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THE WEAKENING OF THE CLAMPED END OF A BEAM AND THE INFLUENCE ON THE DYNAMIC BEHAVIOR (PART I)

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Abstract: *Using analytical equations, the paper aims to solve the dynamic behavior of beams where a clamped end of the beam does not respect the ideal boundary conditions by introducing a weakening coefficient. In the paper, the characteristic equation for determining the eigenvalues and the relationship of the modal function are derived. The results show the first four vibration modes for different values of the weakening coefficient which is considered in the clamped end.*

Keywords: weak clamped end, *eigenvalues, mode shapes*

1. INTRODUCTION

Different types of failures it can be occurred in structures. They can be caused by a lots of factors. It can be mentioned: improper manufacturing conditions, loosening of joints due to shocks and excessive vibrations, degradation caused by environmental conditions, material fatigue and exceeding the expected operating demands [1].

The loss of integrity of structures can be attributed not only to the presence of cracks but also to joint failure, especially for beam-type structures. Methods used of modal parameters prove reliable for the detection and evaluation of damage in beams by applying several techniques like flexibility coefficients, derived stiffness matrix, the frequency response function FRF [2-5]

From a static or dynamic point of view, the analysis of beams with fixed ends involves the consideration of displacements and slopes perfect boundary conditions. Most researchers use measurement of natural frequencies to characterize imperfect boundary conditions, while others consider modal shapes to detect deviation from ideal conditions [6-7].

In this paper, the authors propose an analytical method regarding the dynamic behavior of a beam for which the clamped end is defined by a weakening coefficient k, so that for the zero value of k the support is considered the hinge, and for the 1 value of k, the support becomes clamped.

2. ANALYTICAL APPROACH

In order to analyze the dynamic behavior of a beam with L=1 (normalized beam) under the action of its dead weight (q) and having the constant cross section, it is considered the case of a clamped-hinge supported beam (Fig. 1). On the clamped end it was introduced a weakening coefficient $k_1 \in [0, \ldots, 1]$ which allows us to have both bending moment and slope in this support [8].

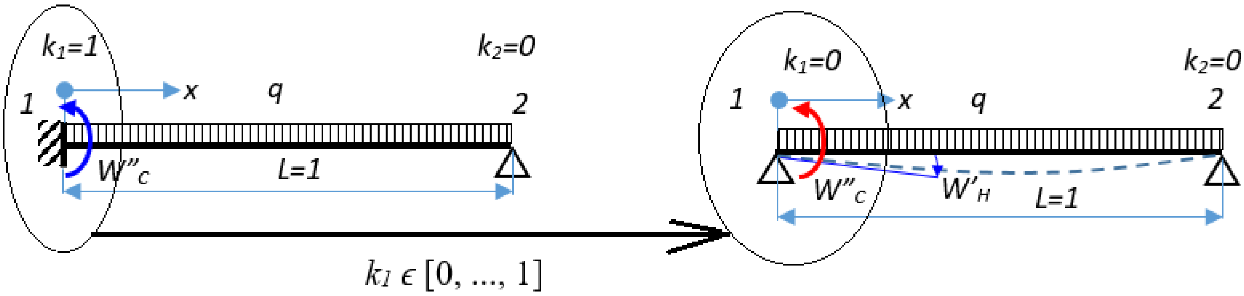

Figure 1: A schematic diagram of a weakened clamped end

It will be considered that for k_1=1, that the support is clamped (Fig. 1 – left) and for k_1=0, the support becomes a hinge (Fig. 1 – right). Any other value of $k_1 \in [0, ..., 1]$ is considered to be a weakened clamped.

It is known from the strength of materials that for a hinge support located at x=0, the boundary conditions for a beam loaded with its dead weight are: the deflection ($W_H(0)=0$) and the bending moment ($W_H(0)=0$) are equals to zero, The slope has the expression:

$$W'_H(0) = \frac{q \cdot L^3}{24E \cdot I}, \quad (1)$$

where,

q [N/m] is the load per unit of length (dead load);

L [m] is the beam length;

E [N/m^2] is the elastic modulus on, or Young's modulus;

I [m^4] is the moment of the inertia of the cross section.

