_1) \tag{2}$$

$$X_B = X_C + L_3 \cdot \cos(\varphi_3) \tag{3}$$

$$Y_B = Y_C + L_3 \cdot \sin(\varphi_3) \tag{4}$$

The angle parameters of the mechanism, depicted in (Table 1), can be calculated in accordance to [1] with the analytical relations (5) to (9):

$$\sin(\varphi) = \frac{Y_A - Y_C}{L} \quad ; \quad \cos(\varphi) = \frac{X_A - X_C}{L} \tag{5}$$

$$\cos(\alpha) = \frac{L^2 + L_2{}^2 - L_3{}^2}{2 \cdot L \cdot L_2} \tag{6}$$

$$\cos(\beta) = \frac{L^2 + L_3{}^2 - L_2{}^2}{2 \cdot L \cdot L_3} \tag{7}$$

$$\varphi_2 = \varphi - \alpha \quad ; \quad \varphi_3 = \varphi + \beta \tag{8}$$

where: $L = \sqrt{(X_A - X_C)^2 + (Y_A - Y_C)^2}$ \tag{9}

Table 1: The positional and angle parameters of the mechanism

Time	XA	YA	XB	YB	α	β	φ	φ₁	φ₂	φ₃
0.00	100.00	0.00	-179.20	-21.21	29.35	49.67	33.69	0.00	4.34	83.363
0.01	100.00	0.63	-179.15	-21.21	29.30	49.57	33.77	0.36	4.47	83.348
0.34	97.68	21.41	-179.05	-21.22	27.88	46.68	36.64	12.36	8.76	83.317
0.68	91.10	41.25	-181.92	-20.91	26.82	44.58	39.65	24.36	12.83	84.234
1.01	80.53	59.29	-187.89	-20.41	26.21	43.40	42.75	36.36	16.54	86.142
1.34	66.44	74.73	-197.03	-20.02	26.10	43.18	45.88	48.36	19.78	89.056
1.68	49.45	86.91	-209.23	-20.24	26.49	43.94	49.00	60.36	22.50	92.940
2.01	30.30	95.30	-224.12	-21.62	27.37	45.65	52.05	72.36	24.68	97.700
2.34	9.83	99.52	-241.09	-24.75	28.64	48.21	54.99	84.36	26.35	103.194

Time	XA	YA	XB	YB	α	β	φ	φ₁	φ₂	φ₃
2.68	-11.08	99.38	-259.36	-30.07	30.21	51.51	57.75	96.36	27.54	109.254
3.01	-31.50	94.91	-278.05	-37.80	31.97	55.44	60.26	108.36	28.29	115.697
3.34	-50.54	86.29	-296.29	-47.92	33.79	59.91	62.43	120.36	28.64	122.338
3.68	-67.38	73.89	-313.24	-60.09	35.58	64.82	64.16	132.36	28.59	128.986
4.01	-81.27	58.27	-328.23	-73.68	37.20	70.12	65.31	144.36	28.12	135.430
4.34	-91.61	40.10	-340.71	-87.75	38.53	75.72	65.70	156.36	27.17	141.421
4.68	-97.94	20.18	-350.35	-101.03	39.48	81.51	65.13	168.36	25.65	146.645
5.01	-100.00	-0.63	-356.95	-111.87	39.95	87.32	63.36	180.36	23.41	150.686
5.34	-97.68	-21.41	-360.39	-118.29	39.95	92.81	60.19	192.36	20.24	153.003
5.68	-91.10	-41.25	-360.32	-118.17	39.60	97.41	55.55	204.36	15.95	152.960
6.01	-80.53	-59.29	-355.90	-110.02	39.23	100.34	49.67	216.36	10.44	150.007
6.34	-66.44	-74.73	-345.75	-94.38	39.14	100.91	43.17	228.36	4.02	144.071
6.68	-49.45	-86.91	-329.19	-74.66	39.42	98.95	36.91	240.36	-2.51	135.865
7.01	-30.30	-95.30	-307.48	-55.61	39.82	94.99	31.67	252.36	-8.15	126.662
7.34	-9.83	-99.52	-283.55	-40.57	40.00	89.81	27.85	264.36	-12.15	117.657
7.68	11.08	-99.38	-260.29	-30.40	39.75	84.08	25.49	276.36	-14.26	109.569
8.01	31.50	-94.91	-239.47	-24.38	39.01	78.25	24.42	288.36	-14.59	102.668
8.34	50.54	-86.29	-221.82	-21.33	37.83	72.55	24.41	300.36	-13.41	96.962
8.68	67.38	-73.89	-207.42	-20.15	36.32	67.11	25.25	312.36	-11.07	92.361
9.01	81.27	-58.27	-196.11	-20.04	34.59	62.02	26.74	324.36	-7.85	88.761
9.34	91.61	-40.10	-187.70	-20.42	32.77	57.34	28.74	336.36	-4.03	86.082
9.68	97.94	-20.18	-182.06	-20.90	30.97	53.17	31.11	348.36	0.15	84.279
10.00	100.00	0.00	-179.20	-21.21	29.35	49.67	33.69	360.00	4.34	83.363

