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GIELIS' TRANSFORMATION AS POTENTIAL APPLICATION FOR CONVEX BILOBE CENTRODES

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Abstract: *In this paper a potential technical application is mentioned, i.e. a non-circular gears train whose pinion pitch curve belongs to the Gielis complex shapes family. In order to insert a Gielis' supershape in the gear theory, the influence of its defining parameters is examined by taking into account i) the conjugated pitch curves geometry and induced gear ratio variation and ii) the pressure angle variation, as an indicator of the motion transmission. Since there are eight parameters involved - the Gielis transformation parameters and the specific gears parameters, highly enlarging the study's hypotheses, the analysis is restricted to bi-lobes gears, with close pitch curves. The analysis results could inspire the non-circular gears designers who are looking for a certain variable gears kinematics.*

Keywords: *Gielis' supershape, non-circular gears, gears ratio, gear pressure angle.*

1. INTRODUCTION

Since it was introduced as a mathematical instrument that would describe various botanical shapes [1], the Gielis' transformation/superformula has been widely spread and implemented in mathematics, science and technology. Varying the transformation parameters, lots of researchers have found a spectacular challenge in identifying and designing simple or complex, symmetric or asymmetric, pinched or bloated forms to be applied in many areas. In terms of technological benefits of Gielis' supershape, mechanics, mechatronics, computer graphics and modeling are just some of the fields that have been used the new geometrical approach [2].

Non-circular gears are those mechanisms parts that exhibit special shapes in order to fulfill specific tasks, such as variable speed or specific law of motion for the driven element. One of the non-circular gears design hypotheses [3] refers to the predefined pinion pitch curve geometry and gears center distance, requiring determination of the conjugate pitch curve and transmission ratio variation; based on this hypothesis, several records are found in literature on modeling and analyzing non-circular gears. Tong and Yang presented a generalized method to design pairs of identical conjugate two-lobe and N-lobe pitch curves, describing the geometry of the driving pitch curve by a monotonically increasing function, with at least C^1 continuity, and two parameters, the minimum and maximum values for the curve radius, respectively [4]. Tsay and Fong introduced Fourier series for the non-circular pitch curve approximation, in order to improve the design flexibility [5]. Yang et al. presented a reshaping algorithm for the design of noncircular internal pitch curves whose outer pitch curve is initially described by desirable monotonic sinusoidal and polynomial functions, further modified to assure correspondence between the curves' kinematics and the number of pitch lobes [6]. Litvin developed a detailed analysis of the non-circular gears design procedure and considered conventional and modified elliptical centrodes, oval centrodes and centrodes with lobes, as examples [7]. Andrei and Vasie introduced the Gielis' supershape as the driving non-circular pitch curve in the general algorithm of the non-circular gear generation [8].

The pressure angle is an important parameter in gears theory, influencing the power transmission capacity and the force transmission conditions, the meshing gears vibrations and noise etc. In case of non-circular gears, the pressure angle varies due to the variable normal direction to the tooth flanks. Litvin analytically expressed the non-circular gears pressure angle and illustrated the change of pressure angle magnitude in some particular types of gears, correlated with geometrical parameters of the specific pitch curves and a predefined tool profile angle [7]. Danieli and Mundo proposed a new methodology of non-circular gears generation, in order to increase the gears contact ratio, considering a constant pressure angle for any given tooth, but variable from one to the next [9].

In this paper, the Gielis complex supershape family is chosen as a geometrical support for the non-circular gears pitch curve generation and an analytical study is developed, focused on the transformation parameters limitations that would enable the generation of a convenient gear shape. Due to numerous parameters involved in the Gielis superformula and the „infinite” combination possibilities, the authors narrowed down the investigations, focusing the analysis on closed convex curves with two lobes. For these geometries, the variations of both the gear ratio and the pressure angle, with consequences on the transmission kinematics and quality, are chosen to filter the non-circular gear pitch curve. The analytical algorithm and graphs are developed based on Java programming original codes and Excel software.