and for the boundary conditions of a clamped end at x=0, the deflection ($W_C(0)=0$) and the slope ($W'_C(0)=0$) are equals to zero. The bending moment can be written as:

$$W''_C(0) = -\frac{q \cdot L^2}{8E \cdot I}, \quad (2)$$

Thus, if we apply the bending moment from relation (2) to the hinged at x=0, it becomes a clamped end, and the slope from relation (1) must be in the opposite direction and depending on the bending moment from (2) can be written:

$$W_H'(0) = -\frac{q \cdot L^3}{24E \cdot I} = -\frac{L}{3}\left(\frac{q \cdot L^2}{8E \cdot I}\right) = -\frac{L}{3}\left(-W_C''(0)\right) = \frac{L}{3}W_C''(0) \tag{3}$$

or, expressing the bending moment from (3) and taking into account the weakened stiffness k_1, we have:

$$k_1 W_C''(0) = k_1 \frac{3}{L} W_H'(0) \tag{4}$$

To satisfy the boundary conditions for x=0 and $k_1 \in [0, ..., 1]$, so that the left support to be a weakened clamped end, we will obtain the relation:

$$(1-k_1)W_H''(0) - k_1 W_C''(0) = (1-k_1)W_H''(0) - k_1 \frac{3}{L} W_H'(0) = 0 \tag{5}$$

From relation (5) it can be seen that for any other values of $k_1 \in [0, ..., 1]$, in the left support we will find both bending moment and slope.

3. MODAL ANALYSIS

For the Euler-Bernoulli model, we started from the spatial solution of the differential equation of bending vibrations, free and undamped:

$$W(x) = \mathrm{A}\sin(\alpha x) + \mathrm{B}\cos(\alpha x) + \mathrm{C}\sinh(\alpha x) + \mathrm{D}\cosh(\alpha x) \tag{6}$$

where,

W(x) is the modal motion function;

A, B, C, D are integration constants that are obtained from the boundary conditions;

α is the eigenvalue;

x is the variable length of the normalized beam.

For clamped end and hinged end, at x=0, the deflection is zero. Substituting x=0 in relation (6), we get:

$$W(0) = 0 = \mathrm{B} + \mathrm{D} \Rightarrow D = -B \tag{7}$$

Entering the result from (7) in the relation (6), the slope and the bending moment become:

$$\begin{cases} W'(0) = \alpha(\mathrm{A} + \mathrm{C}) \\ W''(0) = -2\alpha^2 \mathrm{B} \end{cases} \tag{8}$$

At the right end, for x=L=1, on the hinge support, considering (7) in (6), the deflection and the bending moment are equals to zero:

$$\begin{cases} W(1) = 0 = \mathrm{A}\sin\alpha + \mathrm{B}(\cos\alpha - \cosh\alpha) + \mathrm{C}\sinh\alpha \\ W''(1) = 0 = -\mathrm{A}\sin\alpha - \mathrm{B}(\cos\alpha + \cosh\alpha) + \mathrm{C}\sinh\alpha \end{cases} \tag{9}$$

The constants B and C are obtained from system (9):

$$\begin{cases} B = -A\dfrac{\sin\alpha}{\cos\alpha} \\ C = -A\dfrac{\sin\alpha\cdot\cosh\alpha}{\cos\alpha\cdot\sinh\alpha} \end{cases} \tag{10}$$

By introducing the constants B, C and D in relation (5), the characteristic equation (11) is obtained whose solutions give us the eigenvalues for each vibration mode and the modal function is presented in relationship (12):

$$2\alpha(1-k_1)\sin\alpha\cdot\sinh\alpha + k_1\frac{3}{L}(\sin\alpha\cdot\cosh\alpha - \cos\alpha\cdot\sinh\alpha) = 0 \tag{11}$$

$$W(x) = A\left[\sin(\alpha x) - \frac{\sin\alpha}{\cos\alpha}(\cos(\alpha x) - \cosh(\alpha x)) - \frac{\sin\alpha\cdot\cosh\alpha}{\cos\alpha\cdot\sinh\alpha}\sinh(\alpha x)\right] \tag{12}$$

4. RESULTS

The eigenvalues for the first four vibration modes (n=4) and different values of k_1, solutions of relationship (11), can be found in table 1.

The first 4 (four) normalized vibration modes for the following values of k_1=0.0, 0.25, 0.50, 0.75, 0.85, 0.95 and 1.00 are illustrated in the Fig. 2 – 5.