The kinematic parameters of the mechanism, depicted in (Table 2), are the linear velocities and accelerations, which can be calculated in accordance to [1] with the analytical relations (10) to (18):

$$V_{AX} = -L_1 \cdot \omega_1 \cdot \sin(\varphi_1) \tag{10}$$

$$V_{AY} = L_1 \cdot \omega_1 \cdot \cos(\varphi_1) \tag{11}$$

$$a_{AX} = -L_1 \cdot \omega_1^2 \cdot \cos(\varphi_1) \tag{12}$$

$$a_{AY} = -L_1 \cdot \omega_1^2 \cdot \sin(\varphi_1) \tag{13}$$

$$V_{BY} = \frac{(X_B - X_A) \cdot V_{AX} + (Y_B - Y_A) \cdot V_{AY}}{(Y_B - Y_A) - (X_B - X_A) \cdot \dfrac{(Y_B - Y_C)}{(X_B - X_C)}} \tag{14}$$

$$V_{BX} = -\frac{(Y_B - Y_C)}{(X_B - X_C)} \cdot V_{BY} \tag{15}$$

$$T = (X_B - X_A) \cdot a_{AX} + (Y_B - Y_A) \cdot a_{AY} - (V_{BX} - V_{AX})^2 - (V_{BY} - V_{AY})^2 \tag{16}$$

$$a_{BY} = \frac{T + \dfrac{(X_B - X_A) \cdot (V_{BX}{}^2 + V_{BY}{}^2)}{(X_B - X_C)}}{(Y_B - Y_A) - \dfrac{(X_B - X_A) \cdot (Y_B - Y_C)}{(X_B - X_C)}} \tag{17}$$

$$a_{BY} = -\frac{(V_{BX}{}^2 + V_{BY}{}^2)}{(X_B - X_C)} - \frac{(Y_B - Y_C)}{(X_B - X_C)} \cdot a_{BY} \tag{18}$$

Table 2: The kinematic parameters of the mechanism

Time	V_AX	V_AY	a_AX	a_AY	V_BX	V_BY	a_BX	a_BY
0.00	0.00	62.83	-39.48	0.00	4.81	-0.56	-25.21	2.80
0.01	-0.39	62.83	-39.48	-0.25	4.56	-0.53	-25.25	2.83
0.34	-13.45	61.38	-38.56	-8.45	-4.07	0.48	-26.61	3.02