2. INFLUENCE OF THE SUPERSHAPE'S PARAMETERS ON CENTRODES GEOMETRY AND TRANSMISSION RATIO

One of the requirements for the non-circular gears design includes the definition of one gear pitch curve geometry and the gears center distance. The present study has chosen the Gielis' superformula [1] for the driving centrode polar definition:

$$r_1^*(\varphi_1) = \left(\left| \frac{1}{a} cos\left(n_0 \frac{\varphi_1}{4}\right) \right|^{n_2} + \left| \frac{1}{b} sin\left(n_0 \frac{\varphi_1}{4}\right) \right|^{n_3} \right)^{-\frac{1}{n_1}} \quad (1)$$

where r_1*, φ_1 are the polar coordinates; *a*, *b* – the supershape semi-axes lengths, n_0 – a real parameter that induces rotational symmetry; n_1, n_2, n_3 – real parameters that define the curve shape, $n_1 \neq 0$.

In this paper, a limited analysis is proposed, wherein the parameters involved in the above transformation are chosen as follows: *i*) $a = 1$, $b = 2$, specific to elliptical shapes, as base geometry; *ii*) $n_0 = 2$, generating closed centrodes, with two lobes; *iii*) $n_2 = n_3$, leading to symmetrical centrodes.

Considering the above specified values and relationship between the defining parameters, the Gielis' transformation will define a potential driving centrode from a mating centrodes pair, as follows:

$$r_1^*(\varphi_1) = \left(\left|cos\left(\frac{\varphi_1}{2}\right)\right|^{n_2} + \left|\frac{1}{2}sin\left(\frac{\varphi_1}{2}\right)\right|^{n_2}\right)^{-\frac{1}{n_1}} \quad (2)$$

The driven centrode geometry will be expressed by [7]:

$$r_2^*(\varphi_2(\varphi_1)) = A - r_1^*(\varphi_1) \; ; \; \varphi_2(\varphi_1) = \int_0^{\varphi_1} \frac{1}{i_{12}} d\varphi_1 \quad (3)$$

where r_2*, φ_2 are the polar coordinates of the driven centrode; *A* – the gears center distance; i_{12} – the gear ratio.

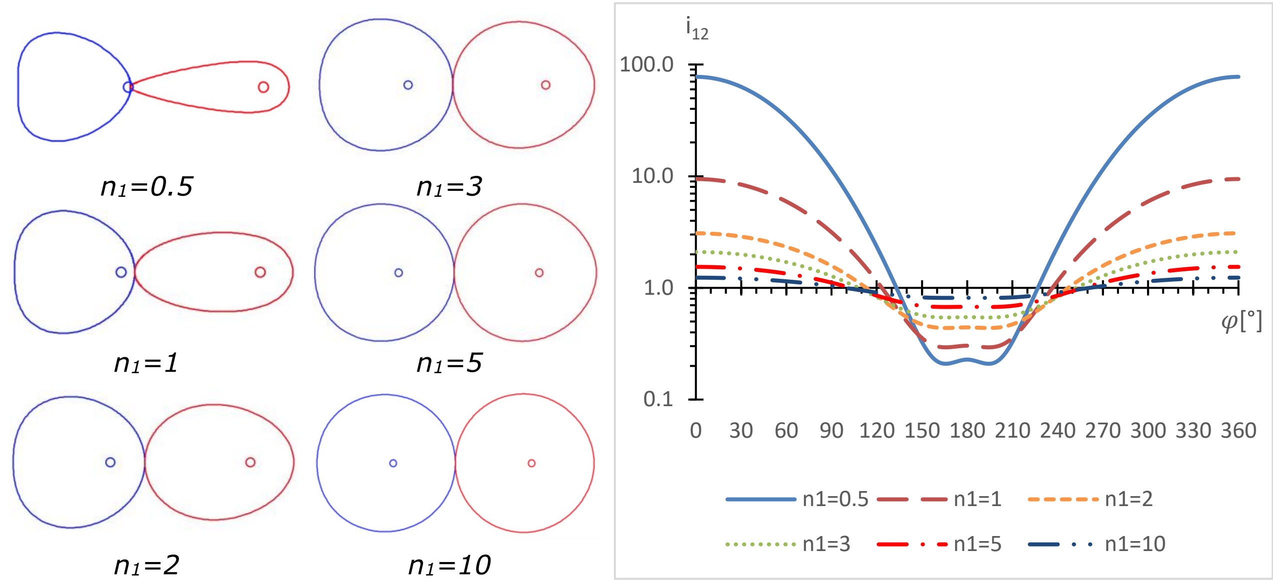
It is also assumed that both centrodes perform one single rotation during the rotational motion period; the unknown center distance *A* will be determined through an iterative algorithm, from equation:

$$2\pi = \int_0^{2\pi} \frac{r_1^*(\varphi_1)}{A - r_1^*(\varphi_1)} d\varphi_1 \quad (5)$$

In Figures 1, 2, the influence of the Gielis' transformation parameters on the conjugated centrodes geometry (Fig. 1a, 2a) and on the gear ratio (Fig. 1b, 2b) are illustrated. For a better view of values $i_{12} < 1$, the gear ratio is represented on logarithmic scale.