Table1. Eigenvalues for the first four vibration modes

k_1	Vibration mode (n)			
	1	2	3	4
1.0	3.9266023	7.0685830	10.2101800	13.3517700
0.95	3.8632168	6.9601886	10.0612560	13.1662817
0.85	3.7479163	6.7905457	9.85839280	12.9437574
0.75	3.6459736	6.6662741	9.73103940	12.8205578
0.50	3.4364156	6.4692320	9.55958400	12.6718200
0.25	3.2447893	6.3389809	9.46287750	12.5952748
0	π	2 π	3 π	4 π

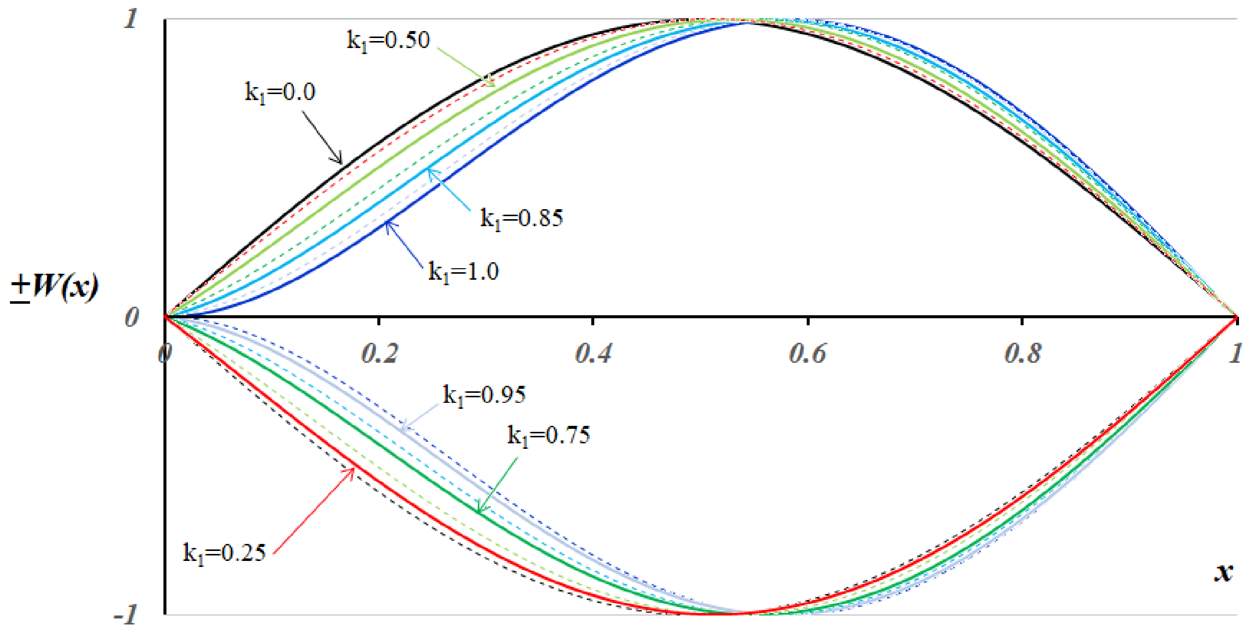

Figure 2: Normalized mode shapes for the first vibration mode

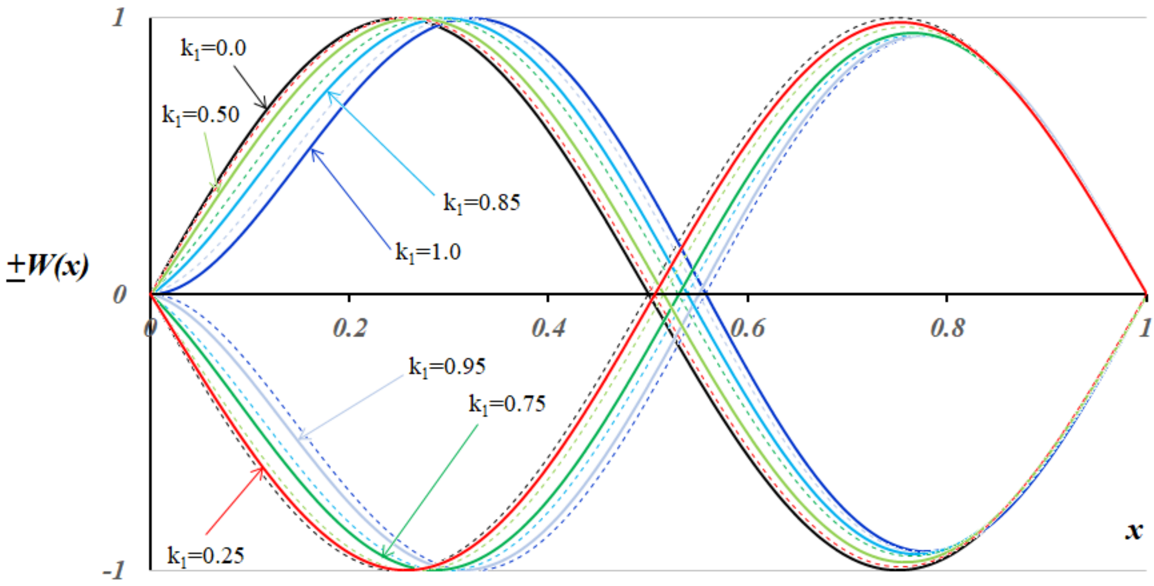

Figure 3: Normalized mode shapes for the second vibration mode

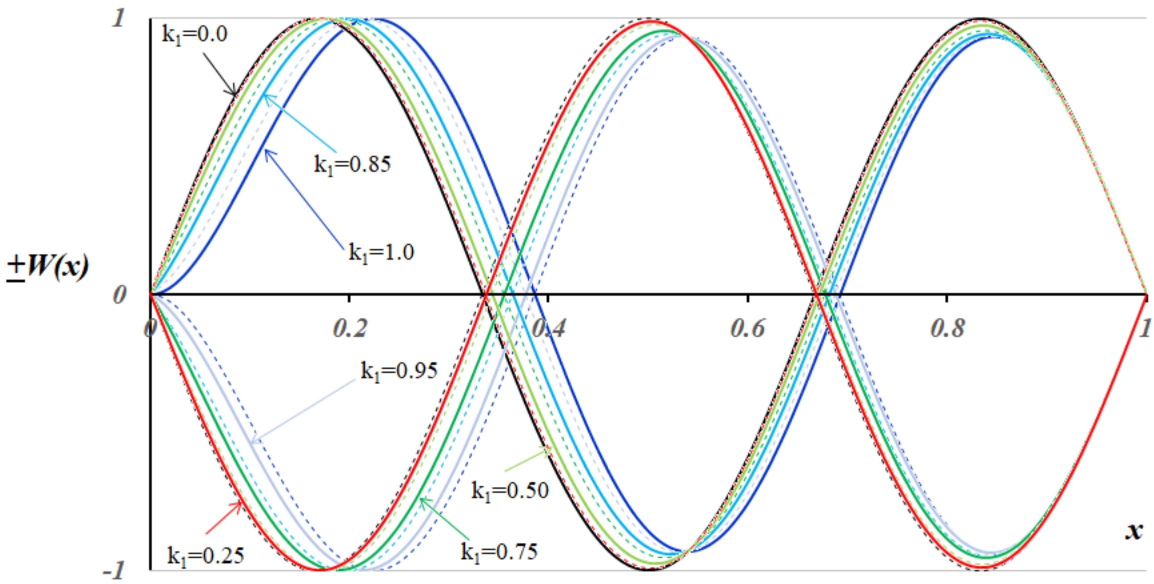

Figure 4: Normalized mode shapes for the third vibration mode

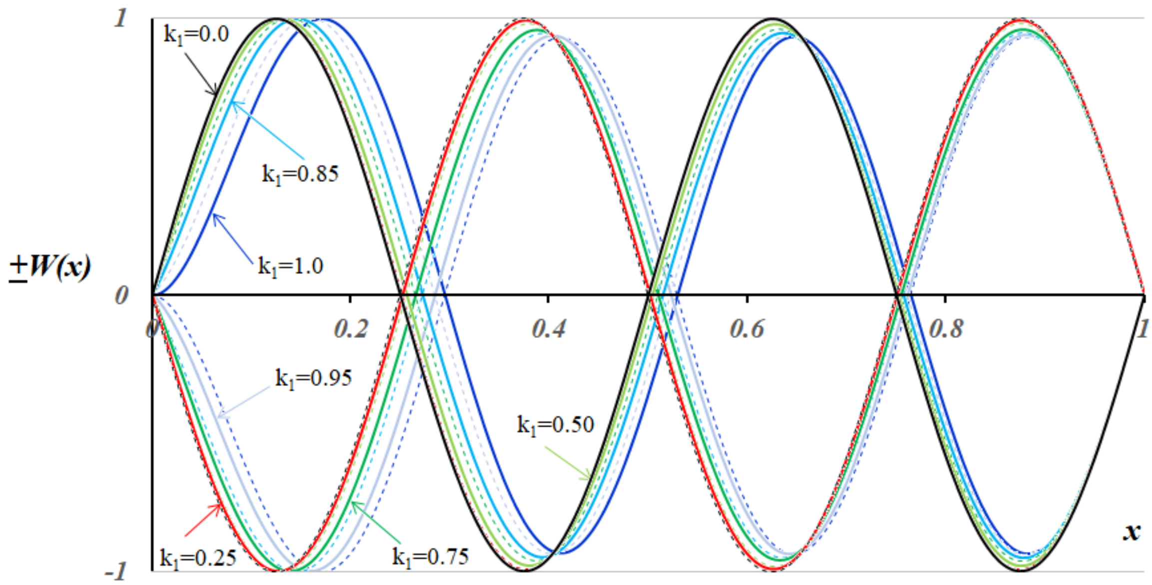

Figure 5: Normalized mode shapes for the fourth vibration mode

1. CONCLUSIONS

The paper presents the eigenvalues and modal shapes for the first four vibration modes for the case where the clamped end of the beam is weakened by the coefficient k_1.

For the extreme cases: $k_1=0$, the eigenvalues (Table 1) were obtained from the simply supported beam (hinged at both ends); respectively for $k_1=1$, we find the eigenvalues for the beam clamped at one end and hinged at the other.

