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THE WEAKENING OF THE CLAMPED END OF A BEAM AND THE INFLUENCE ON THE DYNAMIC BEHAVIOR (PART II)

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Abstract: *In the part II of the paper, the dynamic behavior of a doubly clamped beam is presented, where the right clamped end of the beam is weakened by introducing a weakening coefficient. The analytical calculation is based on the determination of the bending moment from the weakened clamped end expressed as a function of slope, after which the integration coefficients of the modal function and the characteristic equation are determined to obtain the eigenvalues of the first four vibration modes depending on the weakened coefficient of the clamped end. The obtained mode shapes are determined for seven values of the attenuation coefficient.*

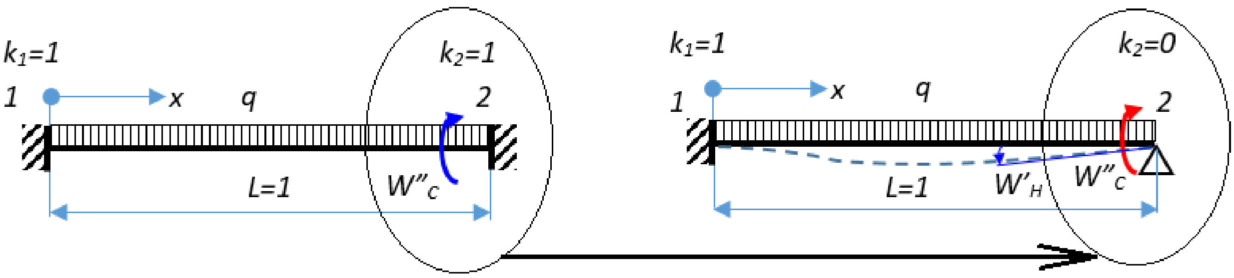
Keywords: weak clamped end, *eigenvalues, mode shapes*

1. INTRODUCTION

The second part of the paper is intended to be a continuation of part 1 and aims to analytically solve a weakened clamped end for a doubly clamped beam in terms of its dynamic behavior. For these reasons, the bibliographic citations are presented in the introduction chapter of part 1.

2. ANALYTICAL APPROACH

As in the previous paper (part I), the beam is of constant cross-section, with the normalized length L=1 (Fig. 1) and loaded with the uniformly distributed load q, which represents the dead weight.



$k_2 \in [0, \ldots, 1]$

Figure 1: A schematic diagram of a weakened clamped end

On the right clamped end, it was introduced a weakening coefficient $k_2 \in [0, \ldots, 1]$ which allows us to have both bending moment and slope in this support.

On the same ideas, k_2=1, means that the support is clamped (Fig. 1 – left) and k_2=0, means that the support becomes a hinge (Fig. 1 – right). Any other value of $k_1 \in [0, \ldots, 1]$ will be considered to be a weakened clamped.

Starting from the known relations from the strength of materials that for a hinge support located at x=L=1, the boundary conditions for a beam loaded with its dead weight are: the deflection ($W_H(L)=0$) and the bending moment ($W_H(L)=0$) are equals to zero. The slope has the expression:

$$W_H'(\mathrm{L}) = -\frac{\mathrm{q} \cdot L^3}{48E \cdot I}, \qquad (1)$$

where,

q [N/m] is the load per unit of length (dead load);

L [m] is the beam length;

E [N/m^2] is the elastic modulus on, or Young's modulus;

I [m^4] is the moment of the inertia of the cross section.