Time	V_{AX}	V_{AY}	a_{AX}	a_{AY}	V_{BX}	V_{BY}	a_{BX}	a_{BY}
0.68	-25.92	57.24	-35.96	-16.28	-13.19	1.33	-28.05	1.85
1.01	-37.25	50.60	-31.79	-23.41	-22.68	1.53	-28.70	-0.94
1.34	-46.96	41.75	-26.23	-29.50	-32.13	0.53	-27.66	-5.28
1.68	-54.61	31.07	-19.52	-34.31	-40.87	-2.10	-24.38	-10.57
2.01	-59.88	19.04	-11.96	-37.62	-48.14	-6.51	-18.89	-15.78
2.34	-62.53	6.17	-3.88	-39.29	-53.28	-12.49	-11.78	-19.85
2.68	-62.45	-6.96	4.37	-39.24	-55.89	-19.52	-3.82	-21.96
3.01	-59.63	-19.79	12.44	-37.47	-55.83	-26.86	4.18	-21.65
3.34	-54.22	-31.76	19.95	-34.06	-53.17	-33.66	11.59	-18.71
3.68	-46.43	-42.34	26.60	-29.17	-48.23	-39.03	17.87	-13.05
4.01	-36.61	-51.06	32.08	-23.00	-41.43	-42.05	22.66	-4.58
4.34	-25.19	-57.56	36.17	-15.83	-33.31	-41.75	25.77	6.89
4.68	-12.68	-61.54	38.67	-7.97	-24.42	-37.09	27.36	21.64
5.01	0.39	-62.83	39.48	0.25	-15.14	-26.96	28.36	39.66
5.34	13.45	-61.38	38.56	8.45	-5.33	-10.46	31.04	59.24
5.68	25.92	-57.24	35.96	16.28	6.13	12.01	38.80	73.79
6.01	37.25	-50.60	31.79	23.41	21.17	36.68	51.83	69.87
6.34	46.96	-41.75	26.23	29.50	40.12	55.37	59.58	37.94
6.68	54.61	-31.07	19.52	34.31	58.62	60.42	47.68	-7.40
7.01	59.88	-19.04	11.96	37.62	70.07	52.16	19.78	-38.12
7.34	62.53	-6.17	3.88	39.29	71.98	37.72	-6.92	-45.05
7.68	62.45	6.96	-4.37	39.24	66.70	23.71	-22.82	-37.66
8.01	59.63	19.79	-12.44	37.47	57.87	13.01	-28.91	-26.53
8.34	54.22	31.76	-19.95	34.06	48.04	5.87	-29.46	-16.71
8.68	46.43	42.34	-26.60	29.17	38.46	1.59	-27.87	-9.39
9.01	36.61	51.06	-32.08	23.00	29.49	-0.64	-26.02	-4.27
9.34	25.19	57.56	-36.17	15.83	21.04	-1.44	-24.81	-0.78
9.68	12.68	61.54	-38.67	7.97	12.84	-1.29	-24.55	1.53
10.00	0.00	62.83	-39.48	0.00	4.81	-0.56	-25.21	2.80

4. THE STAGES OF MOTION STUDY

The 3D assembly of the mechanism is shown in (Fig.4). The components were placed in the assembly with **Insert Components** command from **Assembly** toolbar. Then select a part or assembly from the **Part/Assembly to Insert** list or click **Browse** to open an existing document. The point C with (-200, -200) coordinates was created as a point in a 3D sketch.

The stages of the study are as follows [2] and [3]:

   o Activation of the SolidWorks Motion module;
   o Creation and specification of the study's options;
   o Specify **Rotary Motor**;
   o Specify **Motion Mates**;
   o Running the design study.

To specify **Rotary Motor** click **Motor** to create a new rotary motor; select the face of the beam 1; select **Constant speed** from **Motor Type list** and set 6 rpm value in the speed motor field; click ✓ and the *Rotary Motor1* branch will be created in the **MotionManager** design tree (Fig.5).

The mates indicated in (Table 3) will be applied in **Motion Study** between the assembly components.