From the analysis of the figures 2 – 5, for the first 4 modes of vibration, it can be observed that for stiffness values $k_1<0.5$, from the point of view of dynamic behavior, the weakened clamped end has a behavior very close to that of a hinged support.

For stiffness values $k_1>0.5$, the dynamic behavior of the beam is significantly affected and although the relation (5) that describes the weakened clamped end of the beam is a linear expression of k_1, the effect of k_1 in the modal function does not have a linear behavior.

Also, from the figures 2 – 5, it can be seen the changes in the mode shapes from the case of the clamped-hinge beam to the simply supported beam, respectively to the hinged-hinged beam by modifying the weakened coefficient k_1.

BIBLIOGRAFIE

[1] Lupu D., Tufi;I C., Gillich G.R., Ardeljan M., Detection of transverse cracks in prismatic cantilever beams affected by weak clamping using a machine learning method, Analecta Technica Szegedinenesia, Vol. 16, No. 01, ISSN 2064-7964, pp. 122-128, Szeged, Hungary, 2022.

[2] Tufi;I C., Gillich G.R., Lupu D., Aman A.T., Detection of weak joints and damages for beams using machine learning, Vibroengineering PROCEDIA, Volume 46, ISSN 2345-0533, pp. 8-13, 2022.

[3] Karthikeyan M., Tiwari R., Talukdar S., Identification of crack model parameters in a beam from modal parameters, in 12th National Conference on Machines and Mechanisms (NaCoMM-2005), 2005.

[4] Nahvi H., Jabbari M., Crack detection in beams using experimental modal data and finite element model, International Journal of Mechanical Sciences, Vol. 47, No. 10, pp. 1477–1497, Oct. 2005.

[5] Dems K., Turant J., Structural damage identification using frequency and modal changes, Bulletin of the Polish Academy of Sciences Technical Sciences, Vol. 59, No. 1, pp. 23–32, 2011.

[6] Lupu D., Gillich G.R., Nedelcu D., Gillich N., Mănescu T., A method to detect cracks in the beams with imperfect boundary conditions, International Conference on Applied Science (ICAS 2020), Journal of Physics: Conference Series 1781(2021) 012012, IOP Publishing, pp. 1-13, 2021.

[7] Shi D., Tian Y., Choe K.N., Wang Q., A weak solution for free vibration of multi-span beams with general elastic boundary and coupling condition", JVE International Ltd. Vibroengineering PROCEDIA, vol. 10, ISSN 2345-0533, pp. 298–303, 2016.

[8] P. Bratu, Analiza structurilor elastice. Comportarea la acțiuni statice și dinamice, ISBN 978-973-8132-73-3, Ed. IMPULS, București, România, 2011.