The boundary conditions of a clamped end at x=L, the deflection ($W_C(L)=0$) and the slope ($W'_C(L)=0$) are equals to zero. The bending moment can be written as:

$$W_C''(\mathrm{L}) = -\frac{\mathrm{q} \cdot L^2}{12E \cdot I}, \qquad (2)$$

By applying the bending moment from relation (2) to the hinged at x=L, it becomes a clamped end, the slope from relation (1) must be in the opposite direction and depending on the bending moment from (2) can be written:

$$W_H'(\mathrm{L}) = \frac{\mathrm{q} \cdot L^3}{48E \cdot I} = -\frac{L}{4}\left(-\frac{\mathrm{q} \cdot L^2}{12E \cdot I}\right) = -\frac{L}{4} W_C''(\mathrm{L}) \qquad (3)$$

Expressing the bending moment from (3) and taking into account the weakened stiffness k_2, we have:

$$k_2 W_C''(\mathrm{L}) = -k_2 \frac{4}{L} W_H'(\mathrm{L}) \qquad (4)$$

For x=L and $k_2 \in [0, ..., 1]$, so that the left support to be a weakened clamped end and to satisfy the boundary conditions, we will obtain the relation:

$$(1-k_2)W_H''(\mathrm{L}) - k_2 W_C''(\mathrm{L}) = (1-k_2)W_H''(\mathrm{L}) + k_2\frac{4}{L}W_H'(\mathrm{L}) = 0 \quad (5)$$

For any other values of $k_2 \in [0, ..., 1]$, in the right support, according (5), we will find both bending moment and slope.

3. MODAL ANALYSIS

Euler-Bernoulli model it is considered, and starting from the spatial solution of the differential equation of bending vibrations, free and undamped:

$$W(x) = \mathrm{A}\sin(\alpha x) + \mathrm{B}\cos(\alpha x) + \mathrm{C}\sinh(\alpha x) + \mathrm{D}\cosh(\alpha x) \quad (6)$$

where,

W(x) is the modal motion function;

A, B, C, D are integration constants that are obtained from the boundary conditions;

α is the eigenvalue;

x is the variable length of the normalized beam.

For clamped end, at x=0, the deflection and the slope are equals to zero. Substituting x=0 in relation (6), we get:

$$\begin{cases} W(0) = 0 = \mathrm{B} + \mathrm{D} \Rightarrow D = -B \\ W'(0) = 0 = \mathrm{A} + \mathrm{C} \Rightarrow C = -A \end{cases} \quad (7)$$

At the clamped right end, for x=L=1, the deflection is equal to zero. Entering the result from (7) in the relation (6), become:

$$W(1) = 0 = \mathrm{A}(\sin\alpha - \sinh\alpha) + \mathrm{B}(\cos\alpha - \cosh\alpha) \Rightarrow \mathrm{B} = -\mathrm{A}\frac{\sin\alpha - \sinh\alpha}{\cos\alpha - \cosh\alpha} \quad (8)$$

By introducing the constants B, C and D in relation (5), the characteristic equation (9) is obtained. The solutions of the characteristic equation give us the eigenvalues for each vibration mode.

$$\alpha(1-\mathrm{k}_2)(\sin\alpha\cdot\cosh\alpha - \cos\alpha\cdot\sinh\alpha) + k_2\frac{4}{L}(1-\cos\alpha\cdot\cosh\alpha) = 0 \quad (9)$$

$$W(x) = \mathrm{A}\left[\sin(\alpha x) - \sinh(\alpha x) - \frac{\sin\alpha - \sinh\alpha}{\cos\alpha - \cosh\alpha}(\cos(\alpha x) - \cosh(\alpha x))\right] \quad (10)$$

Also, by introducing the constants B, C and D in relation (6), the modal function is obtained and it is presented in relationship (10).

4. RESULTS

In table 1, the eigenvalues for the first four vibration modes (n=4) and different values of k_2, solutions of relationship (9), are given.

It can be observed that for $k_2=1$, the eigenvalues are for the double clamped beam, and for $k_2=0$, the eigenvalues are from the clamped-hinged beam.

The first 4 (four) normalized vibration modes for the following values of k_1=0.0, 0.25, 0.50, 0.75, 0.85, 0.95 and 1.00 are illustrated in the Fig. 2 – 5.

