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TESTING THE MATHEMATICAL RELATION FOR DERIVING THE PATTERNS OF A TRANSVERSAL CRACK IN CANTILEVER BEAMS

Tufisi C.[1], **Gillich G.R.**[1✉], **Lupu D.**[1], **Pop V.M.**[1]
[1] University Babes-Bolyai, Resita, ROMANIA, cristian.tufisi@ubbcluj.ro, gilbert.gillich@ubbcluj.ro, david.lupu@ubbcluj.ro, marius.pop1@ubbcluj.ro

Abstract: *The paper presents a method for detecting transversal cracks in steel beams of rectangular cross-section, which is centered on the vibration response of the tested component. We have plotted the relative frequency shift curves for two cantilever beams with different thicknesses, both subjected to a transversal crack of 20% depth. The curves were plotted for several crack positions resulted by removing the crack with a step of 5mm along the beam. As a result, we obtained the crack patterns, which consist of a set of frequency shifts for several out-of-plain vibration modes. We also demonstrate a mathematical relation, which allows the prediction of the frequency drop of beams with transversal cracks. The equation is based on the energies stored in the beam's section in the undamaged, respectively, in the damaged state. The severity of the damage was calculated from the stiffness of the slice subjected to the biggest bending moment occurred for the beam loaded with dead mass.*

Keywords: *natural frequency, relative frequency shift, crack, severity, curvature, deflection*

1. INTRODUCTION

The detection of cracks in incipient states is essential for ensuring the overall integrity of engineering structures during operation. Although there are several nondestructive methods for assessing the health of structures, they can be divided into static and dynamic methods [1]. Structural Health Monitoring (SHM) is an automatic method of detecting changes in the integrity of mechanical systems [2]. The objective of SHM is to provide an automatic and real-time assessment of a structure's ability to achieve its desired goal [3]. The presence of damage affects the general stiffness of structures, decreasing their capacity to store energy, thus the natural frequency change can be used for detecting damage, as it offers larger flexibility in measuring the response of structures [4-7].

In prior research, we have found relations to predict the damage severity [8], the natural frequency evolution in the case of known crack parameters [9], and the damage patterns [10] for numerous types to beams in regard of boundary conditions [11] and beam cross-section shape. In this paper, we prove the reliability of the proposed mathematical relations to validate them for damage detection issues.

2. NUMERICAL SIMULATION

2.1 Deriving the damage patterns

Firstly, through FEM simulations, we determine the natural frequencies for two cantilever beams of different cross-sections in a healthy and damaged state. Using the obtained values, the relative frequency shift (RFS) curves for the damage scenarios are plotted and used to validate a simple mathematical relation for predicting the frequency drop in cantilever beams. Moreover, we calculate the damage location coefficients (DLC) and show that these are unique indicators irrespective of the beam thickness.

The analyzed structures consist of two cantilever beams of length L=1000 mm, width B=20 mm, and thickness of the first beam H=5 mm and the second H=4 mm, respectively. The schematic of the beam is presented in Figure 1. The natural frequencies for the two beams both in the intact and damaged state are found using the simulation software ANSYS. The damage geometry resembles a transversal breathing crack across the whole section of the beam having the depth a=1 mm for the first beam and a=0.8 mm for the second, meaning a cross-section reduction of 20%. For all modal studies, we have iteratively replaced the crack by a step of s = 5mm along the length of the beam, considering the crack positions starting from the left side of the beam from 5 mm until it reaches 955 mm.

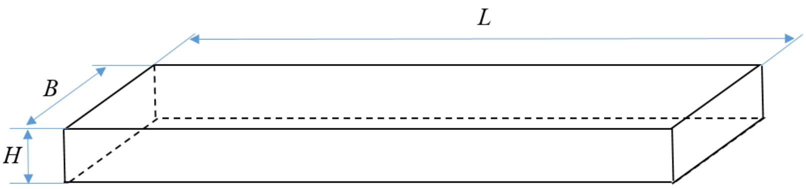


Figure 1: The main dimension of the cantilever beam

The considered beam material is Structural Steel, with its physical-mechanical properties presented in Table 1.

Table 1: Physical-mechanical properties of the two test beams

Analyzed structure	Area *A* [mm^2]	Moment of Inertia *I* [mm^4]	Density *ρ* [kg/m^3]	Young Modulus *E* [N/m2]	Poisson ratio *ν* [-]
Beam 1	100	208.33	7850	$2 \cdot 10^{11}$	0.3
Beam 2	80	106.66			

To evaluate the results of the FEM modal analyses, for the described damage cases, we plot the RFS curves, defined with the relation presented by Gillich et al in the paper [9]:

$$RFS_i(x,a) = \Delta \bar{f}_i(x,a) = \frac{f_{iU} - f_{iD}(x,a)}{f_{iU}} \tag{1}$$

where f_{iU} is the natural frequency for the undamaged beam and f_{iD} is the natural frequency for the damaged one.

