INTERNATIONAL SCIENTIFIC CONFERENCE
CIBv 2010
12 – 13 November 2010, Braşov

VIEWPOINTS ON SOME VIBRATION FEATURES OF THE RAILWAY TRACTION UNITS THE BRACHISTOCRONA PROBLEM

Mugurel-Nicolae GĂMAN[1], Ovidiu VIZANTE[2], Dinu DRĂGAN[3], Iosif SZENTES[4], Valerica PURGE[5], Alexandru GÂTLAN[6], George DUMITRU[7]

[1]PhD. eng., Head of Subsistems for Infrastructure/Instalations Department, The Romanian National Railway Authority - ASFR / AFER
[2] PhD. student, eng., Politechnica University of Bucharest (UPB), Branch Manager - Rail Cargo România, Ein Unternehmen der Österreichische Bundesbahnen
[3]PhD. student, eng., UPB, General Manager of the National Railway Authority- ASFR / AFER
[4]PhD. student, eng., UPB, Manager of the National Railway Training Center in Romania - CENAFER Braşov
[5]Eng., Head of Department - Teaching Technical Support , CENAFER Bucharest
[6] PhD., eng., UPB, Faculty of Transport, Territorial Chief Inspector of ISF - ASFR / AFER Bucharest
[7]PhD. eng., Head of Department of Locomotives & Rolling Stock – CENAFER Bucharest

Corresponding author: George DUMITRU, E-mail: george.dumitru.cfr@gmail.com

Abstract: A Brachistochrone curve (Gr. βραχίστος, brachistos - the shortest, χρόνος, chronos - time), or curve of fastest descent, is the curve between two points that is covered in the least time by a body that starts at the first point with zero speed and is constrained to move along the curve to the second point, under the action of constant gravity and assuming no friction. For less dynamic oscillations of a locomotive in case of jump and galop at high speeds and for limiting damper amplitudes it is a must that for any point on the suspension, the trajectory described during a half oscillation should be as short as possible. The virtual distance covered by a certain point on the suspension is a curve. The point trajectory may be comparative to a epicicloide equation. It is identified the plane curve of the point considered in the suspension plan - the brachistocrona problem, finding that curve in the family of plane curves conecting the two points A and B (the trajectory oscilation extremities). A material point leaving origin A with null initial speed, is subject to a force and reaches point B in the shortest time.

Key words: Brachistochrone curve, wheel-rail dynamic, railway

1. INTRODUCTION

Looking from the point of view of high speed railway transport, particular with more than 200 km/h, if we consider only the dynamic forces appearing at wheel-rail contact level, it was concluded that these may be caused by any irregularity, however small, that may occur in the longitudinal profile of the rail.

Given two points A and B, with A not lower than B, there is just one upside down cycloid that passes through A with infinite slope, passes also through B and does not have maximum points between A and B. This particular inverted cycloid is a brachistochrone curve. The curve does not depend on the body's mass or on the strength of the gravitational constant.

The problem can be solved with the tools from the calculus of variations. If the body is given an initial velocity at A, or if friction is taken into account, the curve that minimizes time will differ from the one described above.

Time passing, it was found that these irregularities multiply because of exploitation wear which leads to the increasing effects of track action / reaction. High dynamic forces caused by these irregularity damage the ride quality of the rail vehicle.

It is also known that once the rails are fastened for high speeds traffic there are no curves with sharp rounded radius which could cause attack shocks on the run. Nevertheless, sinusoidal wear of flank areas may appear on guard rails, particular in the case of a large path run. Also known is the fact that heavy concrete sleepers, imbeded in compact ballast, they increase the chance of big identure.

2. PAPER CONTENTS

The sinusoidal wear of flanc areas amplify sound vibrations, particular an increased noise at train passing. For example at high speeds, even irregularities worth 0.03 mm, can produce important discomfort to passengers and also for sideway residents. In countries such as Japan where it is usual to travel at 300 km/h speed, rail grinding is first intended to reduce noise.

The sinusoidal wear of the wheels flank areas determines an increasing noise vibration but also structural vibrations of the track, thereby the infrastructure is overloading. Accordingly, the study of wheel - rail interaction has a major importance in the high speed rail traffic, for reasons such as large forces transfer in a small contact area and side stability seen from the wheel guidance of the rail vehicle point of view.

Also, if the head rail geometry does not match the wheel rolling surface, then plastic deformations are likely to occur after contact, and material wear limits may be exceeded for both wheel and rail. One of the immediate consequences is that surface cracks appear on the rail head.

Another important aspect is that even for new railways the rail surface is not always perfect. As for the latest rail types, especially those in steel with increased hardness, the rate wear is very low, almost a valent and the geometric profile of running wear can not quickly adapt to the ideal profile. Nevertheless, surface grinding for new rails after installing is used to ensure optimal conditions for the wheel - rail contact from the beginning.