Table1. Eigenvalues for the first four vibration modes

k_1	**Vibration mode (n)**			
	1	**2**	**3**	**4**
1.0	4.7300407	7.8532046	10.9956078	14.1371655
0.95	4.6721294	7.7608209	10.8711137	13.9830025
0.85	4.5634558	7.6070802	10.6855517	13.7757550
0.75	4.4638126	7.4865337	10.5578496	13.6481460
0.50	4.2489669	7.2804336	10.3704785	13.4802534
0.25	4.0732205	7.1534397	10.2710579	13.3991294
0	3.9266023	7.0685830	10.2101800	13.3517700

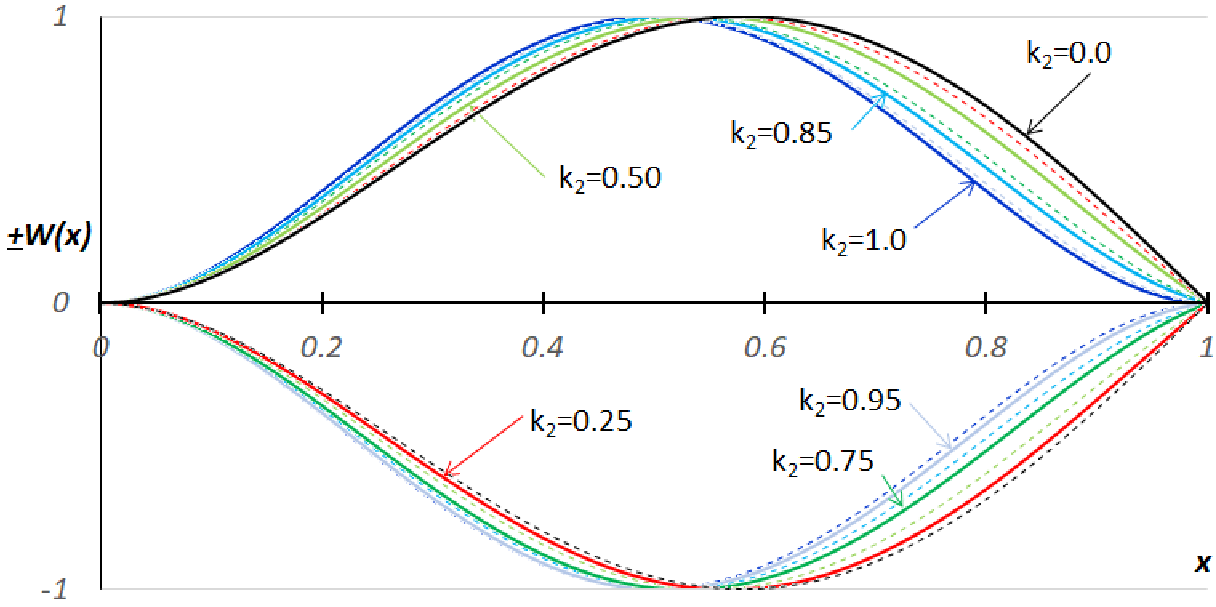


Figure 2: Normalized mode shapes for the first vibration mode

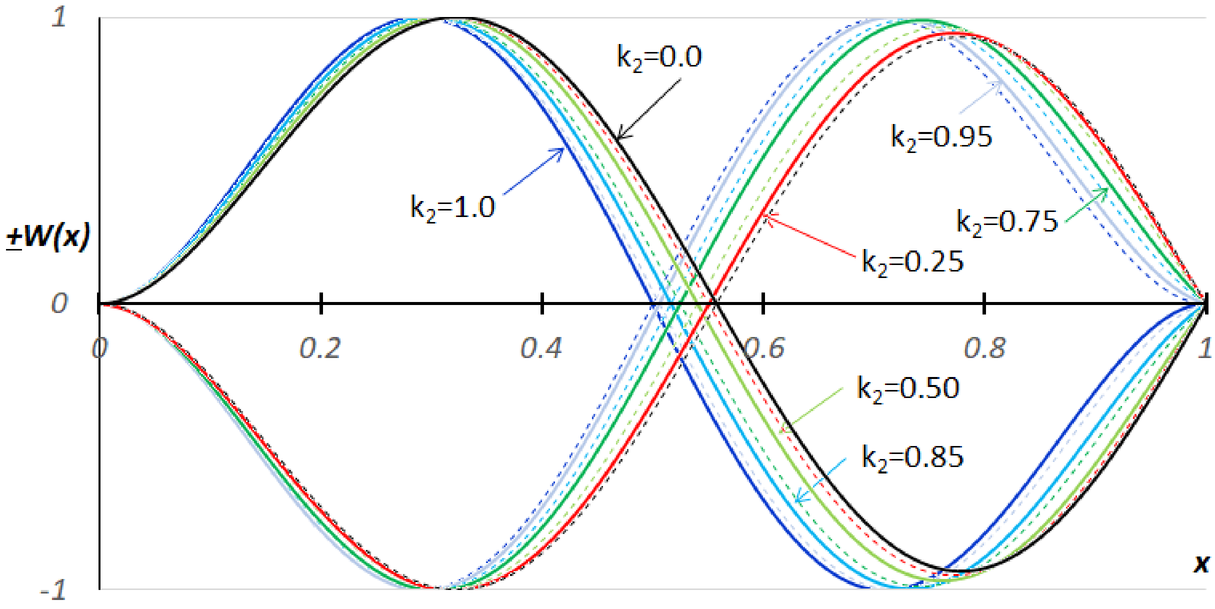


Figure 3: Normalized mode shapes for the second vibration mode

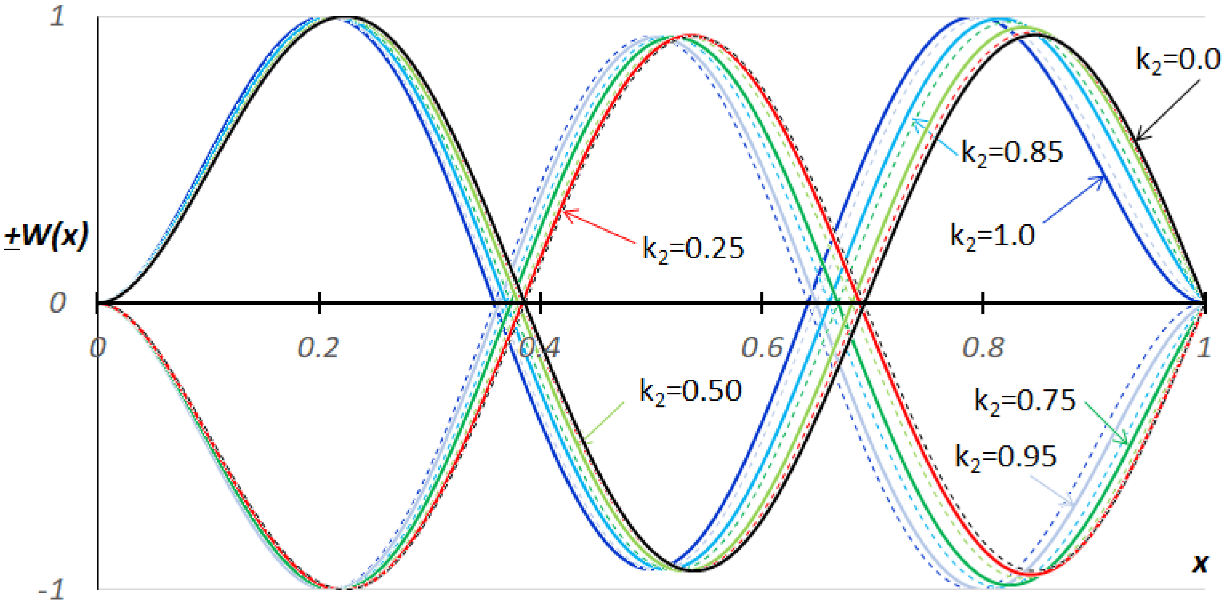