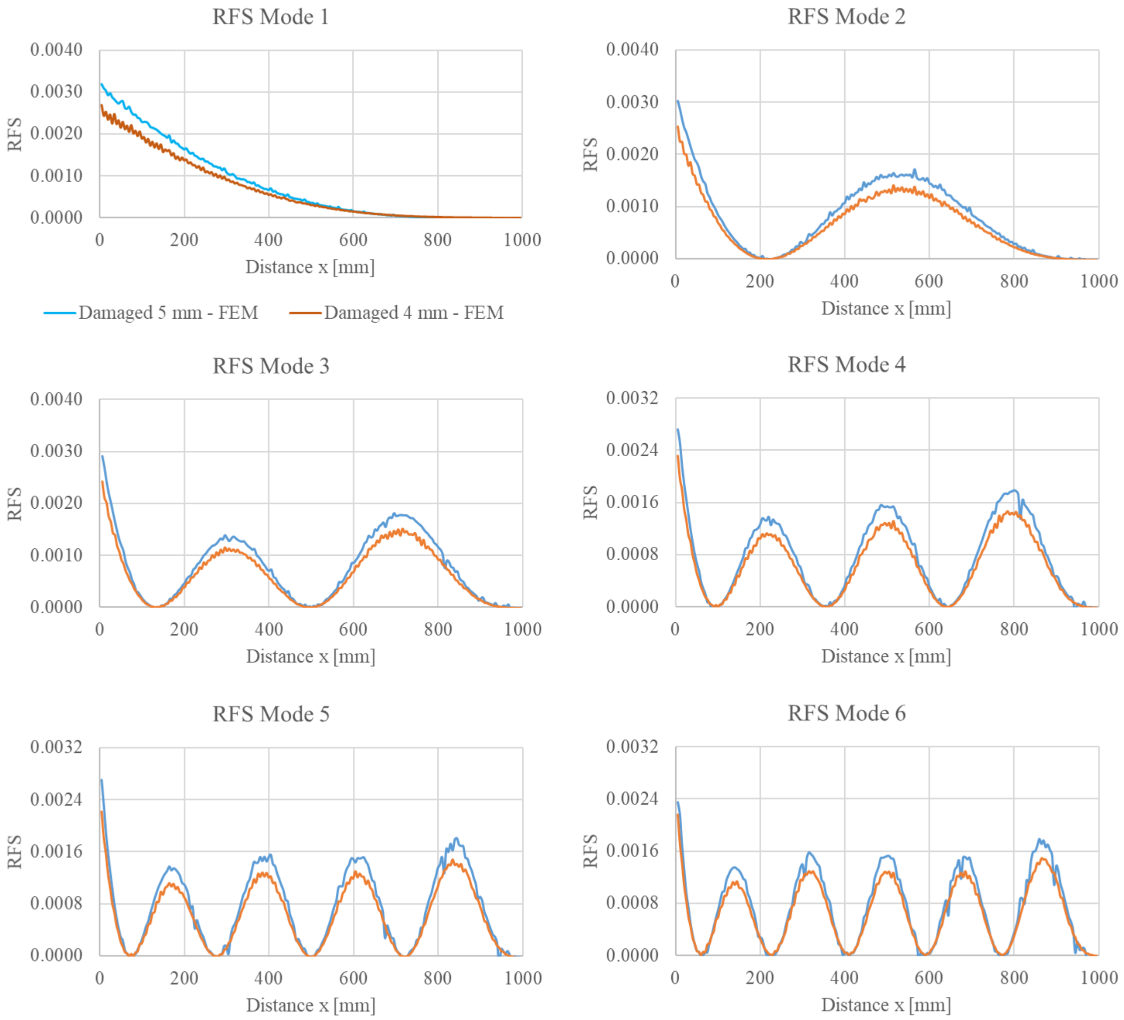


Figure 2: Plotted Relative Frequency Shift curves

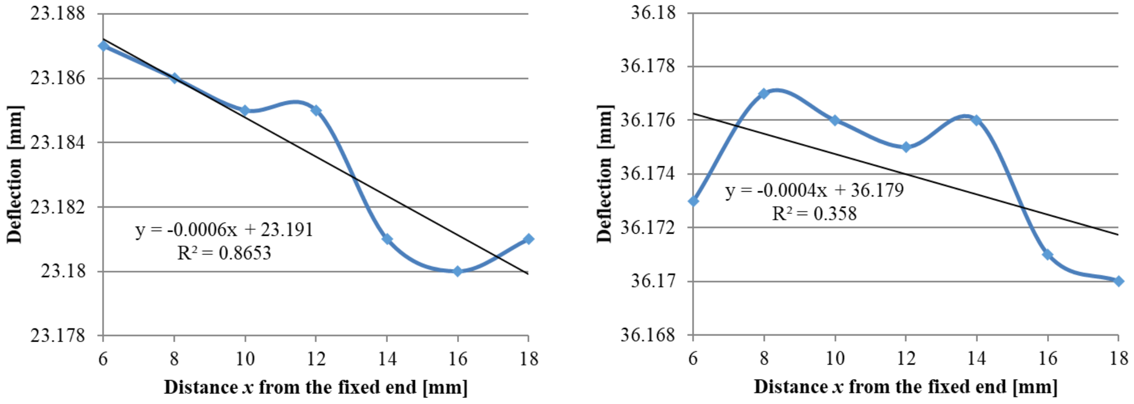
2.2 Deriving the damage severity

A second approach is to find, from the static analysis, the maximum deflections achieved by the two cantilevers under their own mass, both for the damaged and undamaged states. The FEM simulations were performed using all the required loads and constraints, and a fine mesh of hexahedral elements with a maximum edge size of 2 mm was used. With the FEM results, we calculate the damage severity γ(a) with the Gillich-Praisach [12] relation as follows:

$$\gamma(a)=\left[\sqrt{\delta_D(a)}-\sqrt{\delta_U(a)}\right]/\sqrt{\delta_D(a)} \qquad (2)$$

where $\delta_U(a)$ is the deflection under dead mass for the undamaged beam and $\delta_D(a)$ is the deflection under dead mass for the damaged beam.

For this relation we use the so-called theoretical deflection, where, we have estimated the deflection for the two damage cases, such as the obtained values will not be influenced by the end constraint, i.e. fixed beam end. This is achieved by employing a method developed in paper [11] by performing multiple static simulations for the described damages placing the crack at distance *x*=6 to 18 mm and iteratively removing it with a step of 2 mm from the fixed end. The obtained values are plotted in Figure 3; to these values we associated a linear regression curve to find the theoretical deflection values for the beams with a damage at the fixed end. The procedure is necessary for depicting the severity of the crack when it is located at *x*=0 mm [11]. The theoretical deflections and the damage severity are presented in Table 2.



a) Beam 1 b) Beam 2