In curves with sharp rounded radius rail head is exposed to fatigue wear and cracks appear on the rail surface, the so called rail head cracks. To delay this type of fatigue wear it is necessary to adopt a special rail profile which transfers the wheel - rail contact area sideways, where a natural wear occurs anyway. These so called anti - crack profiles have proven to be efficient because it provides a good gauge relief.

Another approach is the negative tolerance, connected to the desired standard profile to avoid constrained wheel - rail contact on a limit surface of the spot contact, by the listing A_{10}. Thus, at high speeds rolling stock is much more sensitive to any side irregularity, the control parameter in this situation is equivalent conicity.

One important aspect to be detached as a conclusion for previous judgment is that, when increasing train speed, the equivalent conicity should be lowered. This is a reverse proportionality between the two parameters outlined above, so as to reduced the premises for side oscillation of the rolling stock. For speeds higher than 200 km/h, the appropriate value for conicity should be of about 0.1 and not exceed 0.3.

To ensure a smooth running along the guard rail the gauge must be kept relatively constant. In this way, side oscillating phenomenon are reduced, and unstable running conditions, often referred to as side motions, do not occur.

Where gauge width is insufficient, this can be corrected by grinding activities to a larger profile. Besides geometric irregularities, rail head may suffer structural changes. While operating, running surface damage and wear fatigue progress, and interventions are required. Some high speed lines have deep cracks on the running surface. High speed trains running produce and air turbulence around and under the train.

In terms of the dynamic forces at wheel - rail contact level, they have a variable nature. They exist between the vehicle and the running surface and can be generated by rolling profile, which is designed to control wheel sideways movements and to improve contact. Side suspension and the running characteristics of the vehicle contribute to the lateral forces, and this form of sideways oscillation can be controlled through an effective design and maintenance.

Running features also depend on the suspension performance, allowed motion between the axle, bogie and vehicle structure, and on the motor axle effect. For all this, running surface geometry is very important and even a critical element. For an effective maintenance, a high standard of construction, a large mass per length unit to reduce sideways oscillation are essential.

Some other rail or vehicles flaws may cause a swinging of the vehicle, a gallop movement or vertical instability, all those enhancing lateral forces. These forces can produce plastic breaks of the rail, causing temporary deformations, or can permanently distort the rail, depending on the size of sideway force. Permanent plastic deformations are directly involved in railway safety.

In technical literature, permanent deformations are also known as null deviations. If these deformations accumulates over time because of the frequent passage of bad maintained rolling stock, it is likely to progressively damage the rails and to cause vehicles sideway oscillations. This represents to a vicious circle that result in a premature wear of both vehicles - their suspensions - and running surface.

Researches undertook by *André Prud'Homme* at the SNCF between 1950 - 1960 led to the conclusion that if the sideway force exerted repeatedly on rail does not exceed a certain H value, null deviations are not sum up endlessly and the final deviation is set out in acceptable limits. This limit value is represented by the H force, where and H is sideway force, and P is static axle load. This relation was originally designed for wooden sleepers track caught in ballast without thermal variations. H force is considered guard rails limit resistance which fulfill criteria for acceptability of null deviation, under the pressure of repeated lateral load that competes with P vertical load. *Prud'Homme* later expanded his research taking count of thermal variations in the welded rail and applied a multiplier factor of 0.85.

The tests results conducted after 1990 by SNCF on concrete sleepers, welded ballast tracks, with dynamic rail stabilizer, on a high speed line as the one between Paris and Lyon, did not brought significant changes on *Prud'homme* formula. The H force has been defined as the maximum repeated lateral force that can be exercised over track by rail vehicle.

A safety reasonable limit, between guard rail resistance and lateral pressure improves running comfort and increases safety, providing in the same time the effective and optimal rail and vehicles maintenance. This issue has a simple solution, first by adopting a system ortogonal Cartesian Oxy with A the point of origin, leading the Ox axis on horizontal direction and focus on Oy vertically downward direction.

The problem can be simplified more, at the design level, starting from the premise that a straight line is the shortest distance between all two points in space up against a material point. Accordingly, all reasoning is confined to identify the function y, as a heteronymous function, which is compulsory holomorphic and of class at least C^1 and follows the differentiability theorem by *Cauchy-Riemann*. Also, the function y is a automorphism group of *Riemann* sphere, a *Möbius* transformation, that is a heteronymous transformation, made of a translation and homotheties, that is a rotation and a dilatation.