Table 3: The mates applied to the shaker mechanism

| Mate name	Mate type	Component 1	Component 2
Mate 1	Coincident1	Assembly origin point	Point 1 of beam 1
Mate 2	Coincident2	Point C	Point 6 of beam 3
Mate 3	Coincident3	Circular edge aligned with point 4	Circular edge aligned with point 5
Mate 4	Coincident4	Circular edge aligned with point 3	Circular edge aligned with point 2
Mate 5	Parallel1	FRONT Plane	Face 2 of beam 2
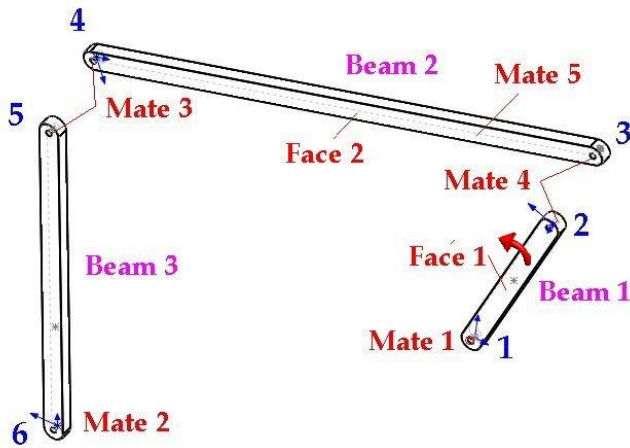
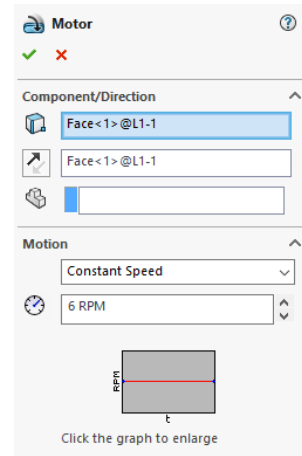**Figure 4:** The 3D assembly of the mechanism

Figure 5: Motor command

5. SIMULATION AND THEORETICAL RESULTS COMPARISON

The results are presented graphically in (Fig.6÷ Fig.14), where the analytical points are placed over the SolidWorks Motion curves. (Fig.6) and (Fig.9) show the positional parameters of the points A and B in X respectively Y direction. (Fig.7) and (Fig.10) show the velocity of the points A and B in X respectively Y direction. (Fig.8) and (Fig.11) show the accelerations of the points A and B in X respectively Y direction. (Fig.12), (Fig.13) and (Fig.14) show the angular parameters of the mechanism.
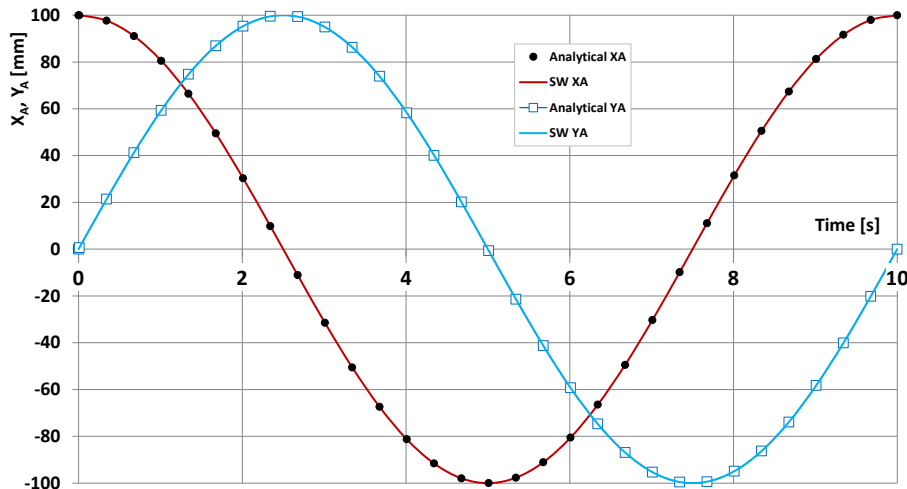
Figure 6: The positional parameters of point A