Figure 4: Normalized mode shapes for the third vibration mode

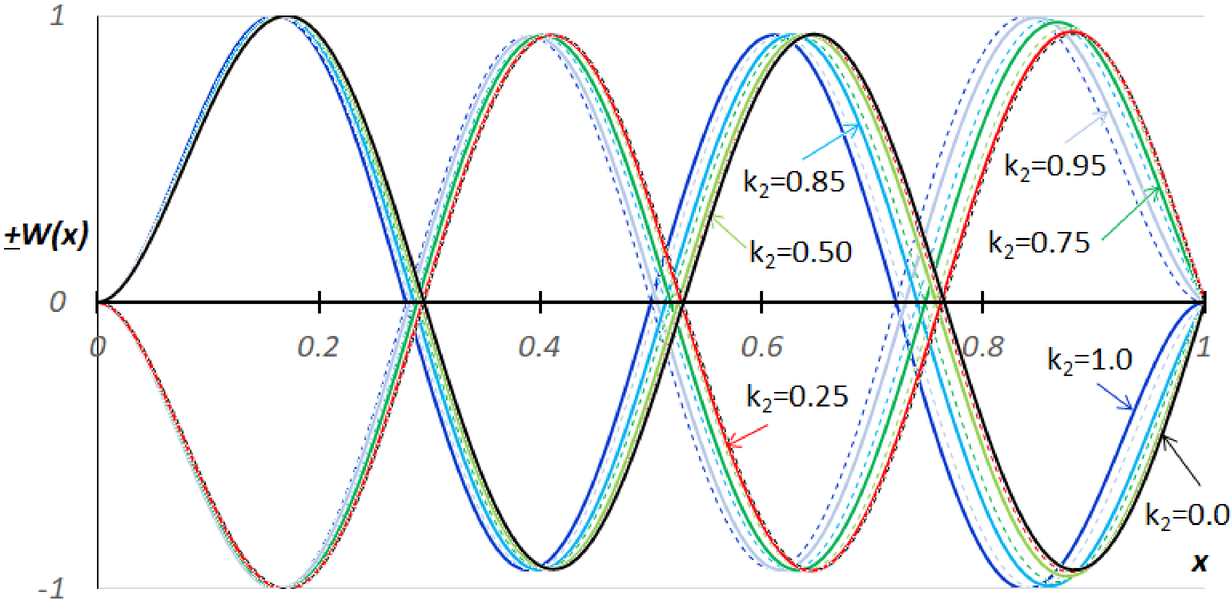


Figure 5: Normalized mode shapes for the fourth vibration mode

1. CONCLUSIONS

The paper presents the eigenvalues and modal shapes for the first four vibration modes for the case where the right clamped end of the beam is weakened by the coefficient k_2.

For the extreme cases: $k_2=0$, the eigenvalues (Table 1) were obtained from the clamped – hinged beam; respectively for $k_2=1$, we find the eigenvalues for the double clamped beam.

From the analysis of the figures 2 – 5, for the first 4 modes of vibration, it can be observed that for stiffness values $k_2<0.5$, from the point of view of dynamic behavior, the weakened clamped end has a behavior very close to that of a hinged support.

For stiffness values $k_2>0.5$, the dynamic behavior of the beam is significantly affected and although the relation (5) that describes the weakened clamped end of the beam is a linear expression of k_2, the effect of k_2 in the modal function does not have a linear behavior.

Also, from the figures 2 – 5, it can be seen the changes in the mode shapes from the case of the double clamped beam to the by clamped-hinge beam by modifying the weakened coefficient k_2.

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