The heteronymous transformations lead circles in the broad sense, in broad circles, where “circles in the broad sense” means a circle or a line regarded as a circle that passes through the infinite point. Last but not least, it is important to note that heteronymous transformation is a group compared to the composing operation and that each heteronymous transformation is a omeomorphism. Going back to the shortest distance that a moving point on the suspension covers, it is easy to identify its canonic form calculating the primitive function $T(y)$.

3. EQUATIONS

$$\mathrm{H}_1 = 10 + 0.33 * \mathrm{P}, [\mathrm{kN}], \quad \mathrm{H}_2 = 0.85\,(10 + 0.33 * \mathrm{P}). \tag{1}$$

$$y:[0,b] \to R, \quad v:[0,b] \to R, \quad s:[0,b] \to R. \tag{2}$$

Also:
$$v^2(x) = 2.gy(x); v(x) = \frac{ds(x)}{dt} = \sqrt{1+y'^2(x)}\frac{dx}{dt}; \quad C_1 = \sqrt{2.g.y}.\sqrt{1+y'^2} \tag{3}$$

$$T(y) = \int_0^{\bar{s}} \frac{ds}{v} = \int_0^b \frac{\sqrt{1+y'^2}}{\sqrt{2.g.y}}dx = \int_0^b \frac{ds}{n(x,y(x))} = \int_0^b \frac{\sqrt{1+y'^2}}{n(x,y)}dx \Rightarrow \tag{4}$$

$$\Rightarrow (x-r)^2 + y^2 = r;$$

$$y' = \tan\varphi \Rightarrow \begin{cases} y = \dfrac{C}{2}\left(2.\varphi + \cos^2\varphi\right) \\ C' = \dfrac{C_1}{2.g} \end{cases} \Rightarrow y' = C.\varphi'.\sin 2.\varphi \Rightarrow$$

$$\Rightarrow \begin{cases} \tan\varphi = -2.C.\varphi'.\sin\varphi.\cos\varphi \\ dx = -2.C.\cos^2\varphi.d\varphi \end{cases} \Rightarrow \begin{cases} x = r.(\theta - \sin\theta) + k \\ y = r.(1-\cos\theta) \end{cases} \tag{5}$$

4. TABLES

Table 1 - Melting points and elemental analyses

A1	Y=27,368	Z=-72,805	A2	Y=-26,211	Z=16,446
B1	Y=28,374	Z=499,194	B2	Y=-58,558	Z=8,835
A3	Y=-7,267	Z=80,709	A4	Y=-55	Z=16
B3	Y=-22,506	Z=27,862	B4	Y=-49,5	Z=9,519

5. FIGURES

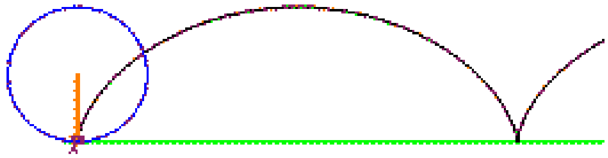

Fig. 1. External load determination - *Prud'Homme* limit

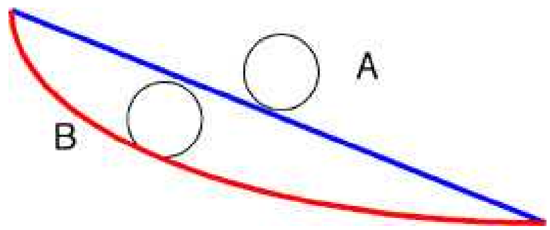

Fig. 2. The above figure demonstrates the difference between a circular path, straight path and cycloidal path.

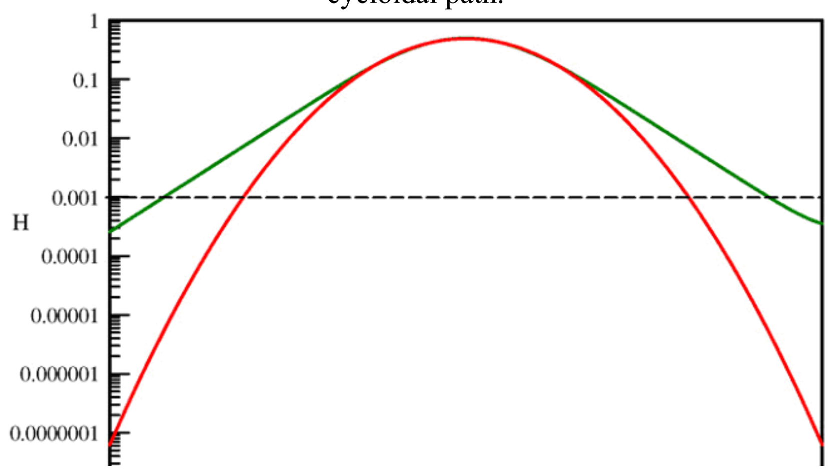

Fig. 3. The amplitudes of the tails $H_0 + H_1$ vs. the width at half-height $w_{1/2}$

6. CONCLUSIONS

The canonic form of the previous equation is the solution to the equation of light motion in strong gravity field, used to determine if there are celestial bodies at big distances that can not be observed directly because they are ecranated by other bodies, which are closer.