Figure 3: Regression curve representing the deflection of the beam with a transverse crack

Table 2: Deflections for the two cantilever beams and the resulted damage severity

Beam 1			Beam 2		
deflection		severity	deflection		severity
Undamaged	Damaged *a*=1mm		Undamaged	Damaged *a*=0.8 mm	
23.045	23.191	0.003153	36.001	36.179	0.002463

From Table 2 one can observe that the severities calculated for the two beam have different values, which demonstrates that even if the cracks are proportional to the thickness of the beam the severity of the damages do not follow the same rule.

2. COMPARISON OF RESULTS

The relation for determining the natural frequency drop, applicable for beams with any boundary conditions, is expressed as [8]:

$$f_{Ci}(x,a)=f_i\left\{1-\gamma(0,a)\left[\bar{\phi}_i''(x)\right]^2\right\} \qquad (3)$$

where $\left[\bar{\phi}_i''(x)\right]$ represents the normalized modal curvature (or bending moment) for a given crack position *x*, and is given by the following relation [8]:

$$\phi''(x) = \cosh(\lambda \frac{x}{L}) + \cos(\lambda \frac{x}{L}) - \frac{\cos\lambda + \cosh\lambda}{\sin\lambda + \sinh\lambda} \cdot \left[\sinh(\lambda \frac{x}{L}) + \sin(\lambda \frac{x}{L}) \right] \tag{4}$$

where λ is the eigenvalue, obtained for om the transcendental equation [11]:

$$\cos(\lambda)\cosh(\lambda) + 1 = 0 \tag{5}$$

which has an infinity of solutions, each of them associated with a specific vibration mode.

Now, we compare the RFS's for the damage cases determined with the help of FEM analysis and the ones calculated using relation (3). The plotted RFS curves are presented in figure 4 for the Beam 1 and in figure 5 for Beam 2. One can observe that these results fit with high precision.