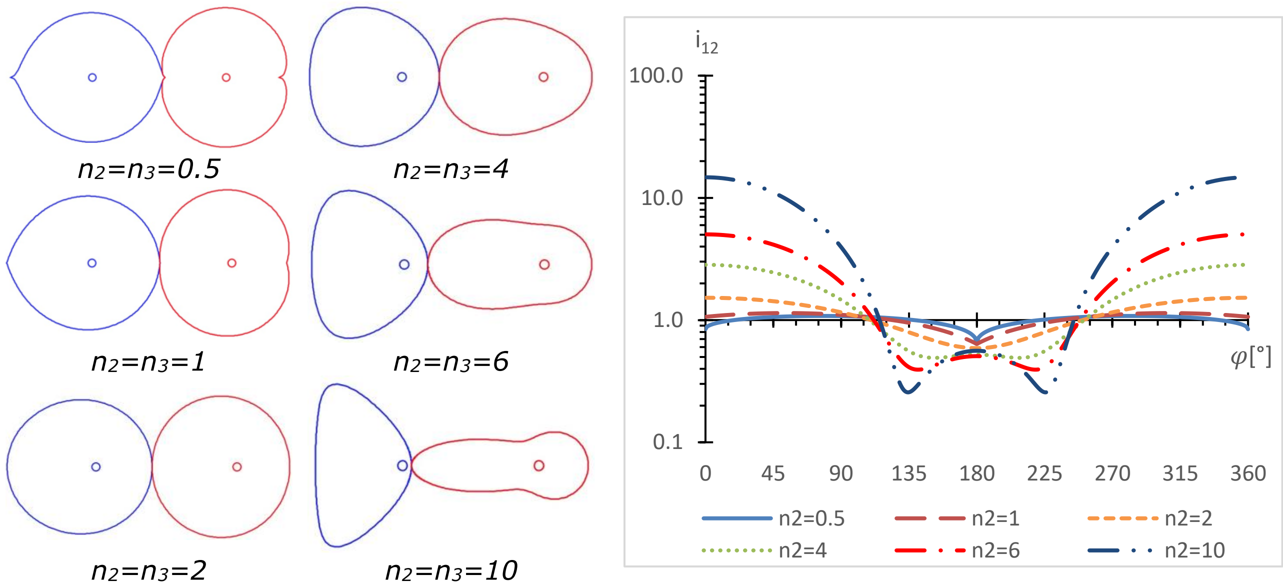
As shown in Figure 1, for the considered case of $n_2 = n_3 = 3$, at values $n_1 < 1$, the driving centrode exhibits slightly concave zones and the rotational center is moved close to the centrode curve, while sharp curves are identified for the driven centrode. Extremely high gear ratio and large variation, with high rate of increasing/decreasing amplitude, are recorded. As n_1 is increased, convex centrodes are generated, that change their geometrical appearance from elliptical to circular shapes. For $n_1 \geq 10$, almost circular shapes are generated for the gears centrodes and the gear ratio amplitude is reduced, getting a small oscillation around value 1, which corresponds to the circular centroids.

While the increase of n_1 parameter has a clear and intuitive influence on the centrodes geometries and kinematics, a different behavior is recorded when n_2 (n_3) parameter is varied. As illustrated in Figure 2, in case of $n_1 = 3$, the mating curves exhibit pairs of turning points while $n_2 < 2$, not being suitable as gears centrodes; convex curves are generated while $2 \leq n_2 \leq 6$; as the parameter is further increased, concave geometries appear on the driven centrode and the displacement of the driving centrode rotational center towards the curve extremity takes place (see n_2 = 10), which is an undesirable effect. Variation of the gear ratio increases with increase of n_2 parameter. Turning points appear in gear ratio representation, due to either sharp geometries or unproper position of the centrode rotational center.