From studies on the vibrating phenomenon may be detached a few conclusions and assumptions for calculation, from the design suspensions perspective. For example, the vibrations absorber creates resistance forces but also dissipates a part of these vibrations energy. To avoid resonances between bogies gallop vibrations and those of flexure in the vehicle box structure, the axles suspension has a strong absorber.

Adopting elastic constants is strictly in interdependence with own high frequency of vertical suspension. This one, together with the vibration frequency of gallop bogie, must be outside own frequencies area of flexure in the vehicle box, which are between 8 and 10 Hz. To reduce these vibrations, which are likely to transmit from the axles to the bogie frame and subsequently to the whole vehicle box, it is used crossing absorber between the bogie and the box, especially when box and bogies oscillation frequencies are sensitive the same.

Note also the fact that dissipated energy on a operation cycle of elastic suspension is equal to the work of friction forces, and at low speeds, and low suspension effort on crosswise direction, the transformation process is one of adiabatic nature, having $n = 1{,}3;1{,}4$ polytrophic coefficient. In this case, damper dissipated energy during a motion period is equal to the work of damping forces. The *Timoshenko* differential equation shows that additional dynamic flexion of the running surface, because of running loads depends on the rail unevenness and on its depth but also on the relation between track vibration pulsation and forced vibration pulsation of the wheal set caused by denivelation itself.

This work opens up additional approaches to the utilization of variational approximations as a modeling technique for physical and mathematical systems. There is no longer the necessity for having exact (or close to exact) solutions a priori in order to determine the validity of a variational approximation. The advantage of this for nonlinear problems is that to obtain an estimate of the correction, one only needs to solve a linear problem, not the fully nonlinear problem.

REFERENCES

1. FUJIMOTO, H., MIYAMOTO, M. and Y. SHIMAMOTO, Y. *Lateral vibration of a Shinkansen and its decreasing measure*, RTRI Report, **Vol. 9**, No. 1, 19-24,1995.
2. FUJIMOTO, H., MIYAMOTO, M., *The vibration of the tail car in a coupled train*, Transactions of the Japan Society of Mechanical Engineers, **Vol. 53**, No. 494, C, 2110-2114, 1987
3. FUJIMOTO, H., MIYAMOTO, M., *A study of lateral dynamic behaviour of rail vehicle in curves*, Transactions of the Japan Society of Mechanical Engineers, **Vol. 58**, No. 548, C, 1067-1074, 2001.
4. ISHIHARA, T., UTSUNOMIYA, M., OKUMURA, M., SAKUMA, Y., and SHIMOMURA, T., *An investigation of lateral vibration caused by aerodynamic continuous force on high speed train running within tunnels*, Proceedings of World Congress on Railway Research, **Vol. E**, 531-538, 1997.
5. KALKER, J. J., *Three dimensional elastic bodies in rolling contact*, Kluwer Academic Publishers, Dordrecht, 1990.
6. ONO, Y. and TAMURA, T., *Numerical study for turbulence effect on aerodynamics of a square cylinder*, Proceedings of the Fifth Asia - Pacific Conference on Wind Engineering, 629-632, 2001.
7. PRUD'HOMME A., *Les problems que pase, pour la voie, la circulation des rames a grande vitesse*, Revue Generale des Chemins de Fer, Nov. 1976.
8. SIRETEANU, T., *The effect of sequential damping on ride comfort improvement*, C145/84, ImechE 1984.
9. SUZUKI, M., *Aerodynamic force acting on train in tunnel*, RTRI Report, **Vol. 14**, No. 9, 37-42, 2000.
10. SUZUKI, M. and H. FUJIMOTO, H., *A wind tunnel test on vehicle vibration in tunnel*, Proceedings of J-RAIL2002, 307-308, 2002.
11. SUZUKI, M., NAKADE, K. and FUJIMOTO, H., *Study on interaction between vehicle dynamics and aerodynamic force on high speed train*, RTRI Report, **Vol. 15**, No. 5, 19-24, 2001.
12. TAKAI, H., *Maintenance of long wave track irregularity on Shinkansen*, RTRI Report, **Vol. 3**, No. 4, 13-20, 1989.
13. VOINEA, R., VOICULESCU, D., SIMION, F., *Dynamic Systems*, University Politehnica of Bucharest's Publishing House, 1993.

Received October 20, 2010