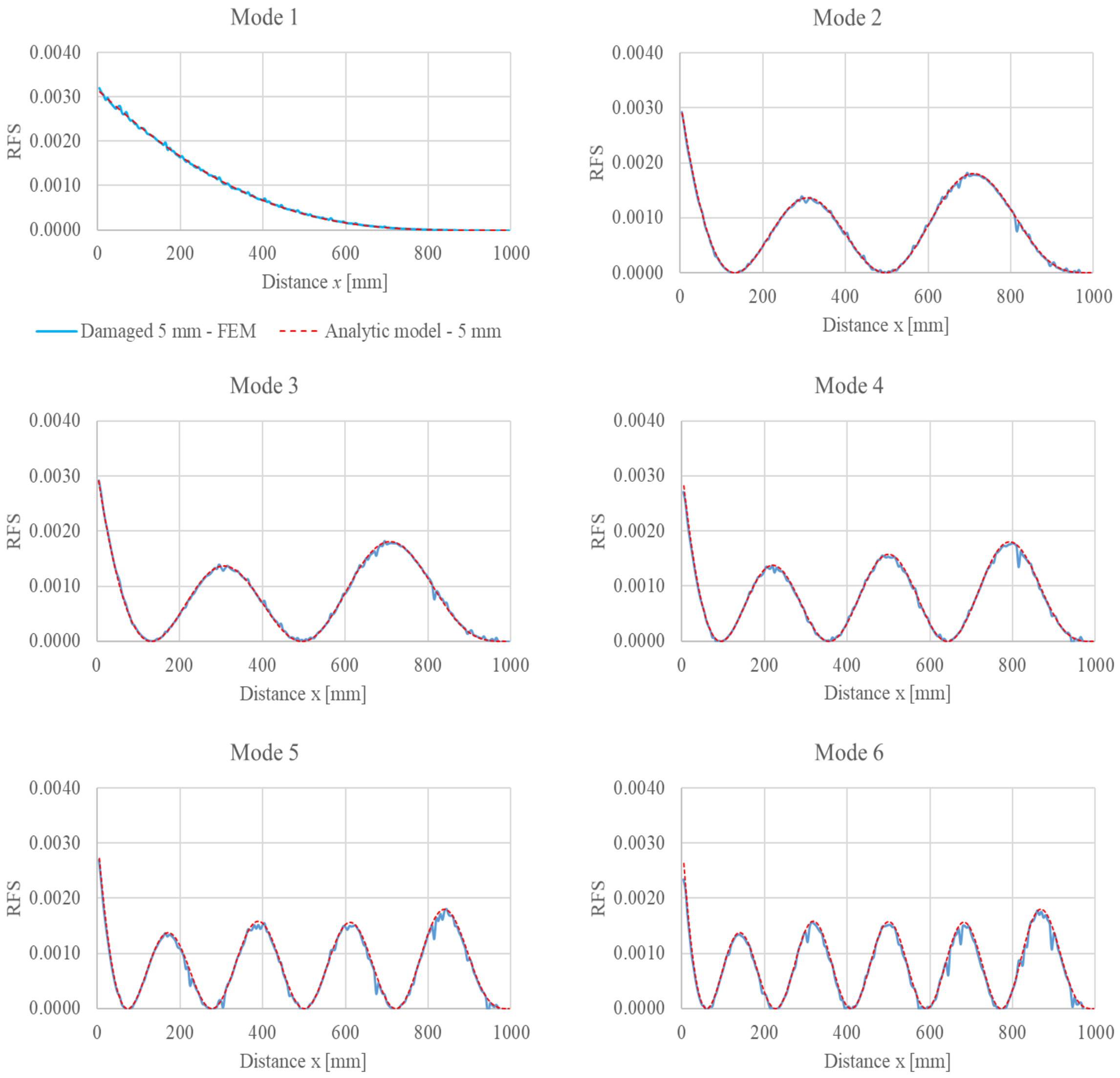


Figure 4: Comparison of the RFS evolution with the crack location for Beam 1 (thickness 5 mm)