X_A, Y_A
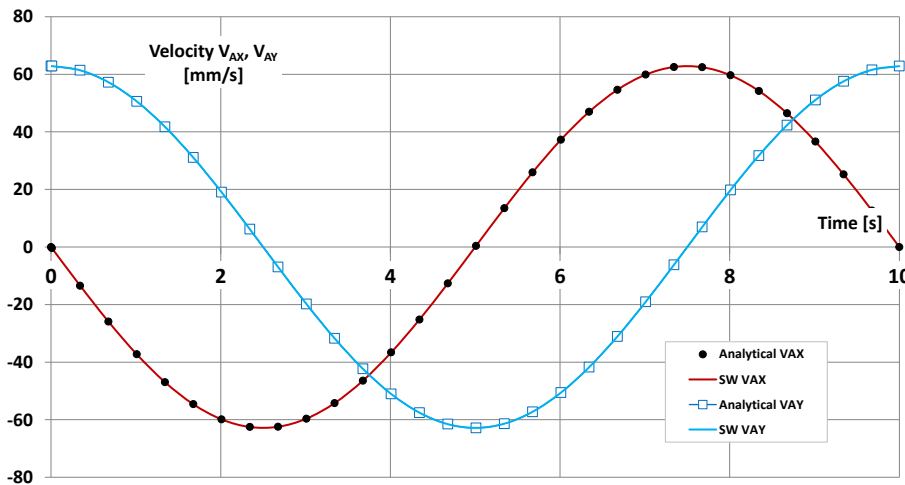
Figure 7: The kinematic parameters of point A

V_{AX}, V_{AY}
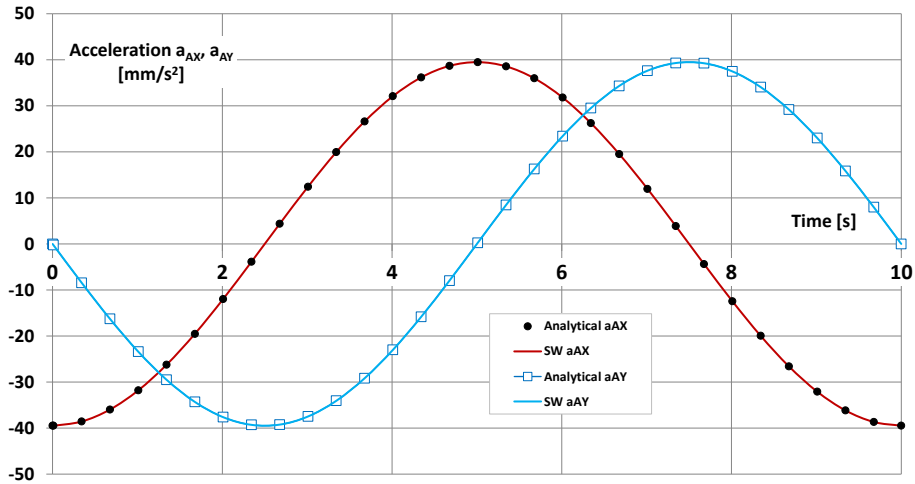
Figure 8: The kinematic parameters of point A