(a) (b)

Figure 1: Influence of n_1 parameter on conjugated centrodes geometry (a) and on gear ratio (b) for $n_2 = n_3 = 3$



(a) (b)

Figure 2: Influence of n_2 parameter on conjugated centrodes geometry (a) and on gear ratio (b) for $n_1 = 3$

3. INFLUENCE OF THE SUPERSHAPE'S PARAMETERS ON PRESSURE ANGLE

In case of non-circular gears, the variation of the pressure angle directly influences the transmission quality through the modifications in both gear teeth flanks geometry and force transmission direction. Considering the Gielis' complex family shapes, the limitation of the pressure angle variation range could be achieved by proper choices of the conjugated pitch curves and tool profile angle; as shown in literature [7], the noncircular gears pressure angle (Fig. 3) is expressed by:

$$\alpha_{12} = \mu_1(\varphi_1) \pm \alpha - \frac{\pi}{2} = \tan^{-1}\frac{r_1(\varphi_1)}{r_1'(\varphi_1)} \pm \alpha - \frac{\pi}{2} \quad (6)$$

where $\mu_1(\varphi_1)$ is the angle between the tangent (t) and the extended vector specific to the pinion tooth active flank current point; α – the tool profile angle; the sign ± corresponds to the right and left side of the tooth flank, respectively.

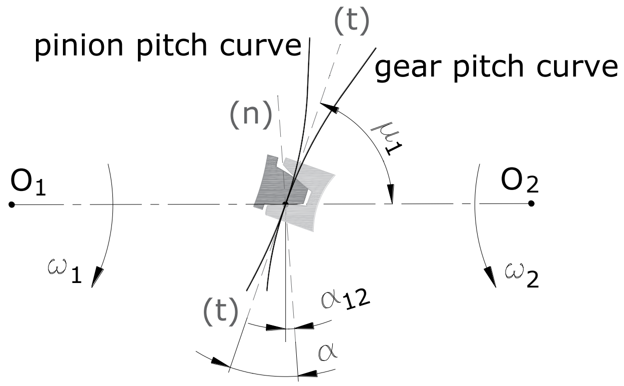
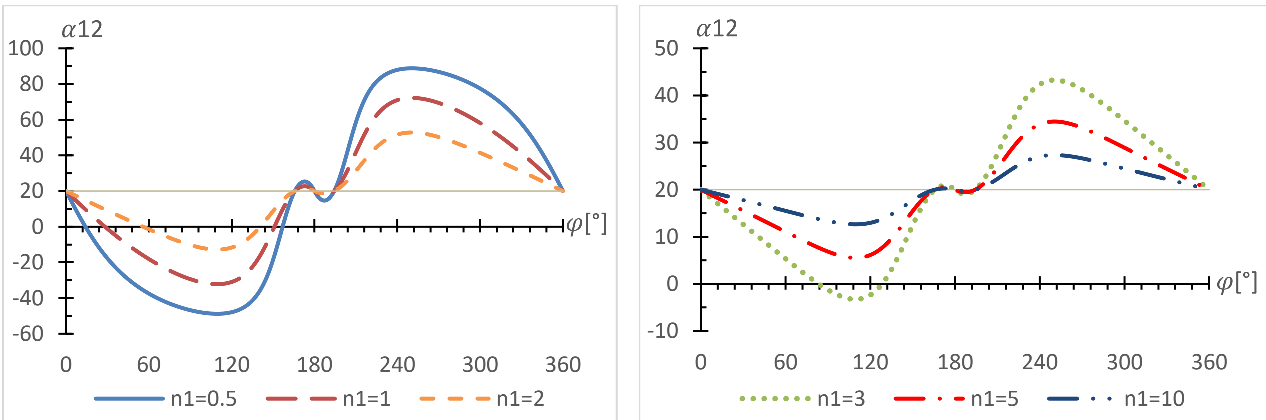
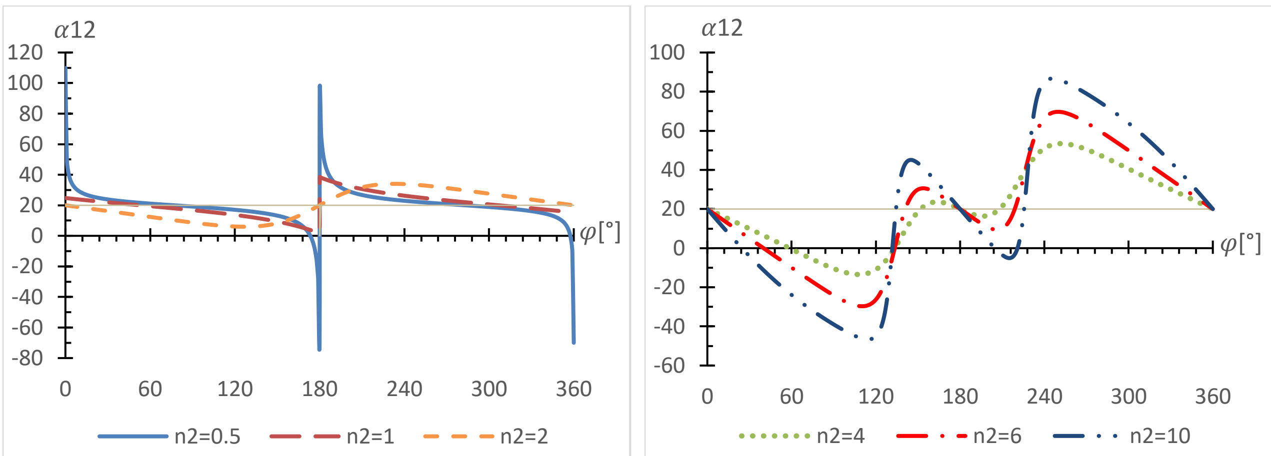