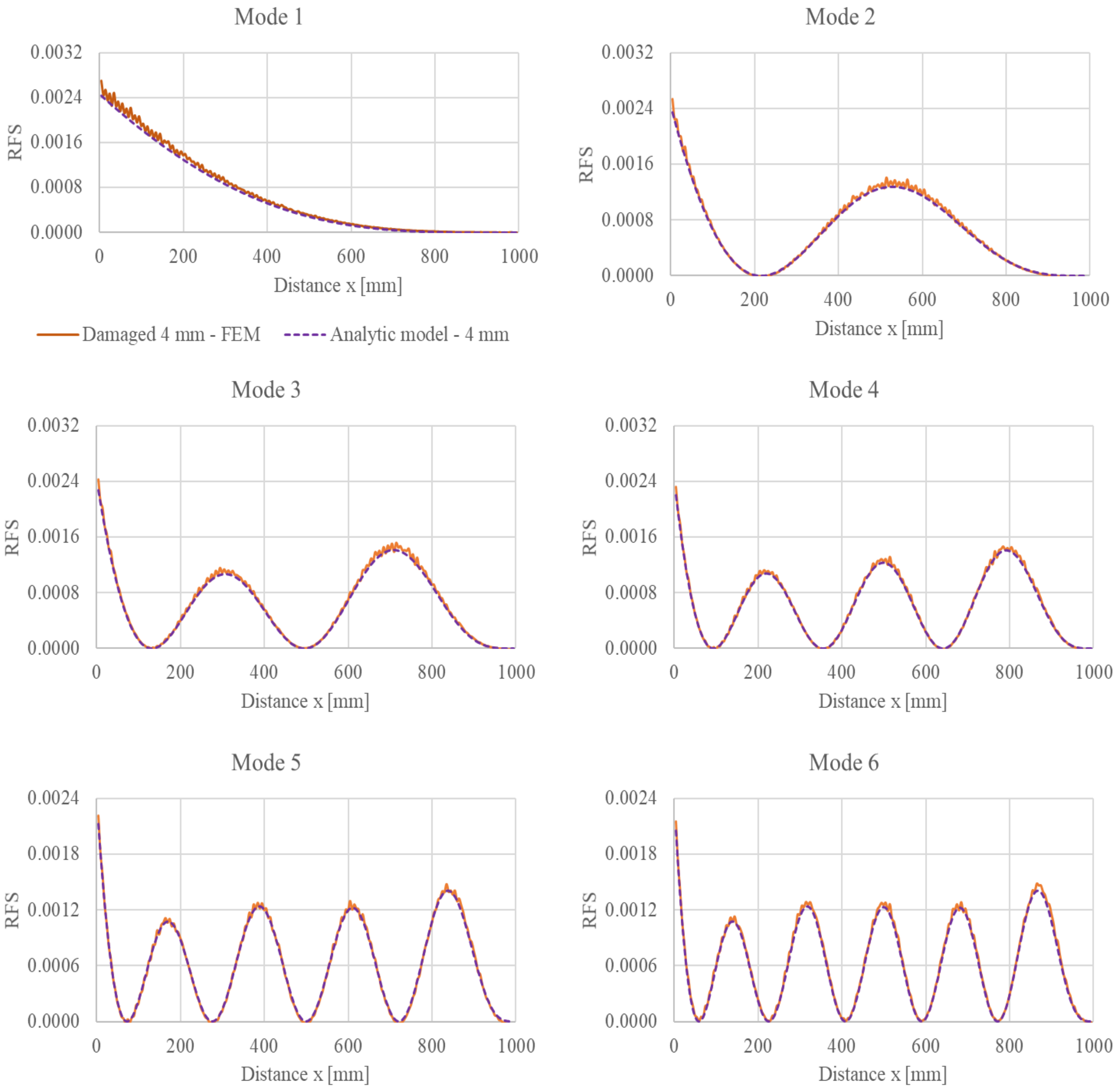


Figure 5: Comparison of the RFS evolution with the crack location for the 4 mm beam

4. CONCLUSION

In the current study, we analyze the behavior of two cantilever beams in order to determine the possibility of obtaining the similar severity values in the case of having proportional transverse cracks relative to the thickness of the cantilevers. It was found that, even if the damage depth is proportional for the two beams, the severity of the two cracks is still different for each beam. This happens because the ratio between length of the beams and the thickness is not similar.

We also verified the method developed in paper [12] and found that it can be trustfully used to calculate the severity of any transversal crack size for different beam sections, when it is located at distance *x*=0 mm from the fixed end.

Finally, by analyzing the RFS curves plotted involving relation (3) and with help of the FEM simulation, we can observe that they are similar. This lead to the conclusion that for beams with proportional dimensions affected by the same extent of damage the RFS curves will be identical. Our further research will also address such damage scenarios with the possibility to employ a mathematical model for detecting proportional cracks in beam-like structures using only the damage signatures for any boundary type.

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