a_{AX}, a_{AY}
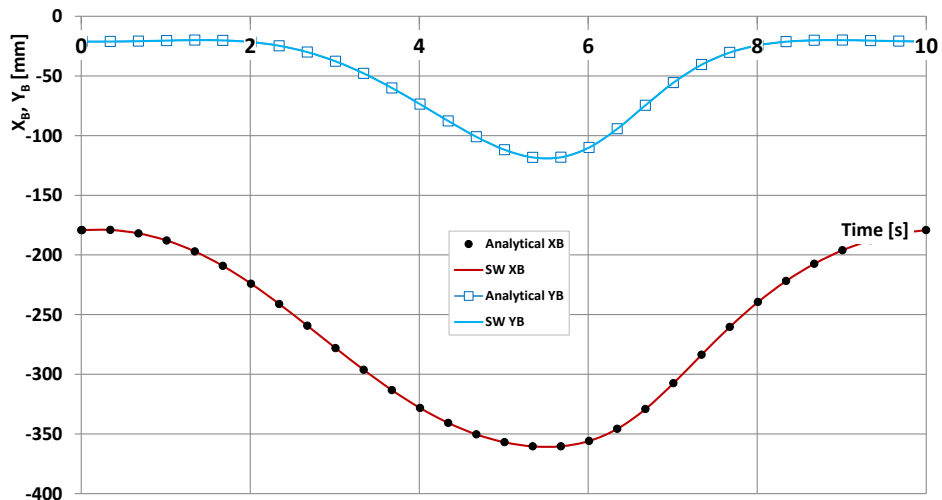
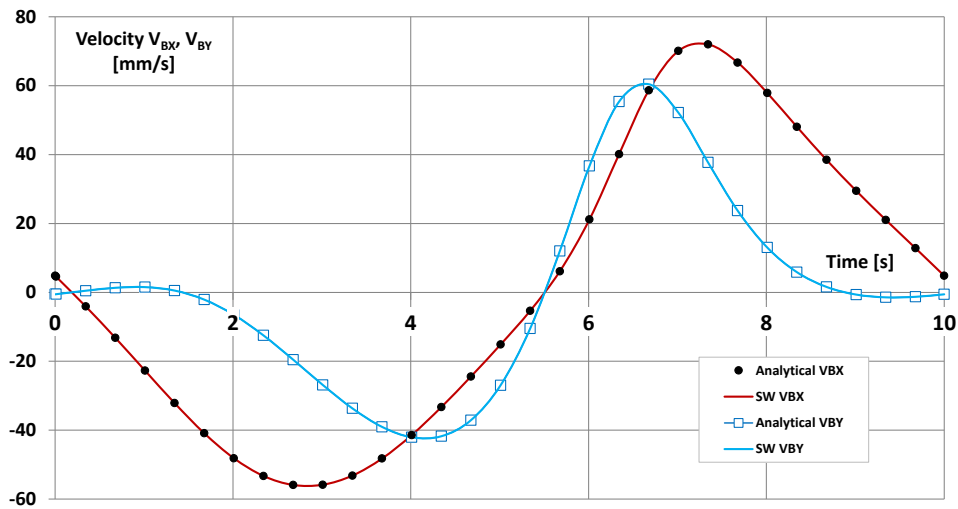**Figure 9:** The positional parameters of point B

X_B, Y_B

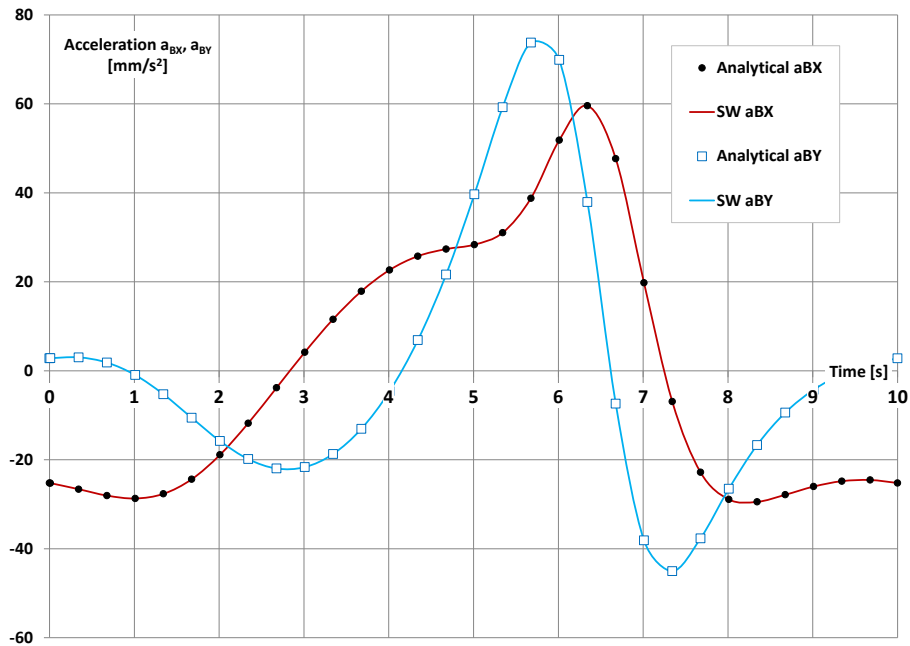**Figure 10:** The kinematic parameters of point B