Figure 3: Pressure angle in non-circular gears



a) b)

Figure 4: Influence of parameters on gear pressure angle, for $n_2=n_3=3$



a) b)

Figure 5: Influence of parameters on gear pressure angle, for $n_1=3$

In Figure 4, the influence of Gielis' transformation parameters on the pressure angle is illustrated for the previously analyzed centrodes geometries, considering the tool profile at standard value 20°. As seen in Figure 4, when increasing n_1 values, the pressure angle variation is reduced; this would be expected due to the centrode geometry tendency towards circular shapes. Undesirable pressure angle variation is reached for n_1 = 0,5, especially due to the position of the centrode's rotational center (Fig. 2a), with influence on the vibrations and motion transmission quality. Values of n_2 (n_3) parameter, in the vicinity of 2, assure a uniform variation of the pressure angle, in convenient range; further increasing values induce non-uniform variation and large amplitude of pressure angle variation (Fig. 5).

4. CONCLUSIONS

The Gielis' transformation exponents influence on the mating gears centrodes geometries, transmission ratio and pressure angle are investigated in case of bi-lobes centrodes, with different axes lengths. It was found out that, for suitable geometries and kinematics of the gears centrodes, the values for the exponents n_1, n_2, n_3 should be chosen greater than 2; still, high values for the exponents of the Gielis' transformation trigonometric terms (n_2, n_3), in comparison to the main exponent n_1, lead to improper positions of the centrodes rotational centers, introduce concave profiles on gears centrodes and increase the amplitude of the pressure angle variation. The present limited study is just a part of a large data base that could be generated and could inspire a gear designer to use Gielis' supershape in order to get a specific non-circular gear ratio variation, controlling the gear pitch curves geometries and transmission smoothness.

REFERENCES

[1] Gielis J., A generic geometric transformation that unifies a large range of natural and abstract shapes, American Journal of Botany, 90(3), 333-338, 2003

[2] Gielis F. & Gielis J., Transformations and their impact on science and technology, Technical report, Belgium, 2021

[3] Sclater N., Mechanisms and Mechanical devices, Sourcebook, The fifth edition, McGraw-Hill, 2011

[4] Tong S.-H. and Yang D.C.H., Generation of identical noncircular pitch curves, Journal of Mechanical Design, 120(2), 1998

[5] Tsay M.-F. and Fong Z.-H., Study on the generalized mathematical model of noncircular gears, Mathematical and computer modelling, 41, 2005

[6] Yan J. et al., On the generation of analytical noncircular multilobe internal pitch curves, Journal of Mechanical Design, 130, 2008

[7] Litvin F. et al., *Noncircular gears. Design and generation*, Cambridge University Press, 2009

[8] Andrei L. and Vasie M., Noncircular gears designed by geometric hypothesis, Innovative Manufacturing Engineering Conference, Applied Mechanics and Materials, 657, 2014

[9] G.A. Danieli, D. Mundo, *New developments in variable radius gears using constant pressure angle teeth*, Mechanism and Machine Theory, Volume 40, Issue 2, February 2005, Pages 203-217