V_{BX}, V_{BY}
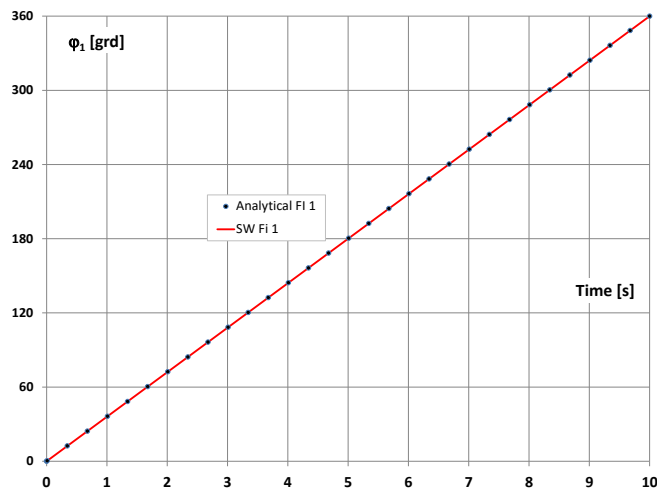
Figure 11: The kinematic parameters of point B

a_{BX}, a_{BY}

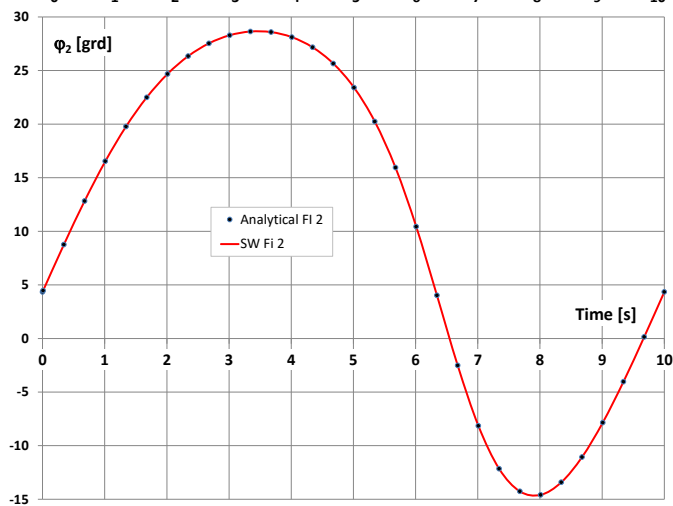**Figure 12:** The angular parameters of the mechanism φ_1

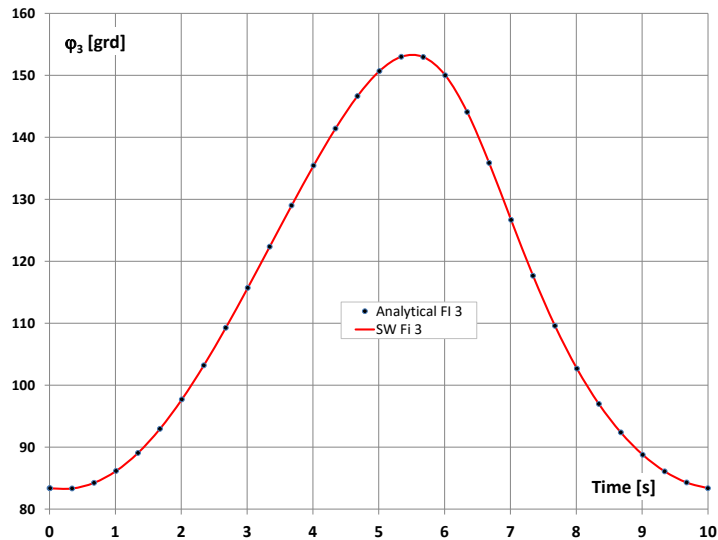**Figure 13:** The angular parameters of the mechanism φ_2

Figure 14: The angular parameters of the mechanism φ_3

4. CONCLUSION

The paper present the required steps to analyze the quadrilateral mechanism and obtain the positional and kinematic parameters through analytical equations and SolidWorks Motion software. The numerical values calculated by theoretical equations were compared graphically with the SolidWorks Motion results. The magnitudes of all results indicated excellent agreement with the values obtained by simulation in SolidWorks Motion software.

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